Quantifying Stability in Deterministic and Stochastic Complex Networks and its Application to Power Grids

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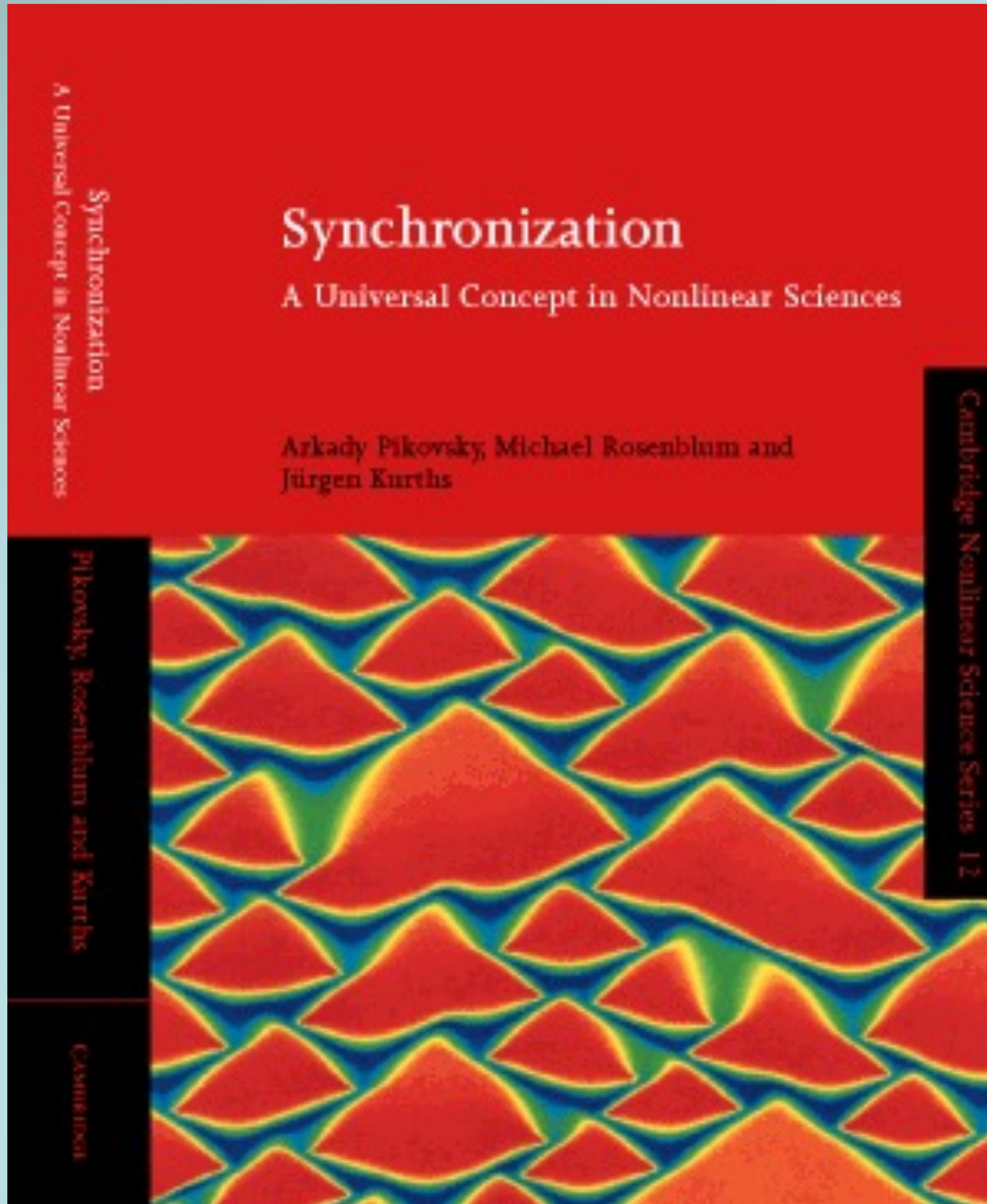
Contents

- Introduction
- Basin stability for complex networks
- Stability of power grids without losses
- Including lossy cases
- Conclusions


Synchronization
A Universal Concept in Nonlinear Sciences
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Synchronization in complex networks

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Power Grids

Intended Solution:

stable synchronized behaviour along the whole network of networks

How to control such networks?

