Coherence and Concentration in Tightly-Connected Networks

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Acknowledgements

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Coherence in Power Networks

• Studied since the 70s
  • Podmore, Price, Chow, Kokotovic, Verghese, Pai, Schweppe,…

• Enables aggregation/model reduction
  • Speed up transient stability analysis

• Many important questions
  • How to identify coherent modes?
  • How to accurately reduce them?
  • What is the cause?

• Many approaches
  • Timescale separations (Chow, Kokotovic,)
  • Krylov subspaces (Chaniotis, Pai ‘01)
  • Balanced truncation (Liu et al ‘09)
  • Selective Modal Analysis (Perez-Arriaga, Verghese, Schweppe ‘82)
This talk

Goal: Characterize the coherence response from a frequency domain perspective
Outline

• Characterization of Coherent Dynamics [Min, M ‘21]

• Reduced-Order Model of Coherent Response [Min, Paganini, M ‘21]

• Grid-forming Frequency Shaping Control [Jiang, Bernstein, Vorobev, M ‘21]
Coherence and Concentration in Tightly-Connected Networks

Hancheng Min and Enrique Mallada

Coherence in networked dynamical systems

Block Diagram:

Node dynamics: \( g_i(s), i = 1, 2, \ldots, n \)

Symmetric Real Network Laplacian: \( L \)

\[
L = V \Lambda V^T, \quad V = \begin{bmatrix} \frac{1}{\sqrt{n}} \end{bmatrix}, V_{\perp} \\
\Lambda = \text{diag}\{0, \lambda_2(L), \ldots, \lambda_n(L)\}
\]

Coupling dynamics: \( f(s) \)

Examples:

• Consensus Networks:

\[
g_i(s) = \frac{1}{s} \\
f(s) = 1
\]

• Power Networks (2\textsuperscript{nd} order generator):

\[
g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}} \\
f(s) = \frac{1}{s}
\]
Coherence in networked dynamical systems

Block Diagram:

1. When does this network exhibit coherence?
2. What is the exact coherent dynamics of this network?

1. Coherence can be understood as a low rank property the closed-loop transfer matrix
2. It emerges as the effective algebraic connectivity increases
3. The coherent dynamics is given by the harmonic mean of nodal dynamics

\[
\bar{g}(s) = \left( \frac{1}{n} \sum_{i=1}^{n} g_i^{-1}(s) \right)^{-1}
\]
Network Coherence: Homogeneous Case

Assume homogeneity: \( g_i(s) = g(s), \ i = 1, \ldots, n \)

Eigendecomposition \( L = V \Lambda V^T \)
Network Coherence: Homogeneous Case

Assume homogeneity:  
\[ g_i(s) = g(s), \quad i = 1, \ldots, n \]
Assume homogeneity:  \( g_i(s) = g(s), \ i = 1, \ldots, n \)

Merge forward path \( V^TV = I \)
Network Coherence: Homogeneous Case

Assume homogeneity: \( g_i(s) = g(s), \ i = 1, \cdots, n \)
Network Coherence: Homogeneous Case

Assume homogeneity: \( g_i(s) = g(s), \ i = 1, \ldots, n \)
Network Coherence: Homogeneous Case

Assume homogeneity: \( g_i(s) = g(s), \ i = 1, \cdots, n \)

The transfer matrix from input \( u \) to output \( y \): 

\[
T(s) = V \text{diag} \left\{ \frac{1}{g^{-1}(s) + f(s)\lambda_i(L)} \right\}_{i=1}^n V^T
\]

\[
V = \left[ \frac{1}{\sqrt{n}}, V_\perp \right], \ \lambda_1(L) = 0
\]

\[
T(s) = \frac{1}{n} g(s) \mathbf{1}\mathbf{1}^T + V_\perp \text{diag} \left\{ \frac{1}{g^{-1}(s) + f(s)\lambda_i(L)} \right\}_{i=2}^n V_{\perp}^T
\]

**Coherent dynamics**

independent of the network structure

**Dynamics dependent of**

the network structure
Network Coherence: Homogeneous Case

\[ T(s) = \frac{1}{n} g(s) \mathbf{1} \mathbf{1}^T + V_{\perp} \text{diag} \left\{ \frac{1}{g^{-1}(s) + f(s) \lambda_i(L)} \right\} V^T \]

The rank-one property of the coherent dynamics leads to:

- **Input aggregation**, for any given input vector \( u(s) \):
  \[
y(s) = \frac{1}{n} g(s) \mathbf{1} \mathbf{1}^T u(s) = \frac{1}{n} g(s) \mathbf{1} \left( \sum_{i=1}^{n} u_i(s) \right)
  \]

