Coherence and Concentration in Tightly-Connected Networks

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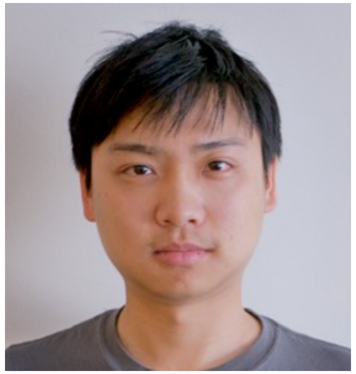
JOHNS HOPKINS UNIVERSITY

ARO-Sponsored Workshop

Synchronization in Natural and Engineering Systems

March 30, 2022

Acknowledgements



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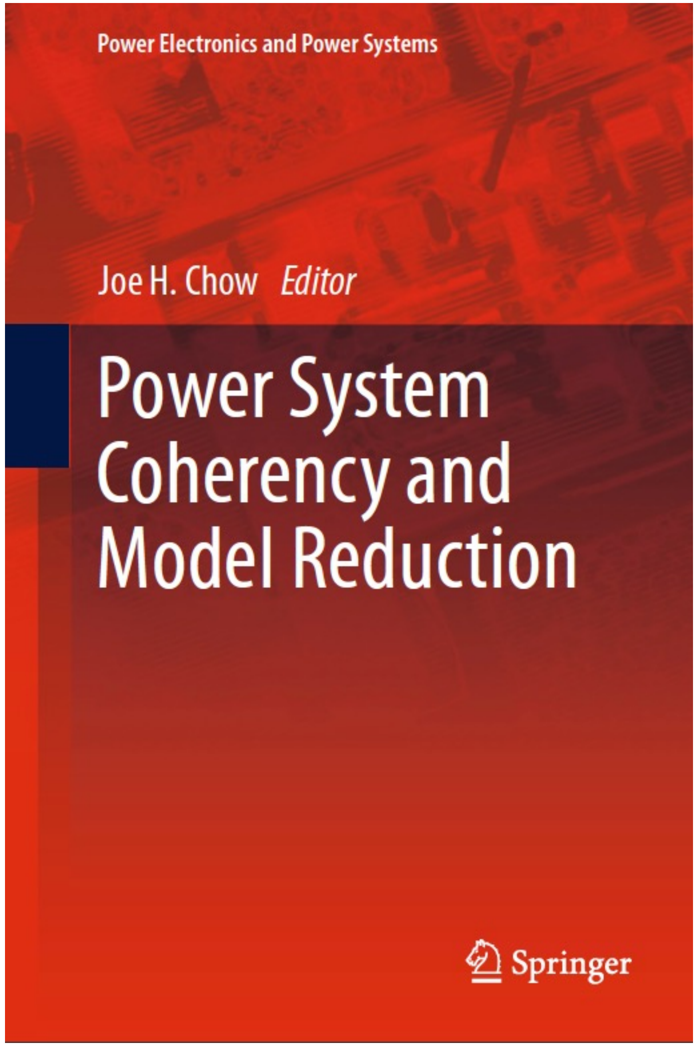
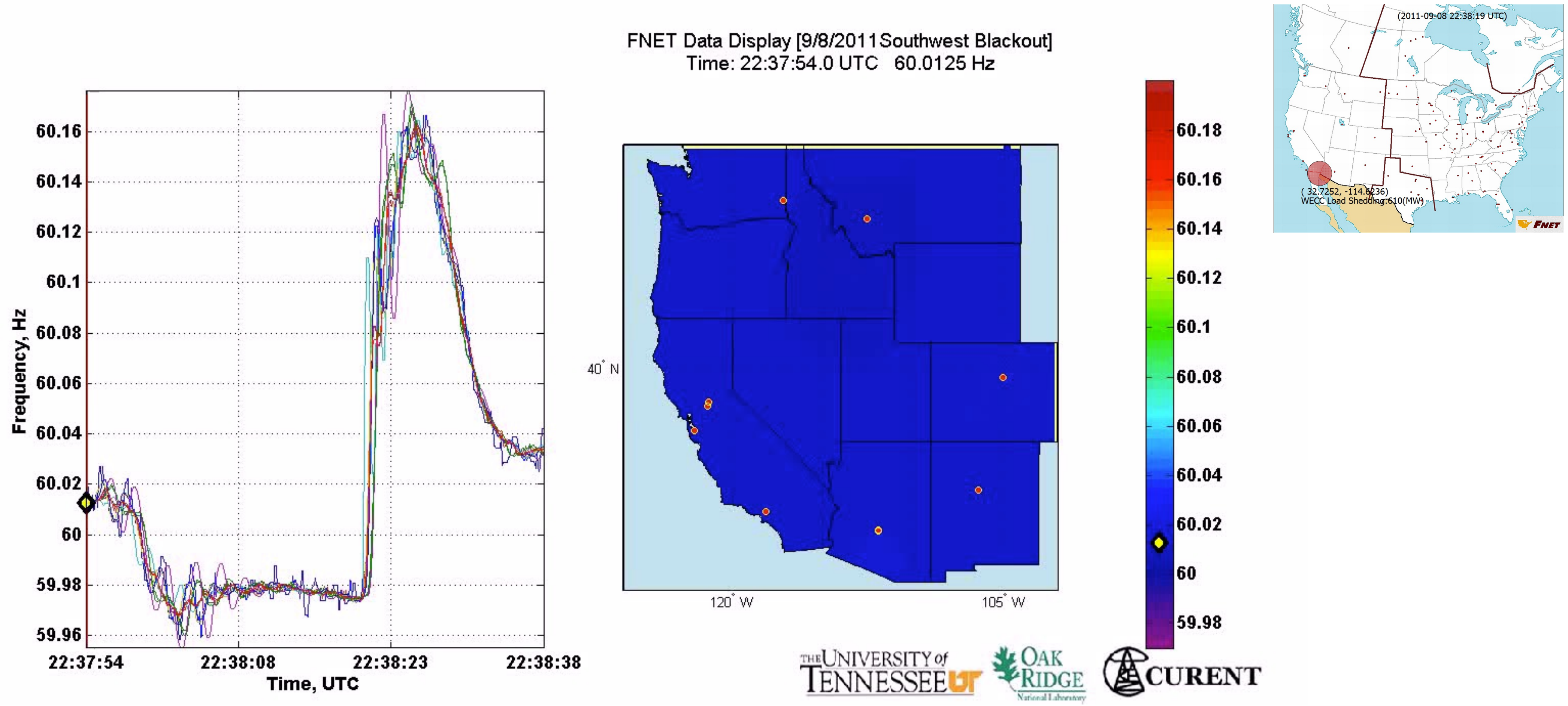
Coherence in Power Networks

- Studied since the 70s
  - Podmore, Price, Chow, Kokotovic, Verghese, Pai, Schweppe,…
- Enables aggregation/model reduction
  - Speed up transient stability analysis
- Many important questions
  - How to identify coherent modes?
  - How to accurately reduce them?
  - What is the cause?
- Many approaches
  - Timescale separations (Chow, Kokotovic,)
  - Krylov subspaces (Chaniotis, Pai ‘01)
  - Balanced truncation (Liu et al ‘09)
  - Selective Modal Analysis (Perez-Arriaga, Verghese, Schweppe ‘82)



This talk



Goal: Characterize the coherence response from a frequency domain perspective

Outline

- Characterization of Coherent Dynamics [Min, M ‘21]
- Reduced-Order Model of Coherent Response [Min, Paganini, M ‘21]
- Grid-forming Frequency Shaping Control [Jiang, Bernstein, Vorobev, M ‘21]

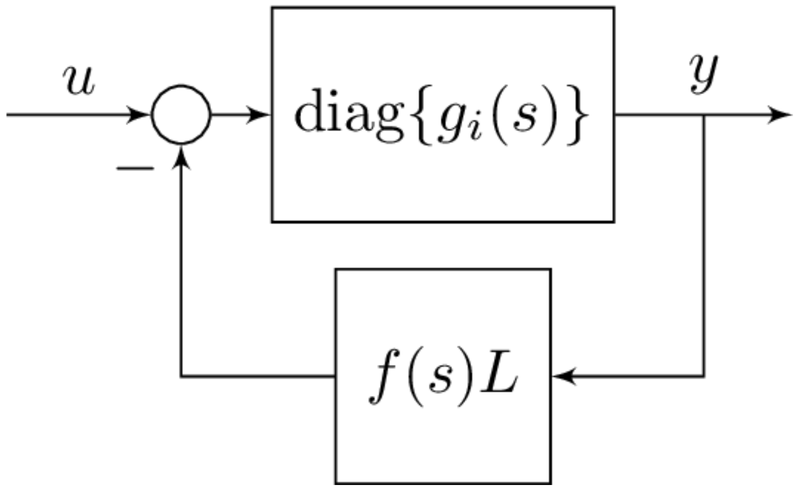
Coherence and Concentration in Tightly-Connected Networks

Hancheng Min and Enrique Mallada

ArXiv preprint: arXiv:2101.00981

Coherence in networked dynamical systems

Block Diagram:



Node dynamics: $g_i(s), i = 1, 2, \cdots, n$

Symmetric Real Network Laplacian: L

$$L = V\Lambda V^T,\ V = [\mathbb{1}/\sqrt{n}, V_\perp]$$
$$\Lambda = \mathrm{diag}\{0, \lambda_2(L), \cdots, \lambda_n(L)\}$$

Coupling dynamics: $f(s)$

Examples:

- Consensus Networks:

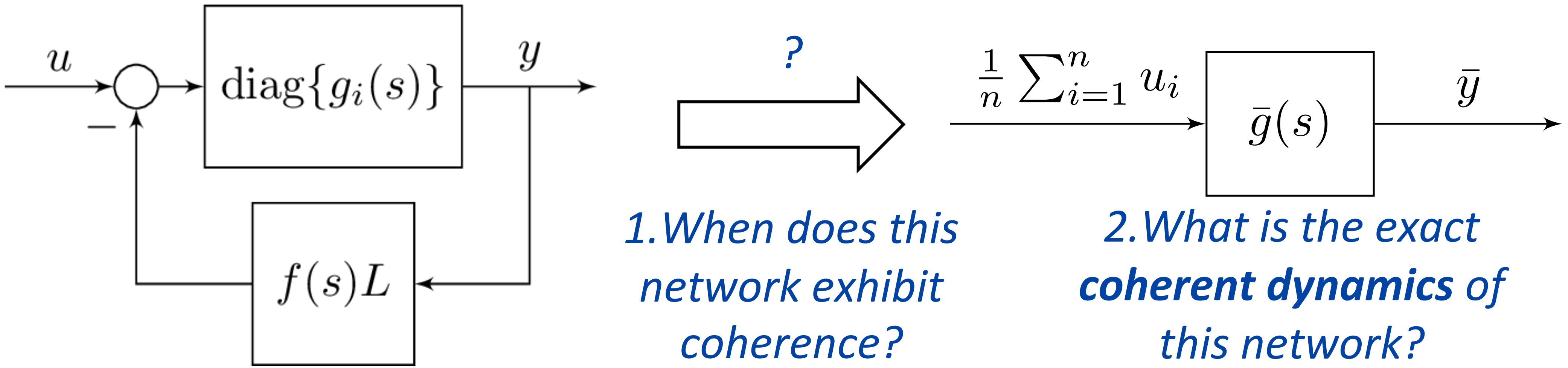
$$g_i(s) = \frac{1}{s}$$
$$f(s) = 1$$

- Power Networks (2nd order generator):

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$
$$f(s) = \frac{1}{s}$$

Coherence in networked dynamical systems

Block Diagram:



1.When does this network exhibit coherence?