Pinning Control (which nodes?)

Highly Non-trivial Task

Monster blackouts

Failing of Control!!!

Man-Made: Germany Papenburg: Monster Black-Out 04-11-2006

- Meyer Werft in Papenburg
- Newly built ship Norwegian Pearl

  length: 294 m, width: 33 m
- Cut one line of the power grid
- Black-out in

  Germany (> 10 Mio people)

  France (5 Mio people)

  Austria, Belgium, Italy, Spain

Danger of **cyberattacks** on the energy supplying systems

(G7 ministers – May 12, 2015)

Challenge for Complex Systems Science

(Physics, Mathematics, Engineering,Computer Science…)

Stability

Alexandr Mikhailovich Lyapunov

- Lyapunov was the first to consider modifications necessary in *nonlinear systems* to the linear theory of stability based on linearizing near a point of equilibrium
- The equilibrium x_ε of the system is said to be *Lyapunov stable*, if for every ($\forall\ \varepsilon > 0$) and ($\forall\ t_0$), there exists a $\delta = \delta(t_0, \varepsilon) > 0$ such that,

  if $|x(t_0)-x_\varepsilon| < \delta$, then $|x(t)-x_\varepsilon| < \varepsilon$, for every $t \geq 0$.
- Extension to asymptotical and exponential stability

Stability of Networks

Synchronizability –
Master Stability Formalism

Pecora&Carrol (1998) –

based on **local stability**

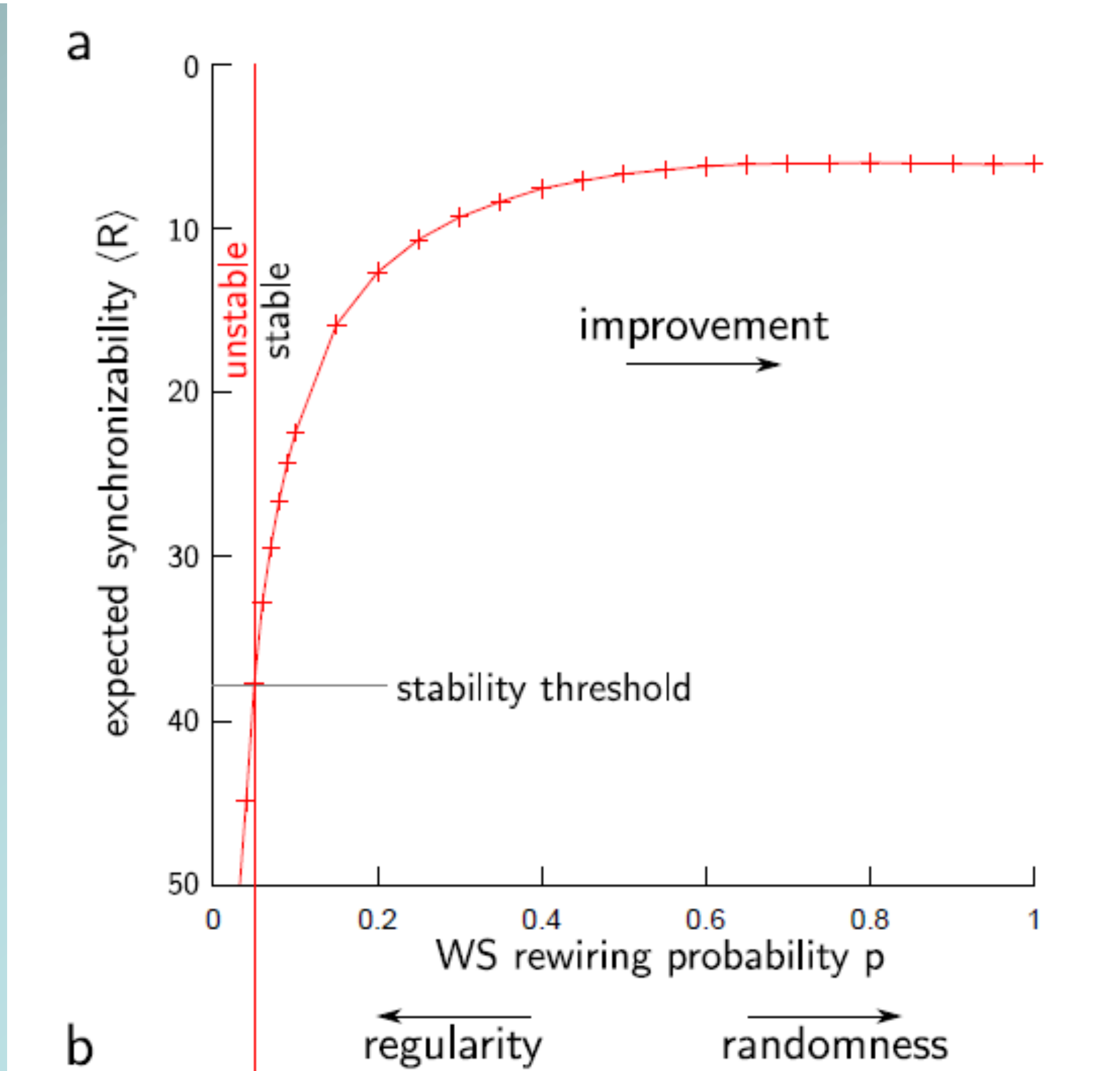
Small world network with Roessler oscillators