- **Output synchronization**, given any two nodes \( i \) and \( j \):
  \[
y_i(s) - y_j(s) = \frac{1}{n} g(s) \mathbf{1} \mathbf{1}^T u(s) - \frac{1}{n} g(s) \mathbf{1} \mathbf{1}^T u(s) = 0
  \]

The **rank-one** coherence dynamics effectively synchronizes the response of every node to that of \( \bar{y}(s) = \frac{1}{n} g(s) \sum_{j=1}^{n} u_j(s) \).
Network Coherence: Homogeneous Case

\[ T(s) = \frac{1}{n} g(s) \mathbf{1}\mathbf{1}^T + V_\perp \text{diag} \left\{ \frac{1}{g^{-1}(s) + f(s)\lambda_i(L)} \right\} V^T \]

The effect of non-coherent dynamics vanishes as:

- The algebraic connectivity \( \lambda_2(L) \) of the network increases
- The \( s \)-region of interest gets close to a pole of \( f(s) \)

For almost any \( s_0 \in \mathbb{C} \)

\[
\lim_{\lambda_2(L) \to +\infty} \left\| T(s_0) - \frac{1}{n} g(s_0) \mathbf{1}\mathbf{1}^T \right\| = 0
\]

For \( s_0 \in \mathbb{C} \), a pole of \( f(s) \)

\[
\lim_{s \to s_0} \left\| T(s) - \frac{1}{n} g(s) \mathbf{1}\mathbf{1}^T \right\| = 0
\]

Our frequency-dependent coherence measure \( \| T(s) - \frac{1}{n} g(s) \mathbf{1}\mathbf{1}^T \| \) is controlled by the effective algebraic connectivity \( |f(s)| \lambda_2(L) \)
Network Coherence: Heterogeneous Case
Network Coherence: Heterogeneous Case

The transfer matrix from input $u$ to output $y$:

$$T(s) = V \left( V^T \text{diag}\{g_i^{-1}(s)\} V + f(s)\Lambda \right)^{-1} V^T$$
The transfer matrix from input $u$ to output $y$:

$$T(s) = V \left( V^T \text{diag} \{ g_i^{-1}(s) \} V + f(s) \Lambda \right)^{-1} V^T$$

Coherent Dynamics?  
Network Dependent?
Informed guess for coherent dynamics: $\bar{g}(s)$

Block Diagram:

Dynamics for node $i$

$$y_i(s) = g_i(s)(u_i(s) - d_i(s)), \quad i = 1, \ldots, n$$

Assume all nodes output are identical as the result of coherence

$$g_i^{-1}(s)\bar{y}(s) = u_i(s) - d_i(s), \quad i = 1, \ldots, n$$

Average equations from $i = 1$ to $n$:

$$\left(\frac{1}{n} \sum_{i=1}^{n} g_i^{-1}(s)\right)\bar{y}(s) = \frac{1}{n} \sum_{i=1}^{n} u_i(s) - \frac{1}{n} \sum_{i=1}^{n} d_i(s)$$

Harmonic mean of all $g_i(s)$

Coherent Dynamics:

$$\bar{y}(s) = \left(\frac{1}{n} \sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1} \frac{1}{n} \sum_{i=1}^{n} u_i(s)$$

$$\bar{g}(s) = \left(\frac{1}{n} \sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$$

Harmonic mean of all $g_i(s)$
Network Coherence: Heterogeneous Case

\[ T(s) = \frac{1}{n} \bar{g}(s) \mathbb{1} \mathbb{1}^T + T(s) - \frac{1}{n} \bar{g}(s) \mathbb{1} \mathbb{1}^T \]

\[ \bar{g}(s) = \left( \frac{1}{n} \sum_{i=1}^{n} g_i^{-1}(s) \right)^{-1} \]

The effect of non-coherent dynamics vanishes as:

- For almost any \( s_0 \in \mathbb{C} \)
  \[ \lim_{\lambda_2(L) \to +\infty} \left\| T(s_0) - \frac{1}{n} \bar{g}(s_0) \mathbb{1} \mathbb{1}^T \right\| = 0 \]
  \[ \lim_{s \to s_0} \left\| T(s) - \frac{1}{n} \bar{g}(s) \mathbb{1} \mathbb{1}^T \right\| = 0 \]

- For \( s_0 \in \mathbb{C} \), a pole of \( f(s) \)

Excluding zeros: the limit holds at zero, but by different convergence result

We can further prove uniform convergence over a compact subset of complex plane, if it doesn’t contain any zero nor pole of \( \bar{g}(s) \)