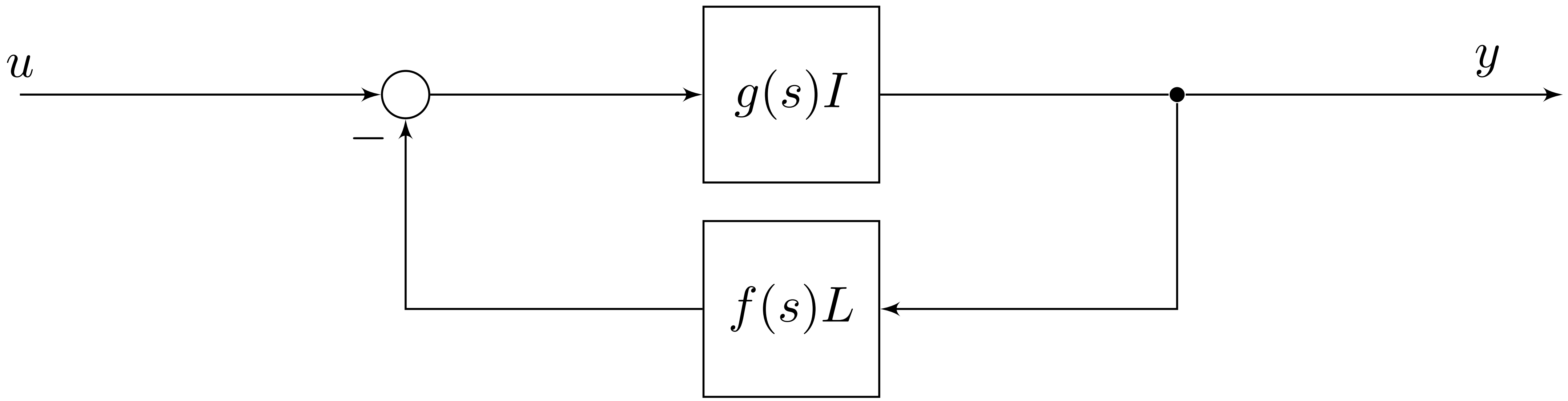
2.What is the exact ***coherent dynamics*** *of this network?*

1. Coherence can be understood as a **low rank** property the **closed-loop transfer matrix**
2. It emerges as the **effective algebraic connectivity** increases
3. The coherent dynamics is given by the **harmonic mean** of nodal dynamics

$$\bar{g}(s) = \left(\frac{1}{n}\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$$

Network Coherence: Homogeneous Case

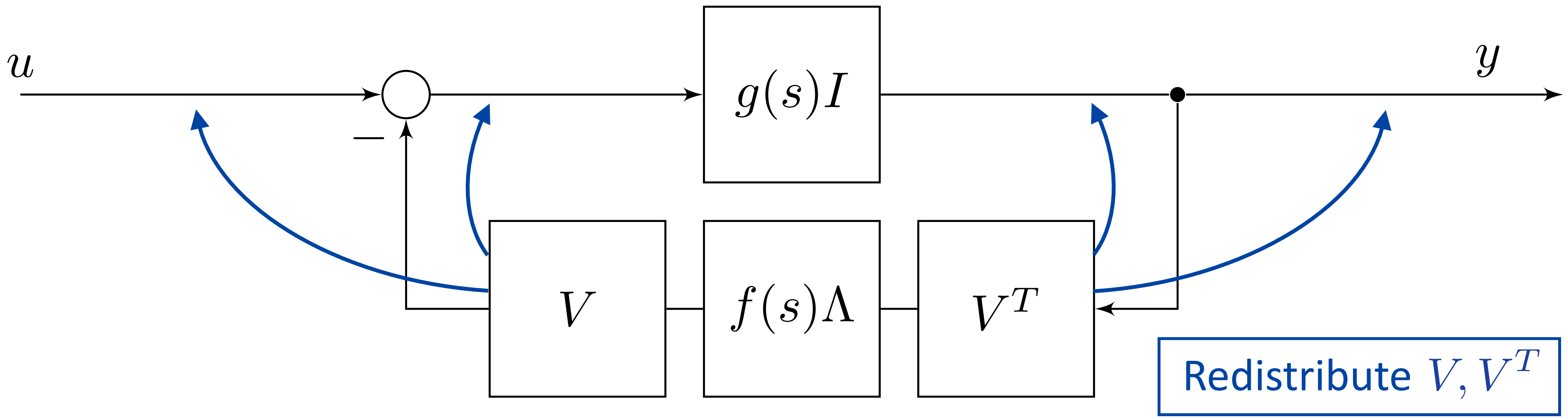
Assume homogeneity: $g_i(s) = g(s),\ i = 1, \cdots, n$



Eigendecomposition $L = V \Lambda V^T$

Network Coherence: Homogeneous Case

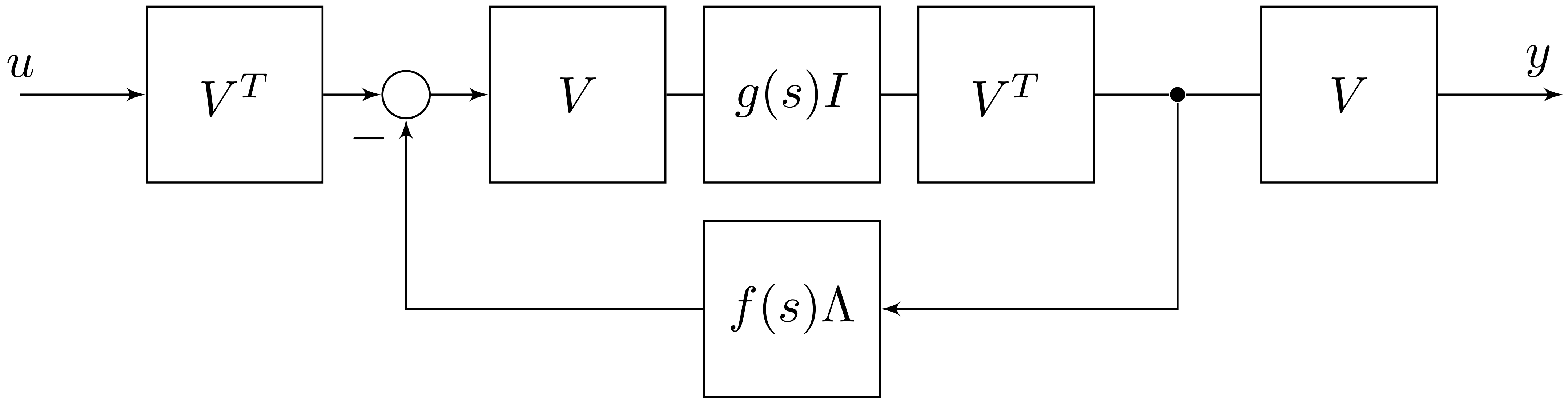
Assume homogeneity: $g_i(s) = g(s),\ i = 1, \cdots, n$



Network Coherence: Homogeneous Case

Assume homogeneity: $g_i(s) = g(s),\ i = 1, \cdots, n$

Merge forward path
$V^T V = I$



Network Coherence: Homogeneous Case

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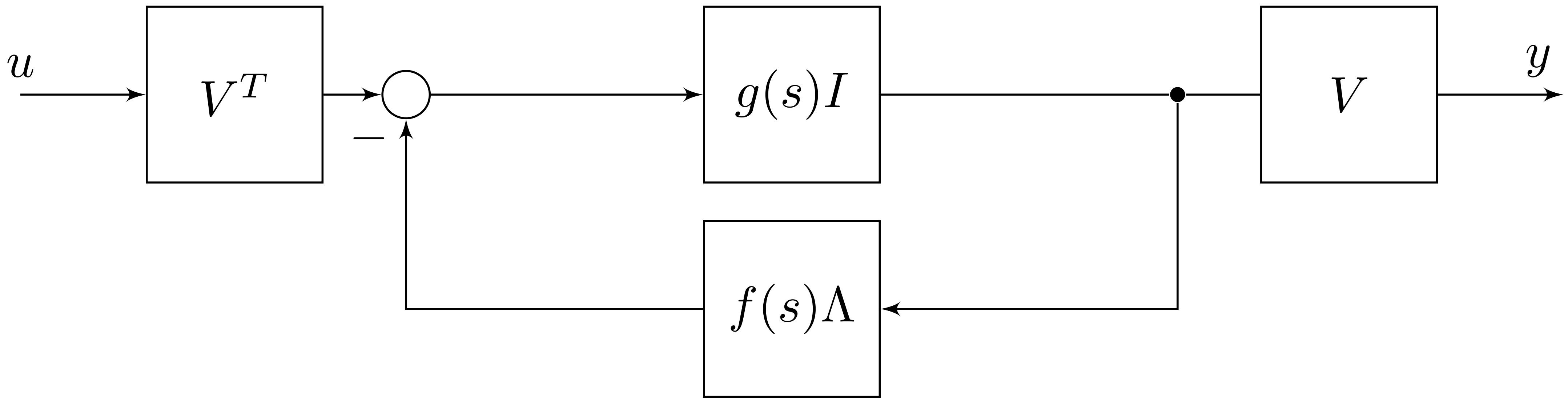
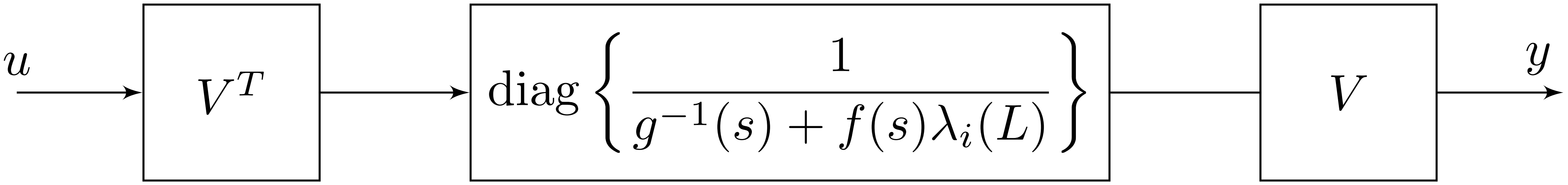