$$\dot{\mathbf{r}}_i = \mathbf{F}(\mathbf{r}_i) + K\sum_j A_{ij}[\mathbf{H}(\mathbf{r}_j) - \mathbf{H}(\mathbf{r}_i)] = \mathbf{F}(\mathbf{r}_i) - K\sum_j L_{ij}\mathbf{H}(\mathbf{r}_j),$$

$$\begin{aligned} \dot{x}_i &= -y_i - z_i - K\sum_{i=1}^{N} L_{ij}x_j \\ \dot{y}_i &= x_i + a y_i \\ \dot{z}_i &= b + z_i(x_i - c) \end{aligned}$$

Chosen: a = b = 0.2, c = 7.0 ➔ R < 37.88

Chaotic Rössler oscillators, N = 100



Main Result: SW-Network **best synchronizable for most random** SW-networks

Puzzle!

MSF – local stability (Lyapunov stability)

How to go beyond (not only small perturbations)?

Lyapunov Functions?

Engineering Term:

Transient Stability

Network´s Basin Stability

basin volume of a state (regime)

measures likelihood of return to this state (regime)

Nature Physics 9, 89 (2013)

Network´s Basin Stability

basin volume of a state (regime) measures the likelihood of

- arrival at this state (regime)
  quantifies its **relevance** (M. Girvan, 2006)
- return to this state after a random perturbation
  quantifies its **stability**
  (Menck, Heitzig, Marwan, Kurths:
  Nat. Phys., 2013)

Normalized Network´s Basin Stability

$\mathcal{B}$ - Synchronous state´s basin of attraction

$$\mathcal{B} = \{\mathrm{x} \in \mathcal{S} \,|\, \Phi_t(\mathrm{x}) \rightarrow \mathcal{I}\}$$

$\mathcal{Q}$ - Subset of state space S covering the system´s (weak) attractor

$$S_{\mathcal{B}\cap\mathcal{Q}} = \mathrm{Vol}(\mathcal{B} \cap \mathcal{Q})/\mathrm{Vol}(\mathcal{Q}) \in [0, 1]$$