Extensions for random network ensembles, \( g_i(s) := g(s, w_i) \) (\( w_i \) random), then \( \bar{g}(s) = (E_w [ g^{-1}(s, w) ])^{-1} \)

Convergence of transfer matrix is related to time-domain response by Inverse Laplace Transform
Connection to Time Domain

If $\bar{g}(s)$ and $T(s)$ stable ($||\bar{g}||_{\infty}, ||T||_{\infty} \leq \gamma$), then there is $\bar{\lambda} = O(\gamma/\varepsilon)$ such that:

- **$\varepsilon$-approximation**, for any network $L$, with $\lambda_2(L) \geq \bar{\lambda}$

  \[
  \sup_{t>0} |y_i(t) - \bar{y}(t)| \leq \varepsilon
  \]

  where $\bar{y}(t)$ is the coherence dynamics response: 
  \[
  y(s) = \bar{g}(s) \frac{1}{n} \sum_{i=1}^{n} u_i(s)
  \]

- **element-wise coherence**, for any pair of nodes $i$ and $j$

  \[
  \sup_{t>0} |y_i(t) - y_j(t)| \leq 2\varepsilon
  \]
Example: Icelandic Power Grid

- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)

\[ g_i(s) = \frac{1}{m_is + d_i + \frac{r_i^{-1}}{\tau_is + 1}} \]

\[ f(s) = \frac{1}{s} \]
Example: Effect of Network Algebraic Connectivity \( \lambda_2(L) \uparrow \)

Coherent dynamics acts as a more accurate version of the Center of Inertia (CoI)
Example: Sinusoidal Disturbances: $\sin(\omega_d t)$

$\omega_d \uparrow$
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• Characterization of Coherent Dynamics [Min, M ‘21]

• Reduced-Order Model of Coherent Response [Min, Paganini, M ’21]

• Grid-forming Frequency Shaping Control [Jiang, Bernstein, Vorobev, M ‘21]
Accurate Reduced-Order Models for Heterogeneous Coherent Generators

Hancheng Min, Fernando Paganini, and Enrique Mallada

IEEE Control Systems Letters, 2021
Aggregation of Coherent Generators

\[ g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}} \]

- \( m_i \): inertia
- \( d_i \): damping coefficient
- \( r_i^{-1} \): droop coefficient
- \( \tau_i \): turbine time constant

Disturbance \( u_1 \)

Angular velocity \( w_i \)

Coherent group of \( n \) generators
Aggregation of Coherent Generators

\[
g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}
\]

**Question:** How to choose the different parameters of \( \hat{g}(s) \)?

\[
\hat{g}(s) = \frac{1}{\hat{m} s + \hat{d} + \hat{r}^{-1}}
\]

\[
\sum_{i=1}^{n} u_i \rightarrow \hat{w}
\]

**Answer:** Use instead

\[
\hat{g}(s) = \frac{1}{n \bar{g}(s)} = \left( \sum_{i=1}^{n} g_i^{-1}(s) \right)^{-1}
\]
Challenges on Aggregating Coherent Generators

For generator dynamics given by a swing model with turbine control:

\[ g_i(s) = \frac{1}{m_is + d_i + \frac{r_i^{-1}}{\tau_is + 1}} \]

The aggregate dynamics:

\[ \hat{g}(s) = \frac{1}{\hat{m}s + \hat{d} + \sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_is + 1}} \]

high-order if \( \tau_i \) are heterogeneous

Need to find a low-order approximation of \( \hat{g}(s) \)
When time constants are heterogeneous:

\[
\frac{r_1^{-1}}{\tau_1 s + 1} \quad \vdots \quad \frac{r_n^{-1}}{\tau_n s + 1}
\]

Aggregating heterogeneous turbine dynamics

\[
\frac{\hat{r}^{-1}}{\tau s + 1}
\]

Time constant \( \tau \) is chosen by:
- Optimization: Germond’78, Guggilam’18
- Weighted harmonic mean: Ourari’06

**Drawbacks:**
- the order of overall approximation model is restricted to 2nd order
- the only “decision variable” is the time constant
- does not consider the effect of inertia or damping in the approx.
Our Approach

Leverage weighted balance truncation to build a hierarchy of approximations

\[
\hat{g}(s) = \frac{1}{\hat{m}s + \hat{d} + \sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_i s + 1}} 
\]

\[
\tilde{g}_k(s) = \frac{1}{\tilde{m}s + \tilde{d} + \tilde{g}_{tb,k-1}(s)}
\]