Network Coherence: Homogeneous Case

Assume homogeneity: $g_i(s) = g(s),\ i = 1, \cdots, n$



Network Coherence: Homogeneous Case

Assume homogeneity: $g_i(s) = g(s),\ i = 1, \cdots, n$

The transfer matrix from input u to output y :

$$T(s) = V \operatorname{diag}\left\{\frac{1}{g^{-1}(s) + f(s)\lambda_i(L)}\right\}_{i=1}^{n} V^T$$

$$V = [\mathbb{1}/\sqrt{n}, V_\perp],\ \lambda_1(L) = 0$$

$$\Downarrow$$

$$T(s) = \boxed{\frac{1}{n} g(s)\mathbb{1}\mathbb{1}^T} + \boxed{V_\perp \operatorname{diag}\left\{\frac{1}{g^{-1}(s) + f(s)\lambda_i(L)}\right\}_{i=2}^{n} V_\perp^T}$$

Coherent dynamics independent of the network structure

Dynamics dependent of the network structure

Network Coherence: Homogeneous Case

$$T(s) = \boxed{\frac{1}{n} g(s) \mathbb{1}\mathbb{1}^T} + V_\perp \mathrm{diag}\left\{ \frac{1}{g^{-1}(s) + f(s)\lambda_i(L)} \right\} V^T$$

The rank-one property of the coherent dynamics leads to:

- **Input aggregation**, for any given input vector $u(s)$:

$$y(s) = \frac{1}{n} g(s) \mathbb{1}\mathbb{1}^T u(s) = \frac{1}{n} g(s) \mathbb{1} \left(\sum_{i=1}^{n} u_i(s) \right)$$

- **Output synchronization**, given any two nodes i and j:

$$y_i(s) - y_j(s) = \frac{1}{n} g(s) \mathbb{1}^T u(s) - \frac{1}{n} g(s) \mathbb{1}^T u(s) = 0$$

The **rank-one** coherence dynamics effectively synchronizes the response of every node to that of $\bar{y}(s) = \frac{1}{n} g(s) \sum_{j=1}^{n} u_j(s)$

Network Coherence: Homogeneous Case

$$T(s) = \frac{1}{n} g(s) \mathbb{1}\mathbb{1}^T + \boxed{V_\perp \operatorname{diag}\left\{ \frac{1}{g^{-1}(s) + f(s)\lambda_i(L)} \right\} V^T}$$

The effect of non-coherent dynamics vanishes as:

- The **algebraic connectivity** $\lambda_2(L)$ of the network increases

For almost any $s_0 \in \mathbb{C}$

$$\lim_{\lambda_2(L)\to+\infty} \left\| T(s_0) - \frac{1}{n} g(s_0) \mathbb{1}\mathbb{1}^T \right\| = 0$$

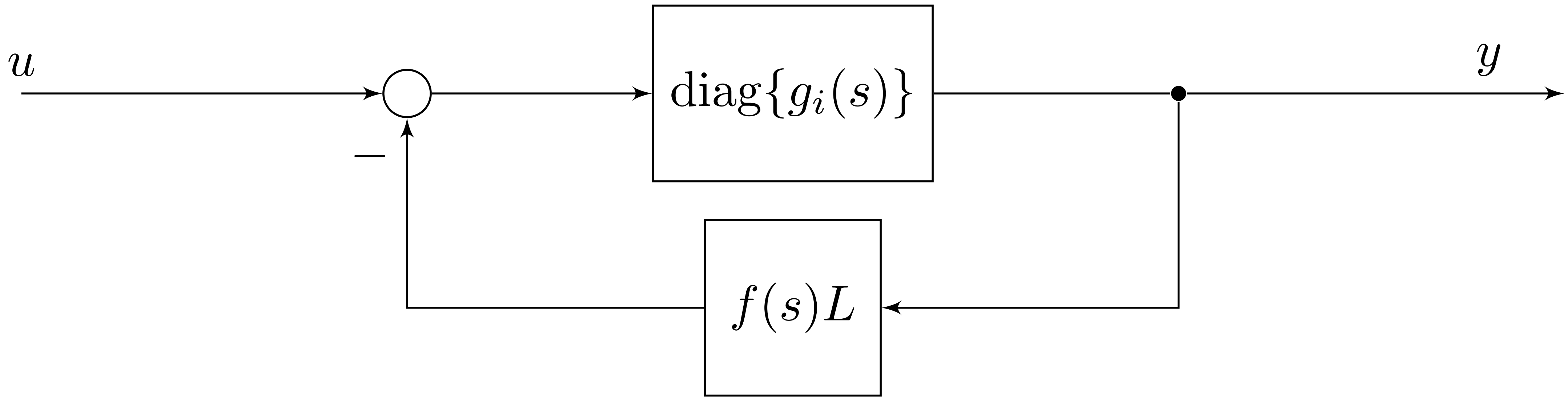
- The s-region of interest gets close to a **pole** of $f(s)$

For $s_0 \in \mathbb{C}$, a pole of $f(s)$

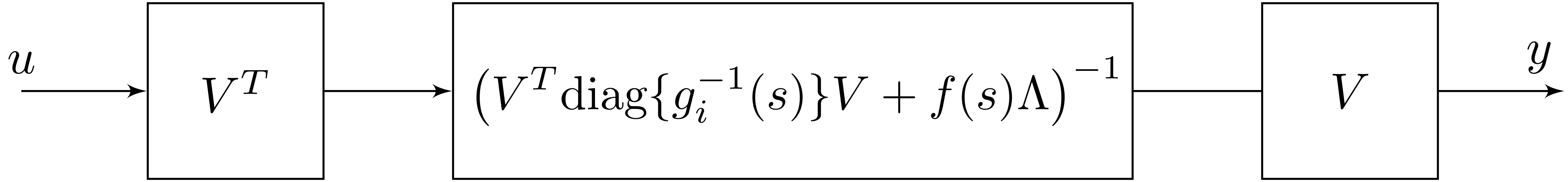
$$\lim_{s\to s_0} \left\| T(s) - \frac{1}{n} g(s) \mathbb{1}\mathbb{1}^T \right\| = 0$$

Our **frequency-dependent** coherence measure $\left\| T(s) - \frac{1}{n} g(s) \mathbb{1}\mathbb{1}^T \right\|$ is controlled by the **effective algebraic connectivity** $|f(s)|\lambda_2(L)$

Network Coherence: Heterogeneous Case



Network Coherence: Heterogeneous Case



The transfer matrix from input u to output y :

$$T(s) = V\left(V^T \mathrm{diag}\{g_i^{-1}(s)\}V + f(s)\Lambda\right)^{-1} V^T$$

Network Coherence: Heterogeneous Case

The transfer matrix from input u to output y :

$$T(s) = V\left(V^T \mathrm{diag}\{g_i^{-1}(s)\}V + f(s)\Lambda\right)^{-1} V^T$$

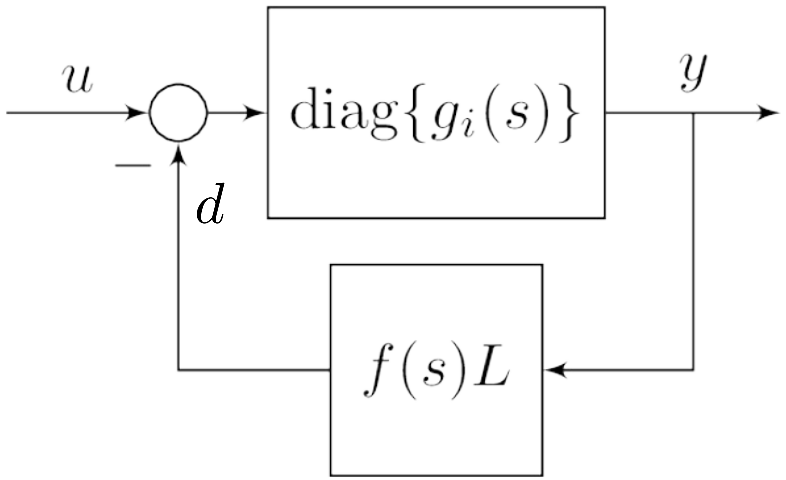
$\Downarrow$

$$T(s) = \boxed{\frac{1}{n}\bar{g}(s)\mathbb{1}\mathbb{1}^T} + \boxed{N(s)}$$

Coherent Dynamics? **Network Dependent?**

Informed guess for coherent dynamics: $\bar{g}(s)$

Block Diagram:



Dynamics for node i

$$y_i(s) = g_i(s)(u_i(s) - d_i(s)),\ i = 1, \cdots, n$$

Assume all nodes output are **identical** as the result of **coherence**

$$y_i(s) = \bar{y}(s)$$

$$g_i^{-1}(s)\bar{y}(s) = u_i(s) - d_i(s),\ i = 1, \cdots, n$$

Average equations from $i = 1$ to n:

$\mathbb{1}^T L = \mathbb{0}$

$$\left(\frac{1}{n}\sum_{i=1}^{n} g_i^{-1}(s)\right)\bar{y}(s) = \frac{1}{n}\sum_{i=1}^{n} u_i(s) - \boxed{\frac{1}{n}\sum_{i=1}^{n} d_i(s)}$$

=0

Coherent Dynamics:

$$\bar{y}(s) = \left(\frac{1}{n}\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1} \frac{1}{n}\sum_{i=1}^{n} u_i(s)$$

$$\bar{g}(s) = \left(\frac{1}{n}\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$$

Harmonic mean of all $g_i(s)$

Network Coherence: Heterogeneous Case

$$T(s) = \frac{1}{n}\bar{g}(s)\mathbb{1}\mathbb{1}^T + \boxed{T(s) - \frac{1}{n}\bar{g}(s)\mathbb{1}\mathbb{1}^T}$$

$$\bar{g}(s) = \left(\frac{1}{n}\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$$