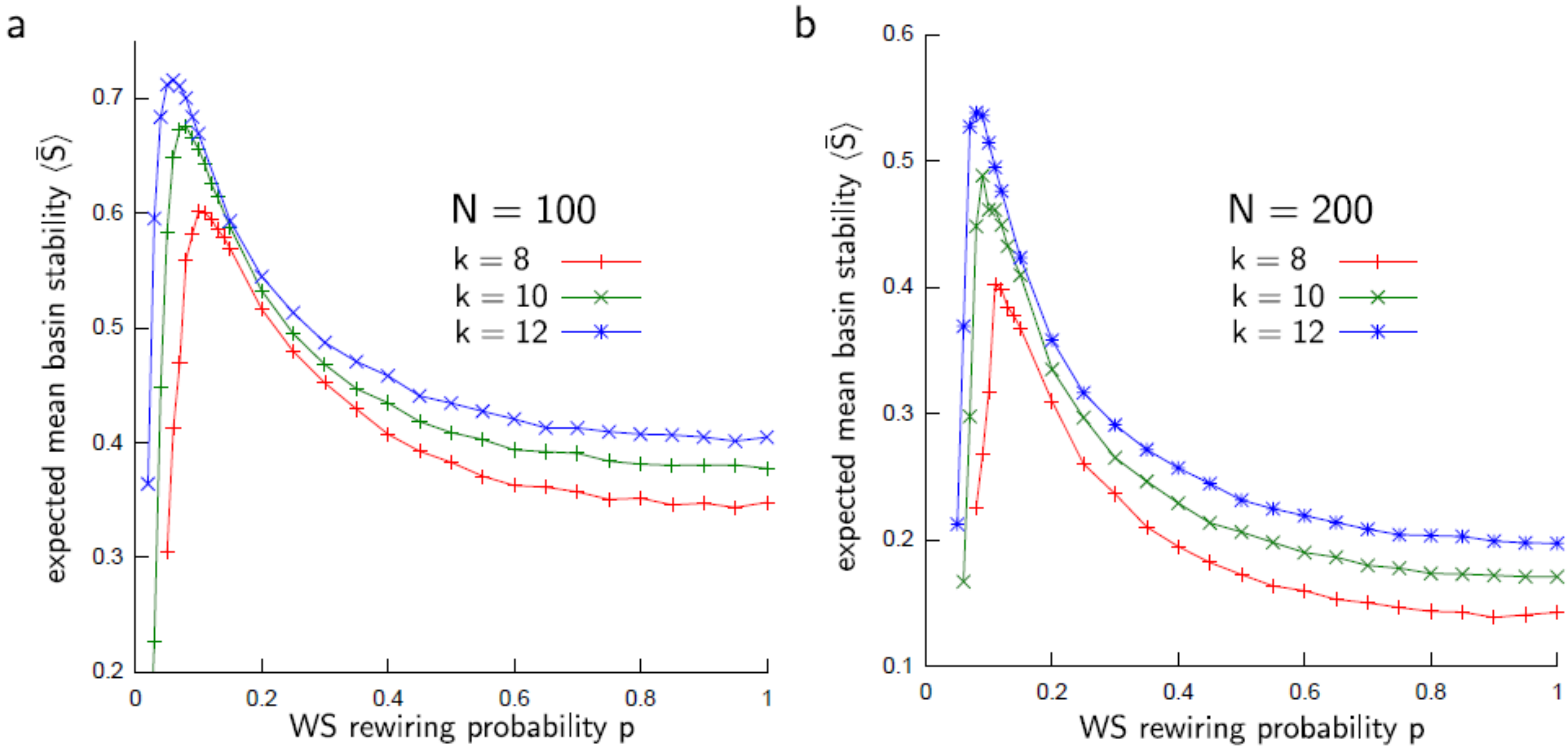
Normalized Basin Stability

Bernoulli-like experiment

- T experiments (different initial conditions – randomly distributed)
- M states converge to $\mathcal{I}$
- Estimate M / T

➔ standard error $$e := \frac{\sqrt{S_B(1 - S_B)}}{\sqrt{T}}$$

- T=500 ➔ error < 0.023



Supplementary Figure S1: **Basin Stability in Rössler networks.** Expected basin stability $\langle \bar{S} \rangle$ versus p. The grey shade indicates $\pm$ one standard deviation. The dashed line shows an exponential fitted to the ensemble results for $p \geq 0.15$. Solid lines are guides to the eye. **a**: $N = 100$, **b**: N=200.