The case \( k = 3 \), leads to a more flexible approximation
Comparison with (Some) Existing Methods

By essentially relaxing the restrictions on reduced order model:
- increase the model order to 3rd order,
- reduction on closed-loop dynamics,
our proposed models outperform models by conventional approach.
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• Grid-forming Frequency Shaping Control [Jiang, Bernstein, Vorobev, M ‘21]
Storage-Based Frequency Shaping Control
Yan Jiang, Eliza Cohn, Petr Vorobev, Member, IEEE, and Enrique Mallada, Senior Member, IEEE

IEEE Transactions on Power Systems, 2021

Grid-forming frequency shaping control
Yan Jiang¹, Andrey Bernstein², Petr Vorobev³, and Enrique Mallada¹

IEEE Control Systems Letters, 2021
Grid-following Frequency Shaping Control

**Key idea:** use model matching control (at each bus/area)

\[ u_i \rightarrow \frac{1}{f_i ms + d} \rightarrow \frac{1}{f_i ms + d} \rightarrow w_i \]

\[ f_i \frac{r^{-1}}{r + 1} \rightarrow \frac{f_i c_{fs}(s)}{\tau s + 1} \rightarrow w_i \]

\[ c_{fs}(s) := \frac{A_1 s^2 + A_2 s + A_3}{\tau s + 1} \]

\[ A_1 = \tau (a - m) \]
\[ A_2 = b\tau + a - m \]
\[ A_3 = b - r_g - d \]

Leads to CoI Frequency \( \bar{w} \) with:

**RoCoF:**
\[ \|\dot{\bar{w}}\|_\infty = \frac{1}{\sum_i f_i} \frac{\sum_i u_{0i}}{a} \]

**Steady-state:**
\[ \bar{w}(\infty) = \frac{1}{\sum_i f_i} \frac{\sum_i u_{0i}}{b} \]

[TPS 21] Jiang, Cohn, Vorobev, M, Storage-based frequency shaping control, IEEE Transactions on Power Systems, under review
Trading off Control Effort and RoCoF

[TPS 21] Jiang, Cohn, Vorobev, M. Storage-based frequency shaping control, IEEE Transactions on Power Systems, accepted
Trading off Control Effort and RoCoF

**Challenge:** Solution Limited to Grid-following Inverters

[TPS 21] Jiang, Cohn, Vorobev, M, Storage-based frequency shaping control, IEEE Transactions on Power Systems, accepted
Grid-forming Frequency Shaping Control

**Key idea:** use model matching control on coherent dynamics

\[ \sum_i u_i \rightarrow \hat{g}(s) = \left( \sum_{i \in G} g_i^{-1}(s) + \sum_{i \in I} h_i^{-1}(s) \right)^{-1} \rightarrow \bar{w} \]

**Generation:**
\[ g_i(s) = \frac{1}{m_is + d_i + \frac{r_i^{-1}}{\tau_is+1}}, \quad i \in G \]

**Inverters:**
\[ h_i(s) = \frac{1}{m_is + d_i + c_i(s)}, \quad i \in I \]

\[ a := \sum_{i \in G} m_i + \sum_{i \in I} m_i \]
\[ b := \sum_{i \in G} (d_i + r_i^{-1}) + \sum_{i \in I} d_i \]
\[ \sum_i c_i(s) = \sum_{i \in G} \frac{r_i^{-1}\tau_is}{\tau_is+1} \]

**RoCoF:**
\[ ||\dot{\bar{w}}||_\infty = \frac{\sum_i u_{0i}}{a} \]

**Steady-state:**
\[ \bar{w}(\infty) = \frac{\sum_i u_{0i}}{b} \]

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Summary

- Frequency domain characterization of **coherent dynamics**, as a low rank property of the transfer function.

- **Coherence is a frequency dependent** property:
  - Effective algebraic connectivity $f(s)\lambda_2(L)$
  - Disturbance frequency spectrum

- We use frequency **weighted balanced truncation** to suggest possible improvements to obtain accurate reduced order model of aggregated dynamics of coherent generators:
  - increase model complexity (3rd order/two turbines)
  - model reduction on closed-loop dynamics

- Grid-forming Frequency Shaping Control
Thanks!

Related Publications:
• Min, M, “Coherence and Concentration in Tightly Connected Networks,” submitted
• Min, Paganini, M, “Accurate Reduced Order Models for Coherent Synchronous Generators,” L-CSS 2021
• Jiang, Bernstein, Vorobev, M, “Grid-forming Frequency Shaping Control,” L-CSS 2021

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Backup Slides

Numerical Examples
Modal Decomposition
Coherence