The effect of non-coherent dynamics vanishes as:

- For almost any $s_0 \in \mathbb{C}$

$$\lim_{\lambda_2(L)\to+\infty} \left\| T(s_0) - \frac{1}{n}\bar{g}(s_0)\mathbb{1}\mathbb{1}^T \right\| = 0$$

- For $s_0 \in \mathbb{C}$, a pole of $f(s)$

$$\lim_{s\to s_0} \left\| T(s) - \frac{1}{n}\bar{g}(s)\mathbb{1}\mathbb{1}^T \right\| = 0$$

- Excluding zeros: the limit holds at zero, but by different convergence result
- We can further prove **uniform convergence** over a compact subset of complex plane, if it doesn't contain any zero nor pole of $\bar{g}(s)$
- Extensions for random network ensembles, $g_i(s) \coloneqq g(s, w_i)$ (w_i random), then $\bar{g}(s) = (E_w[\, g^{-1}(s, w)])^{-1}$
- Convergence of transfer matrix is **related to time-domain response** by Inverse Laplace Transform

Connection to Time Domain

If $\bar{g}(s)$ and $T(s)$ stable $(||\bar{g}||_\infty, ||T||_\infty \leq \gamma)$, then there is $\bar{\lambda} = O(\gamma/\varepsilon)$ such that:

- ***ε*-approximation,** for any network L, with $\lambda_2(L) \geq \bar{\lambda}$

$$\sup_{t>0} |y_i(t) - \bar{y}(t)| \leq \varepsilon$$

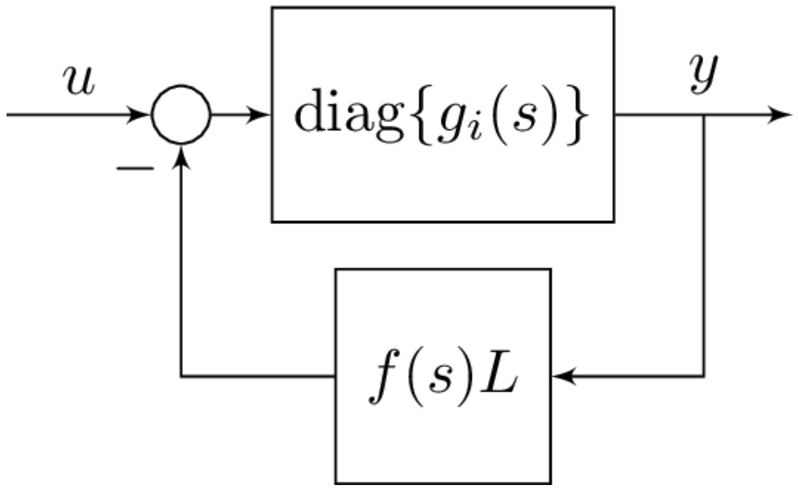
where $\bar{y}(t)$ is the coherence dynamics response: $y(s) = \bar{g}(s)\frac{1}{n}\sum_{i=1}^{n} u_i(s)$

- **element-wise coherence**, for any pair of nodes i and j

$$\sup_{t>0} |y_i(t) - y_j(t)| \leq 2\varepsilon$$

Example: Icelandic Power Grid

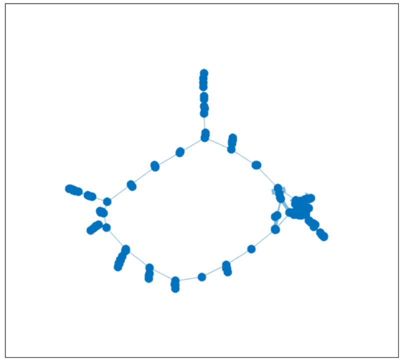
- Iceland power network: 189 buses, 35 generators, load 1.3GW (PSAT)



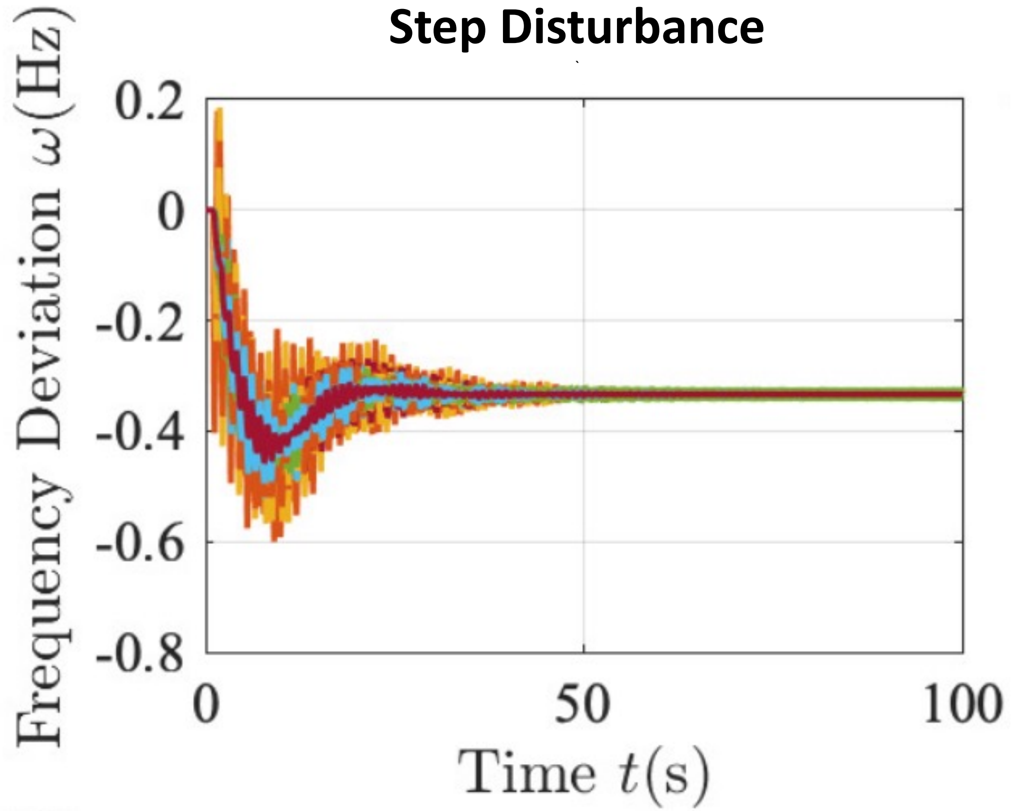
$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$