$$\bar{S}_{\mathcal{B}} = \mathrm{mean}_{K \in I_s} S_{\mathcal{B}}(K)$$

averaged over coupling strengths K

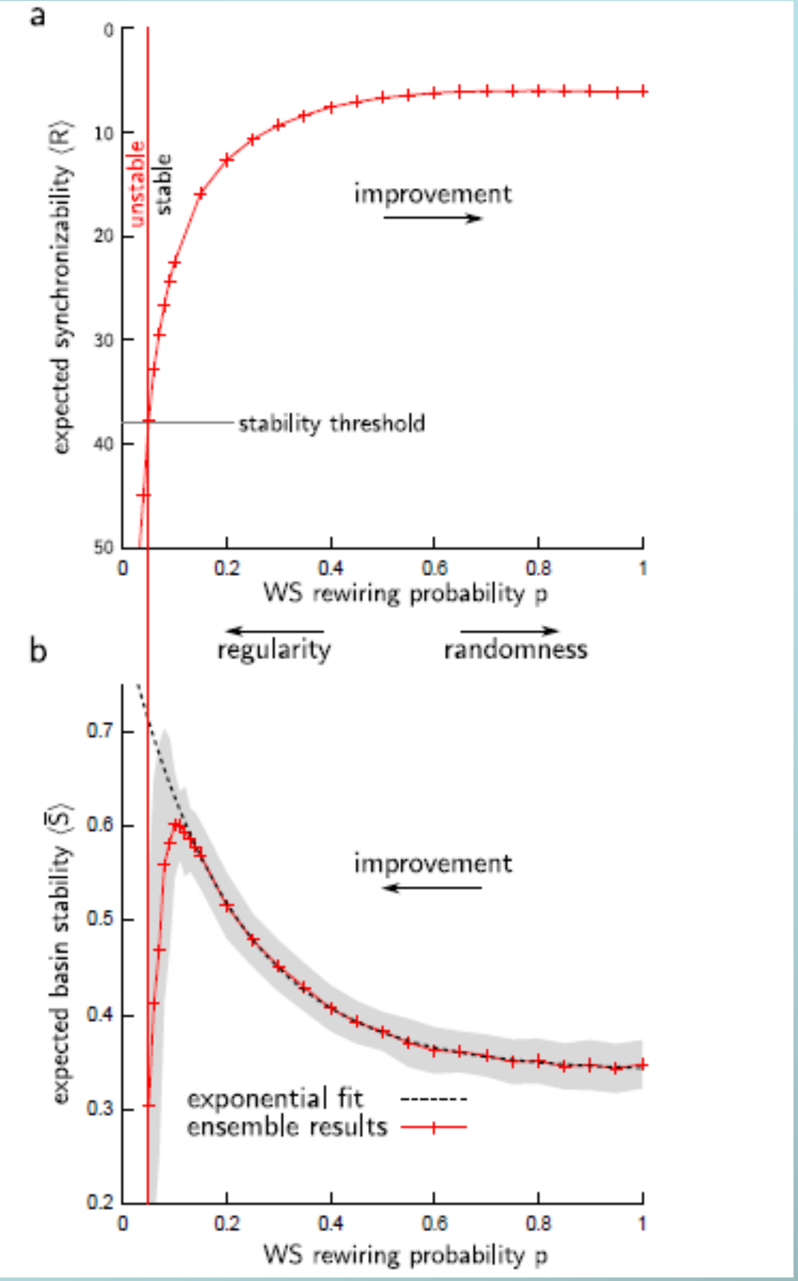
Synchronizability and basin stability inWatts-Strogatz (WS) networks of chaotic oscillators.

a: Expected synchronizability R versus the WS model's parameter p.

The scale of the y-axis was reversed to indicate improvement upon increase in p.

b: Expected basin stability S versus p. The grey shade indicates one standard deviation.

The dashed line shows an exponential fitted to the ensemble results for p > 0.15. Solid lines are guides to the eye. The plots shown were obtained for N = 100 oscillators of Roessler type, each having on average k = 8 neighbours. Choices of larger N and different k produce results that are qualitatively the same.