$$f(s) = \frac{1}{s}$$

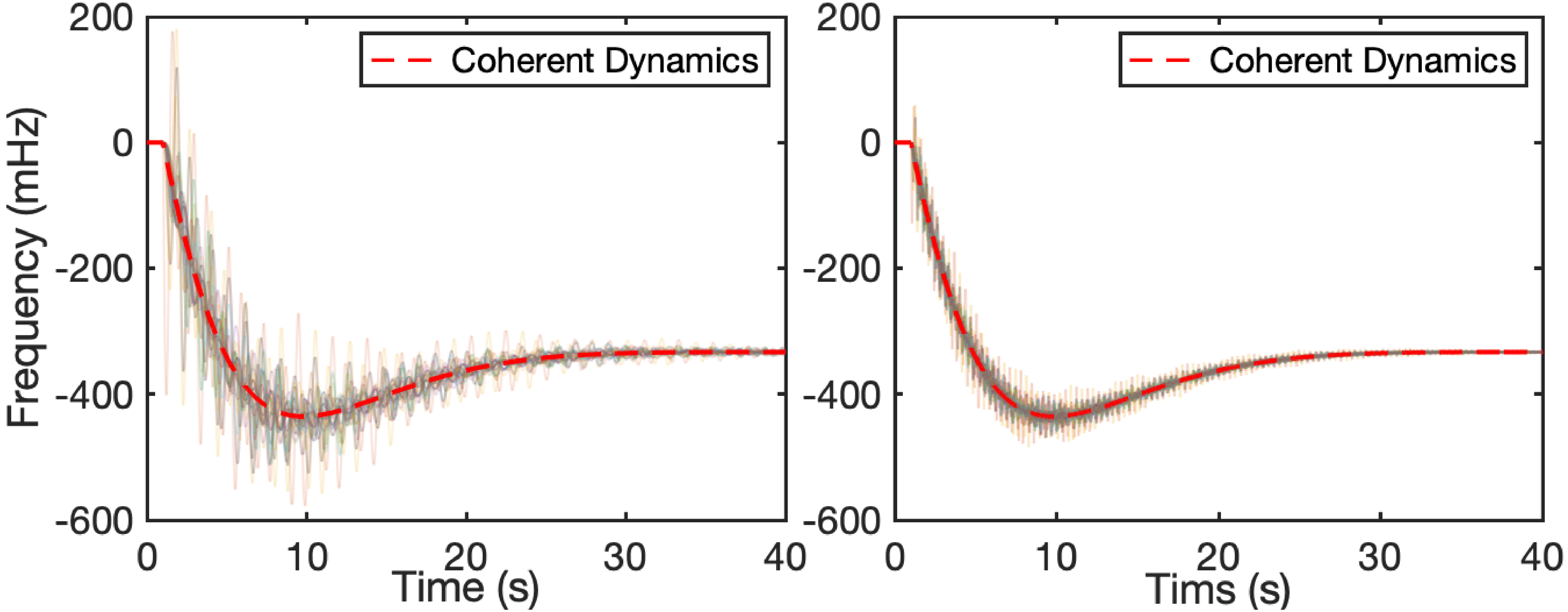
Icelandic Grid



Step Disturbance



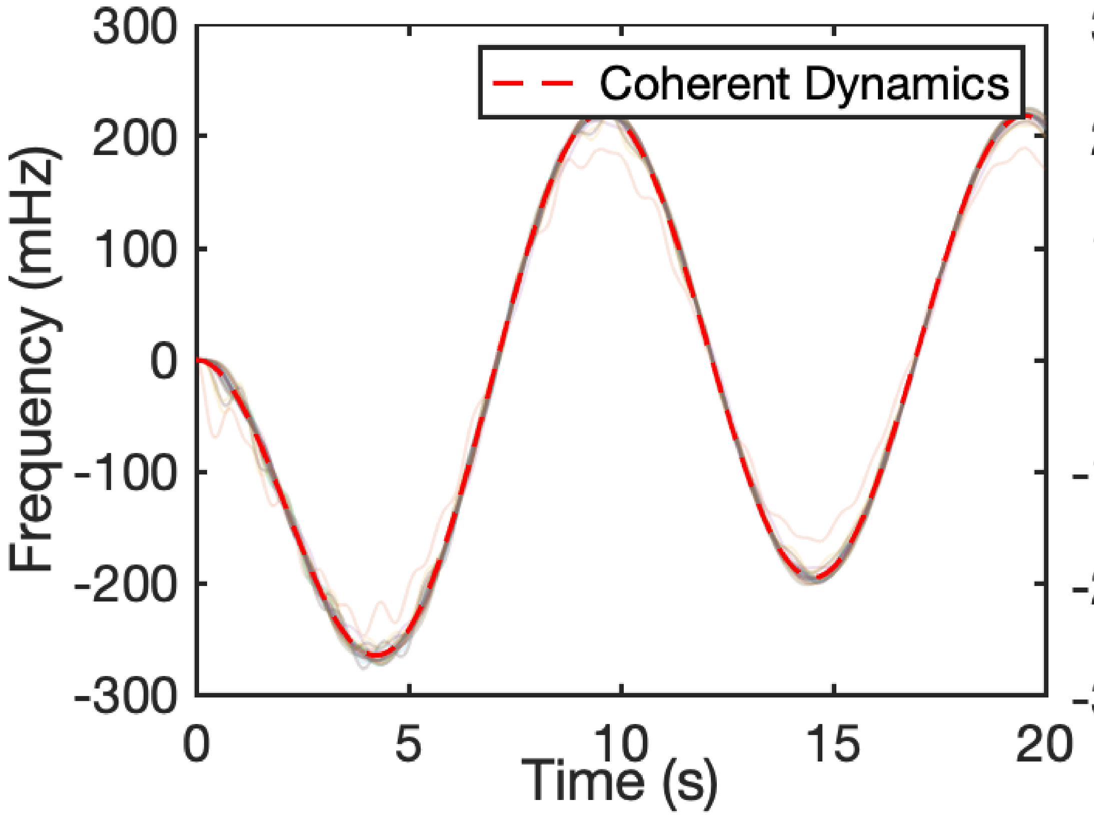
Example: Effect of Network Algebraic Connectivity $\lambda_2(L) \uparrow$

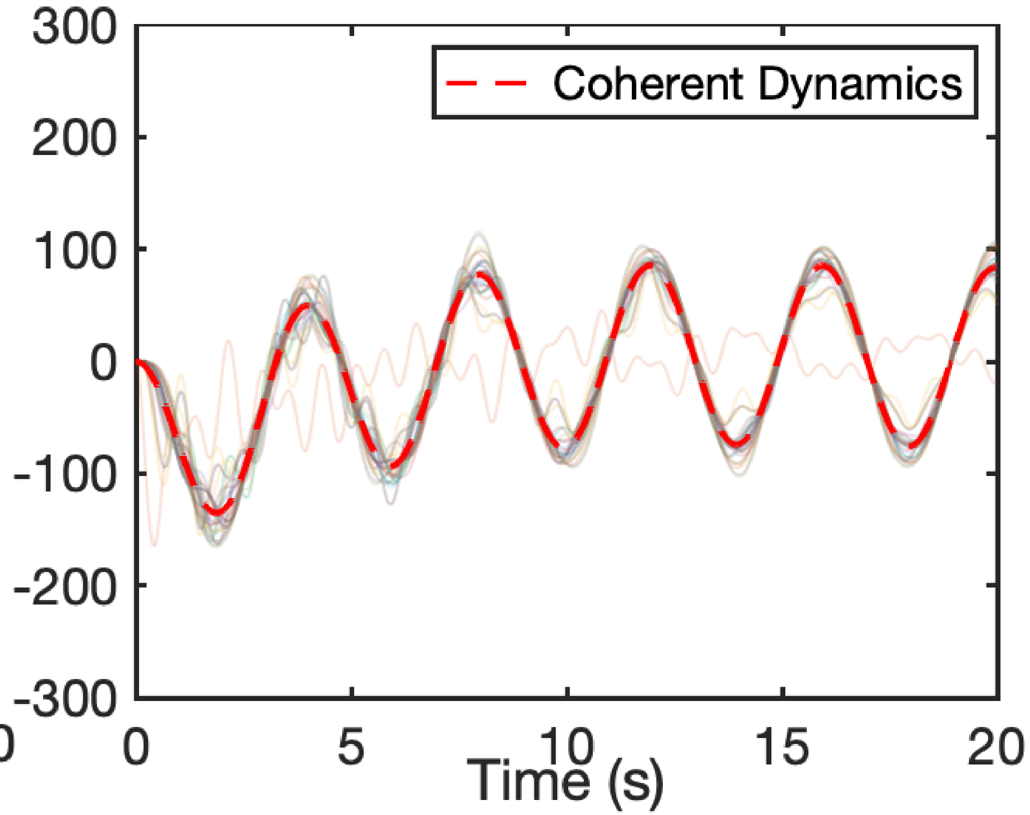


Coherent dynamics acts as a more accurate version of the Center of Inertia (CoI)

Example: Sinusoidal Disturbances: $\sin(\boldsymbol{\omega_d t})$

$\omega_d \uparrow$





Outline

- Characterization of Coherent Dynamics [Min, M '21]
- Reduced-Order Model of Coherent Response [Min, Paganini, M '21]
- Grid-forming Frequency Shaping Control [Jiang, Bernstein, Vorobev, M '21]

Accurate Reduced-Order Models for Heterogeneous Coherent Generators

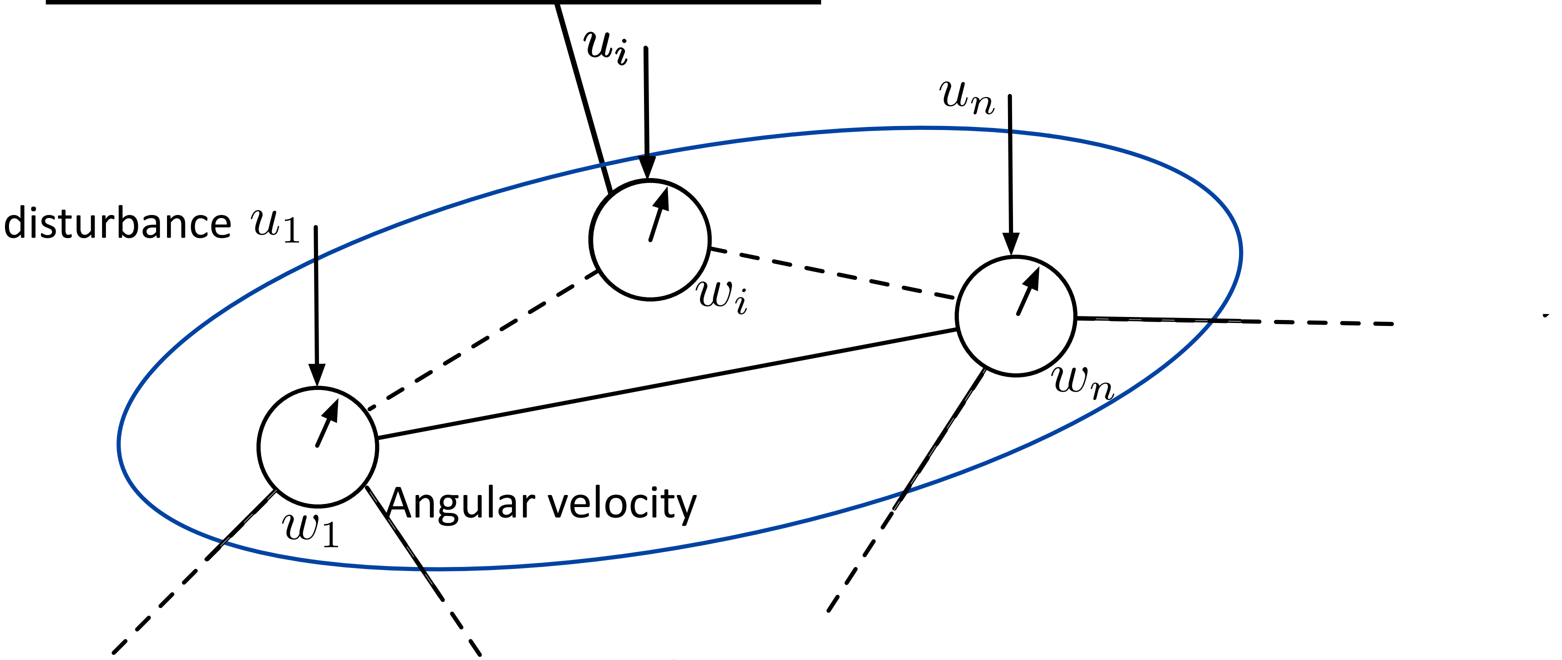
Hancheng Min, Fernando Paganini, and Enrique Mallada

IEEE Control Systems Letters, 2021

Aggregation of Coherent Generators

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$

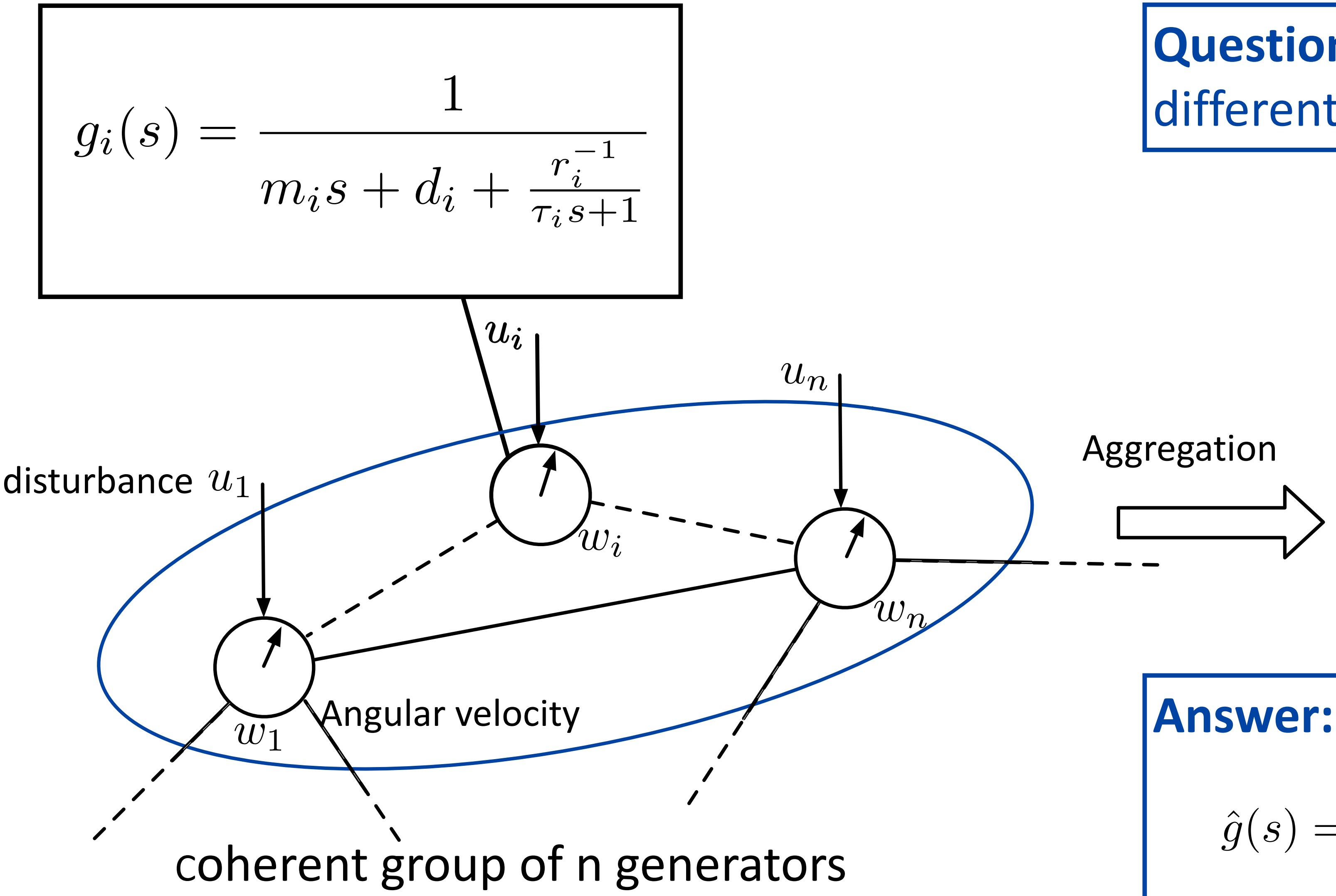
m_i: inertia
d_i: damping coefficient
r_i^{-1}: droop coefficient
τ_i: turbine time constant

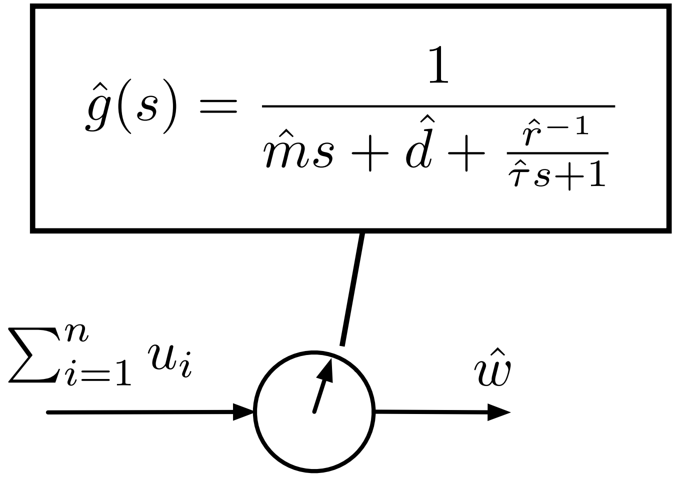


coherent group of n generators

Aggregation of Coherent Generators

Question: How to choose the different parameters of $\hat{g}(s)$?





Answer: Use instead

$$\hat{g}(s) = \frac{1}{n}\bar{g}(s) = \left(\sum_{i=1}^{n} g_i^{-1}(s)\right)^{-1}$$

Challenges on Aggregating Coherent Generators

For generator dynamics given by a swing model with turbine control:

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}$$

The aggregate dynamics:

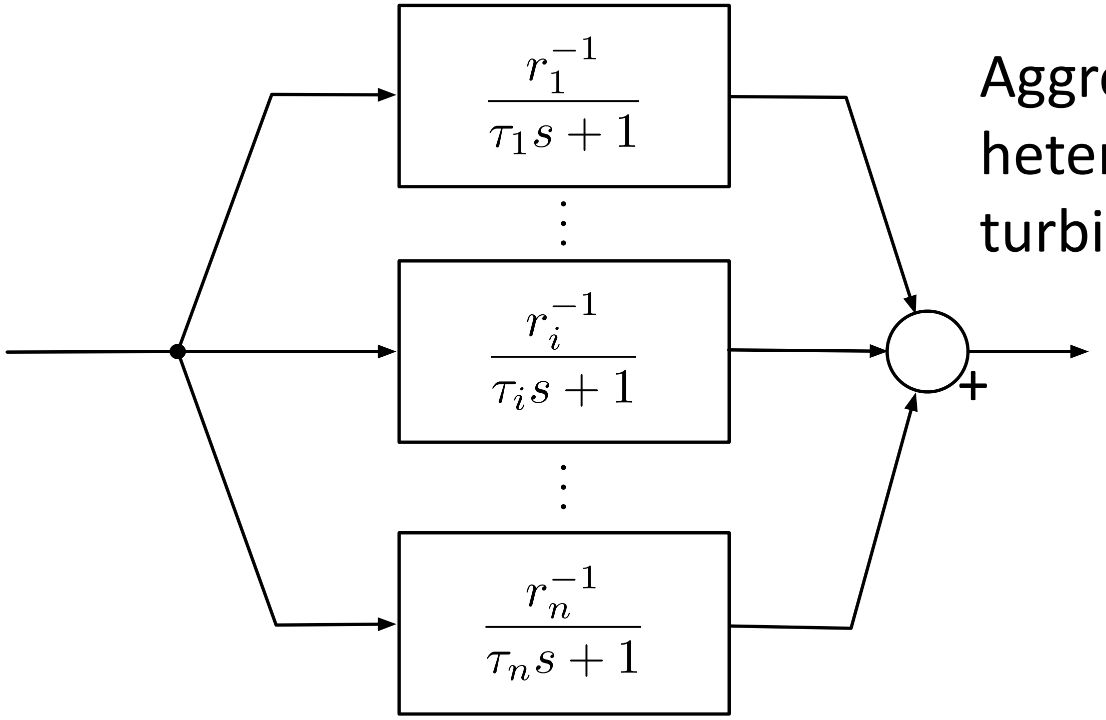
$$\hat{g}(s) = \frac{1}{\hat{m}s + \hat{d} + \boxed{\sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_i s + 1}}}$$

high-order if τ_i are heterogeneous

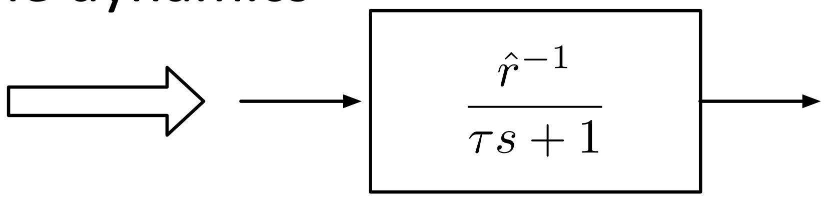
Need to find a low-order approximation of $\hat{g}(s)$

Prior Work: Aggregation for heterogeneous τ_is

When time constants are **heterogenous**:



Aggregating heterogenous turbine dynamics



Time constant τ is chosen by:

- Optimization: Germond'78, Guggilam'18
- Weighted harmonic mean: Ourari'06

Drawbacks:

- the order of overall approximation model is restricted to 2nd order
- the only "decision variable" is the time constant
- does not consider the effect of inertia or damping in the approx.