Analysis of Power Grids

Single generator´s dynamics

$$\dot{\theta} = \omega$$

$$\dot{\omega} = -\alpha\omega + P - \underbrace{K\sin(\theta - \theta_{\text{grid}})}_{=:P_{\text{trans}}}$$

θ and ω - phase and angular frequency of the generator´s voltage vector in a reference frame that co-rotates with the grid´s rated frequency

➔ ω = 0 synchrony

P – net power input

- αω - damping

P_{trans} - power flow to the grid across the transmission line

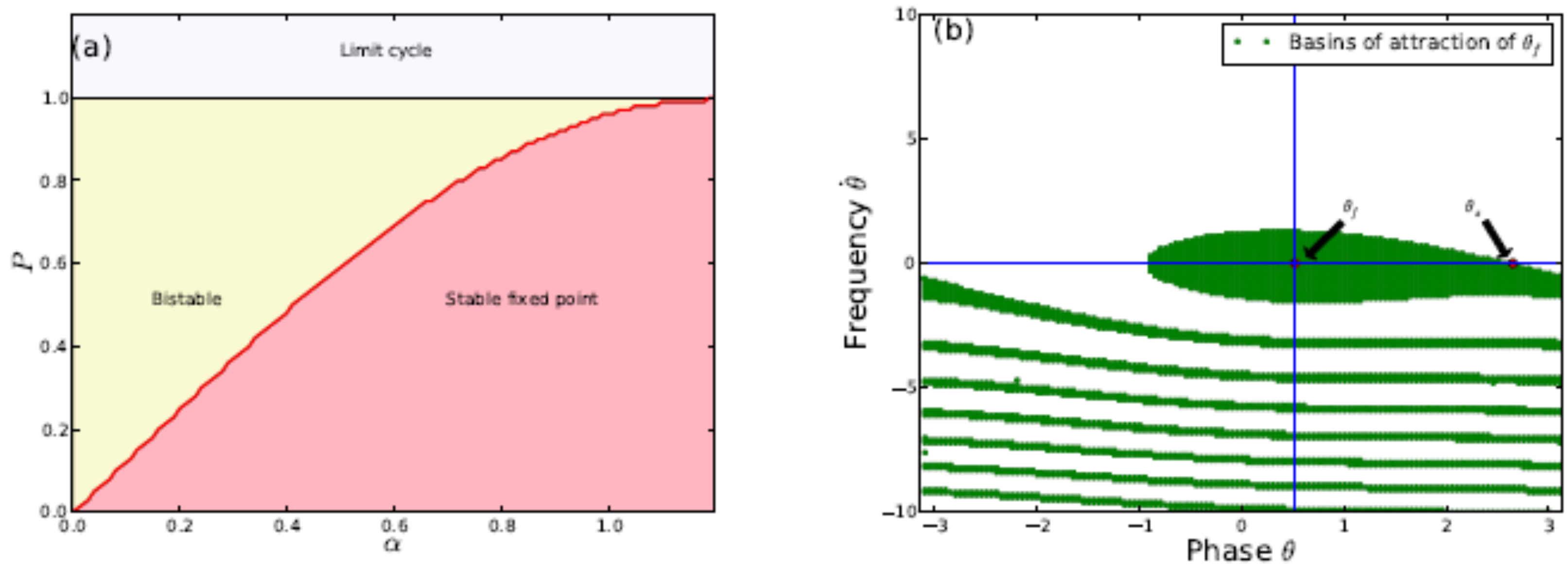


Fig. 1. (color online). Parameter space and state space of the one-machine infinite bus system. In the left panel (a), red indicates the area of stable fixed point. In the yellow area, the oscillator either converges to stable fixed point or rotates periodically depending crucially on initial values of θ and $\dot{\theta}$. White area indicates the existence of stable limit cycle. (b): Basin of attraction of the stable fixed point θ_f is indicated in the green area with $\alpha = 0.1$, $P = 0.5$ and $K = 1$. The stable fixed point and saddle are also plotted in red. The saddle is at the right side of the stable fixed point.