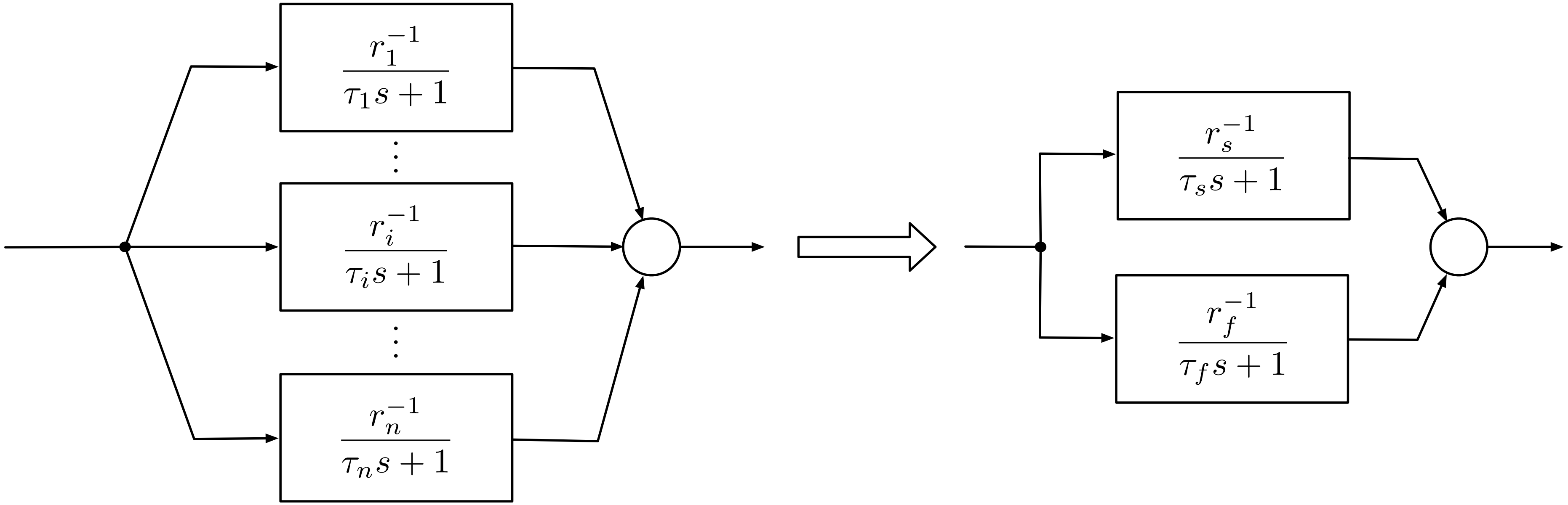
Inaccurate Approximation

Our Approach

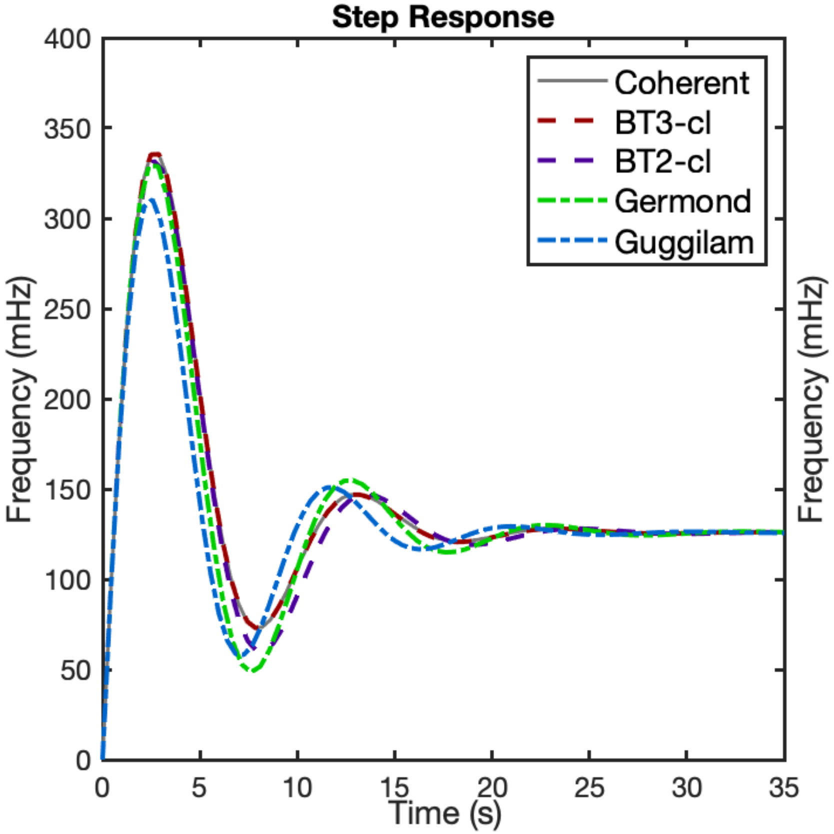
Leverage **weighted balance truncation** to build a hierarchy of approximations

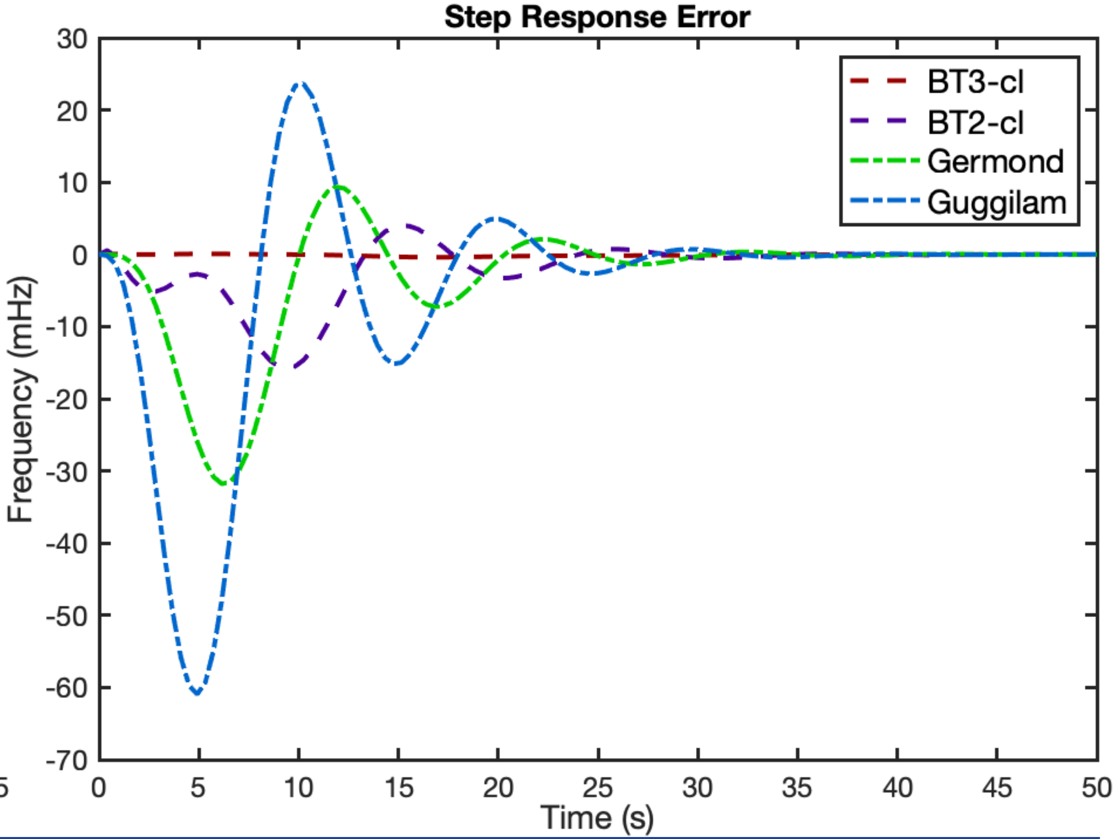
$$\hat{g}(s) = \frac{1}{\hat{m}s + \hat{d} + \sum_{i=1}^{n} \frac{r_i^{-1}}{\tau_i s+1}} \Longrightarrow \tilde{g}_k(s) = \frac{1}{\tilde{m}s + \tilde{d} + \tilde{g}_{tb,k-1}(s)}$$

The case $k = 3$, leads to a more flexible approximation



Comparison with (Some) Existing Methods





By essentially relaxing the restrictions on reduced order model:

- **increase the model order to 3rd order,**
- **reduction on closed-loop dynamics,**

our proposed models outperform models by conventional approach

Outline

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Storage-Based Frequency Shaping Control

Yan Jiang, Eliza Cohn, Petr Vorobev, *Member, IEEE*, and Enrique Mallada, *Senior Member, IEEE*

[TPS 21] ***IEEE Transactions on Power Systems**, 2021*

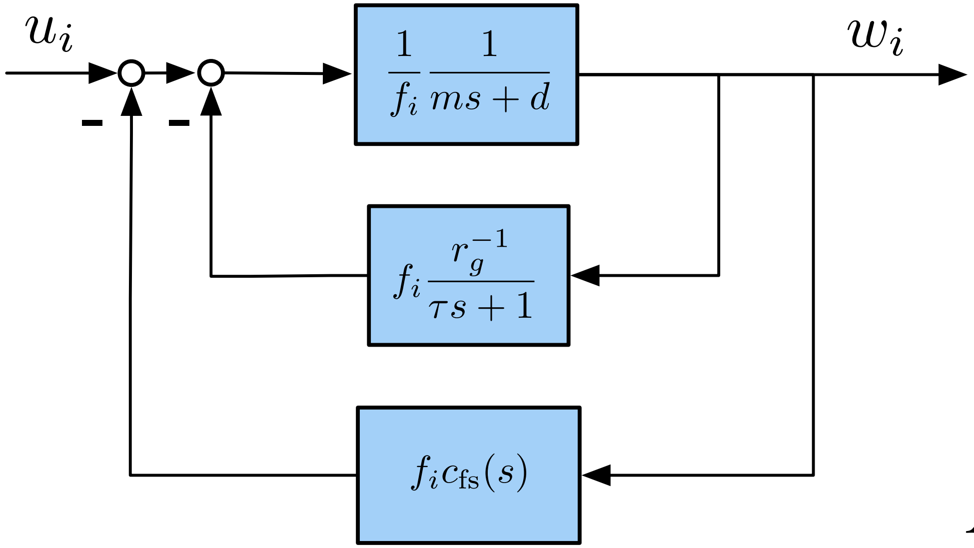
Grid-forming frequency shaping control

Yan Jiang[1], Andrey Bernstein[2], Petr Vorobev[3], and Enrique Mallada[1]

[L-CSS 21] ***IEEE Control Systems Letters,** 2021*

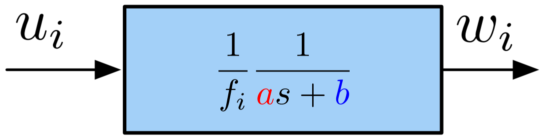
Grid-following Frequency Shaping Control

Key idea: use model matching control (at each bus/area)



$$c_{\text{fs}}(s) := \frac{A_1 s^2 + A_2 s + A_3}{\tau s + 1}$$

$$A_1 = \tau\,(a - m)$$
$$A_2 = b\tau + a - m$$
$$A_3 = b - r_{\text{g}} - d$$



Leads to CoI Frequency $\overline{w}$ with:

RoCoF:

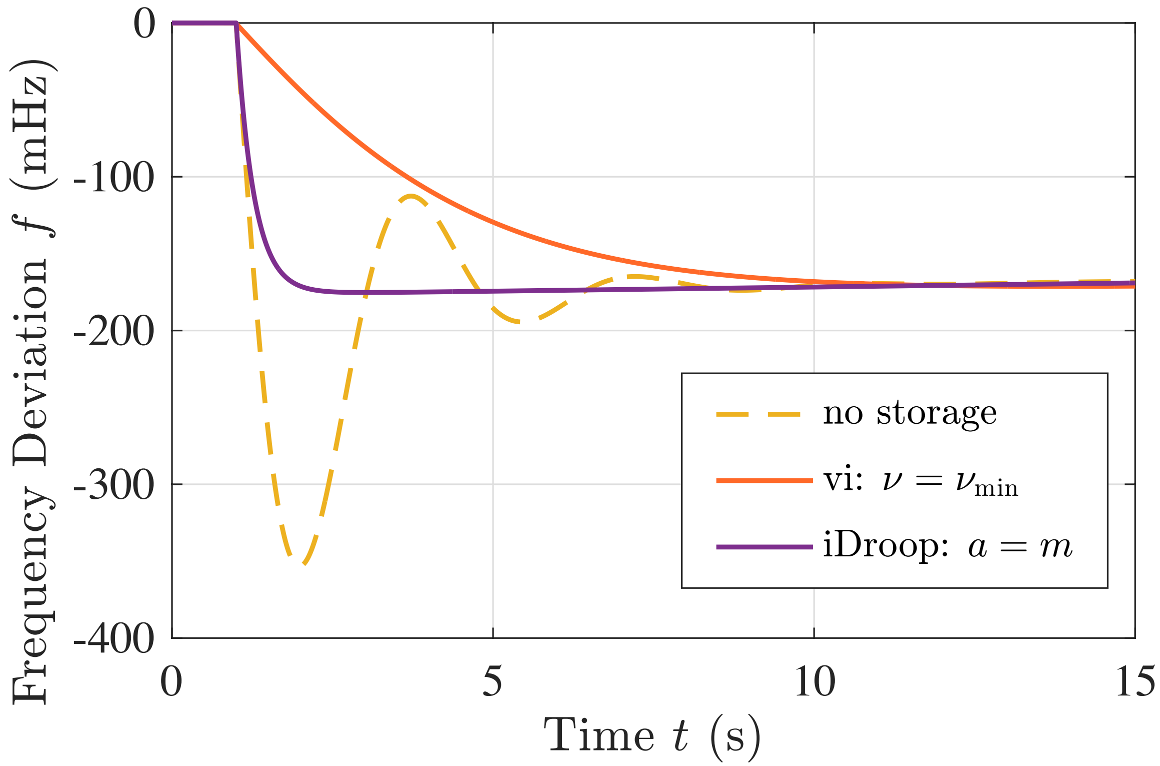
$$||\dot{\bar{w}}||_\infty = \frac{|\sum_i u_{0i}|}{\sum_i f_i}\frac{1}{a}$$

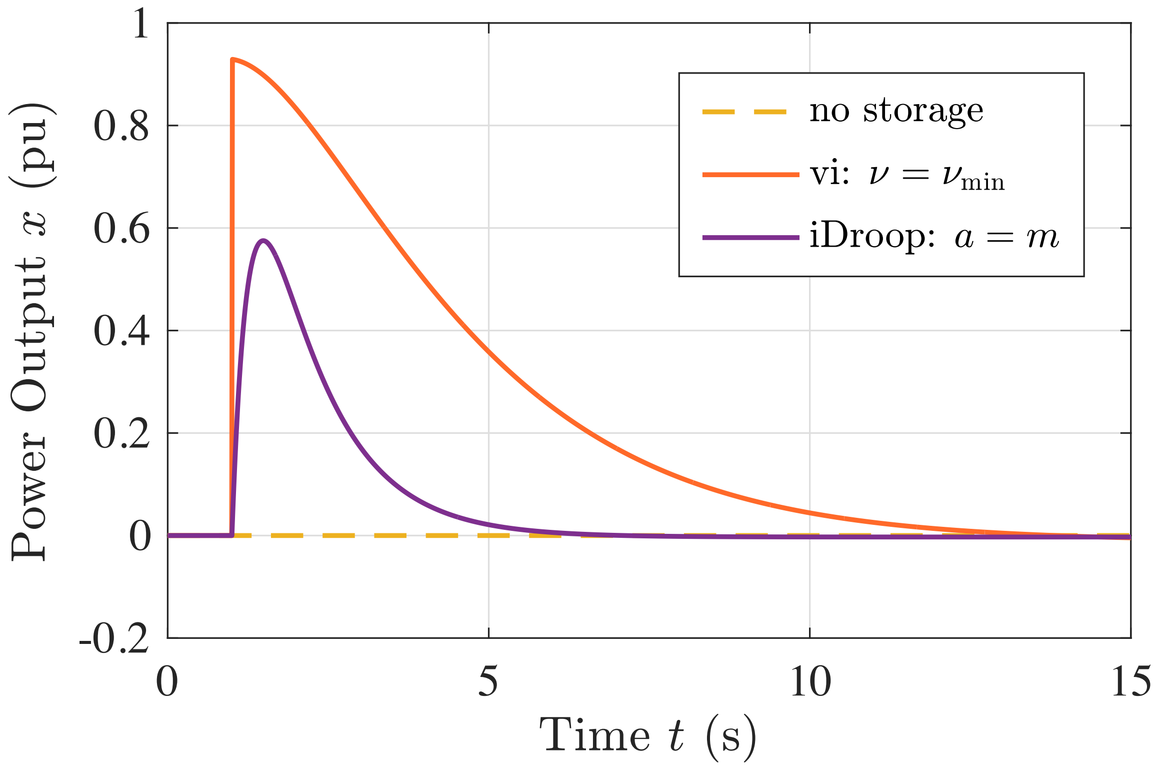
Steady-state:

$$\bar{w}(\infty) = \frac{\sum_i u_{0i}}{\sum_i f_i}\frac{1}{b}$$

[TPS 21] Jiang, Cohn, Vorobev, M, *Storage-based frequency shaping control*, **IEEE Transactions on Power Systems**, under review

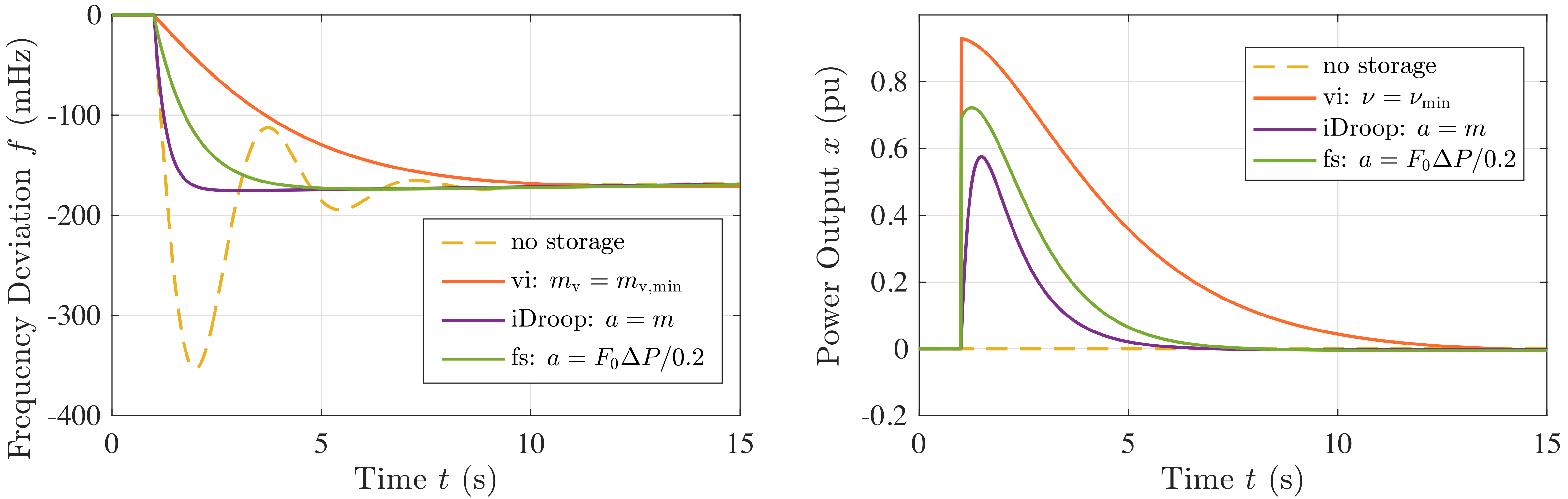
Trading off Control Effort and RoCoF





[TPS 21] Jiang, Cohn, Vorobev, M, *Storage-based frequency shaping control*, **IEEE Transactions on Power Systems**, accepted

Trading off Control Effort and RoCoF

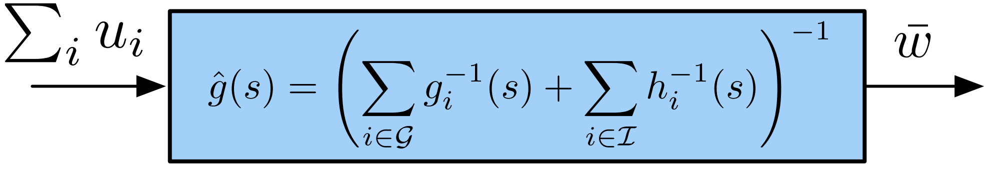


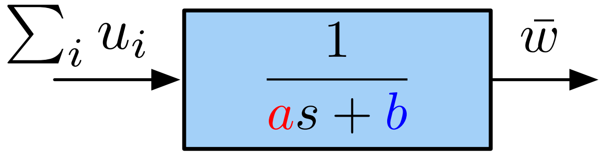
Challenge: Solution Limited to Grid-following Inverters

[TPS 21] Jiang, Cohn, Vorobev, M, *Storage-based frequency shaping control*, **IEEE Transactions on Power Systems**, accepted

Grid-forming Frequency Shaping Control

Key idea: use model matching control on coherent dynamics





Generation:

$$g_i(s) = \frac{1}{m_i s + d_i + \frac{r_i^{-1}}{\tau_i s + 1}}, \quad i \in \mathcal{G}$$

Inverters:

$$h_i(s) = \frac{1}{m_i s + d_i + c_i(s)}, \quad i \in \mathcal{I}$$

$$a := \sum_{i\in\mathcal{G}} m_i + \sum_{i\in\mathcal{I}} m_i$$

$$b := \sum_{i\in\mathcal{G}} (d_i + r_i^{-1}) + \sum_{i\in\mathcal{I}} d_i$$

$$\sum_{i\in\mathcal{I}} c_i(s) = \sum_{i\in\mathcal{G}} \frac{r_i^{-1}\tau_i s}{\tau_i s + 1}$$

RoCoF:

$$||\dot{\bar{w}}||_\infty = \frac{|\sum_i u_{0i}|}{a}$$

Steady-state:

$$\bar{w}(\infty) = \frac{\sum_i u_{0i}}{b}$$

[L-CSS 21] Jiang, Bernstein, Vorobev, M, *Grid-forming frequency shaping control*, **IEEE Control Systems Letters**, 2021

Summary

- Frequency domain characterization of **coherent dynamics**, as a low rank property of the transfer function.

- **Coherence is** a **frequency dependent** property:
  - Effective algebraic connectivity $f(s)\lambda_2(L)$
  - Disturbance frequency spectrum

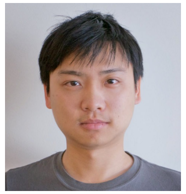
- We use frequency **weighted balanced truncation** to suggest possible improvements to obtain accurate reduced order model of aggregated dynamics of coherent generators:
  - increase model complexity (3^{rd} order/two turbines)
  - model reduction on closed-loop dynamics

- Grid-forming Frequency Shaping Control

Thanks!

Related Publications:

- Min, M, "Coherence and Concentration in Tightly Connected Networks," **submitted**
- Min, Paganini, M, "Accurate Reduced Order Models for Coherent Synchronous Generators," **L-CSS 2021**
- Jiang, Bernstein, Vorobev, M, "Grid-forming Frequency Shaping Control," **L-CSS 2021**



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Backup Slides

Numerical Examples

Modal Decomposition

Coherence