Power Grid Model

$$\dot{\theta}_i = \omega_i$$

$$\dot{\omega}_i = -\alpha_i \omega_i + P_i - \sum_{j=1}^{N} K_{ij} \sin(\theta_i - \theta_j)$$

θ_i and ω_i denote phase and frequency of the generator at node i

Node i net generator if $P_i > 0$

Node i net consumer if $P_i < 0$

α_i - damping constant

P_i - net power input

Main Question:
How stable is the synchronized regime?

$$\omega_i = 0, \ \dot{\omega}_i = 0$$

Stability even in case of large perturbations at one node

➔Concept of basin stability

Nature Commun. 5, 3969 (2014)

Single-Node Basin Stability

$$S_i \in [0, 1]$$

Probability that the grid will return to its synchronous state after node i has been hit by a random large perturbation

Single node basin stability $S_i := \mathrm{Vol}(\mathcal{B}_i \cap \mathcal{Q})/\mathrm{Vol}(\mathcal{Q}) \in [0, 1]$

$$\mathcal{B}_i := \{(\theta_i, \omega_i) | (\theta_j, \omega_j)_{j=1,\ldots,N} \in \mathcal{B} \text{ with } \theta_j = \theta_j^s \text{ and } \omega_j = 0 \text{ for all } j \neq i\}$$

Perturbed Initial Conditions **only** at Node i

$$\begin{pmatrix} \theta_1(0) \\ \omega_1(0) \\ \vdots \\ \theta_i(0) \\ \omega_i(0) \\ \vdots \\ \theta_N(0) \\ \omega_N(0) \end{pmatrix} = \begin{pmatrix} \theta_1^s \\ 0 \\ \vdots \\ \theta_i^s \\ 0 \\ \vdots \\ \theta_N^s \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \Delta\theta_i \\ \Delta\omega_i \\ 0 \\ \vdots \\ 0 \end{pmatrix}.$$

Application to the Scandinavian Power Grid

Figure 4: **Northern European Power Grid.** The grid has $N = 236$ nodes and $E = 320$ transmission lines. The load scenario was chosen randomly, with squares (circles) depicting $N/2$ net consumers with $P_i = -P$ (net generators with $P_i = +P$). The colour scale indicates how large a node's basin stability S_i is. Insets I-III show re-computed basin stability values after 27 lines have been added in order to 'heal' dead trees (see Methods). New lines are coloured red. Our simulation parameters, $\alpha = 0.1$, $P = 1$, and $K = 8$, imply the simplifying assumptions that all generators in the grid are of the same making and that all transmission lines are of the same voltage and impedance. These assumptions enable us to focus on the effects of the (unweighted) topology. For details, see Methods.

First Conclusions

- Concept of basin stability enables important new insights and principles for the design of (Smart) Power Grids
- Dead ends and dead trees strongly diminish stability (**trouble makers**) ➔ to be avoided
- „Healing“ dead ends by addition of a few transmission lines enhances substantially stability
- For the Scandinavian power grid: addition of 27 lines (8 % of the total) suffice to substantially improve stability – rather low-cost solution)

Power grids with losses

$$H_i \ddot{\phi}_i = P_i - D_i \dot{\phi}_i - \sum_{j=1}^{n} P_{ij} \; ,$$

$$P_{ij} = K_{ij}\Big(\sin\left(\alpha_{ij}\right) + \sin\left(\phi_i - \phi_j - \alpha_{ij}\right)\Big)$$

Complex Admittance $Y_{ij} = -iK_{ij} \exp(i\alpha_{ij})$

X reactance
R resistance

$$Y_{ij} = \frac{1}{R_{ij} + iX_{ij}}$$

Loss-free correct?

- In most power grid studies considered (as above):

  $\alpha = 0$ (because $R = 0$)

- But in reality:

  $\alpha = 0.24$ – high-voltage power grids

  $\alpha = 1.4$ – medium voltage…

Crucial question:

Which consequences have losses for solutions and stability?

Study Cases

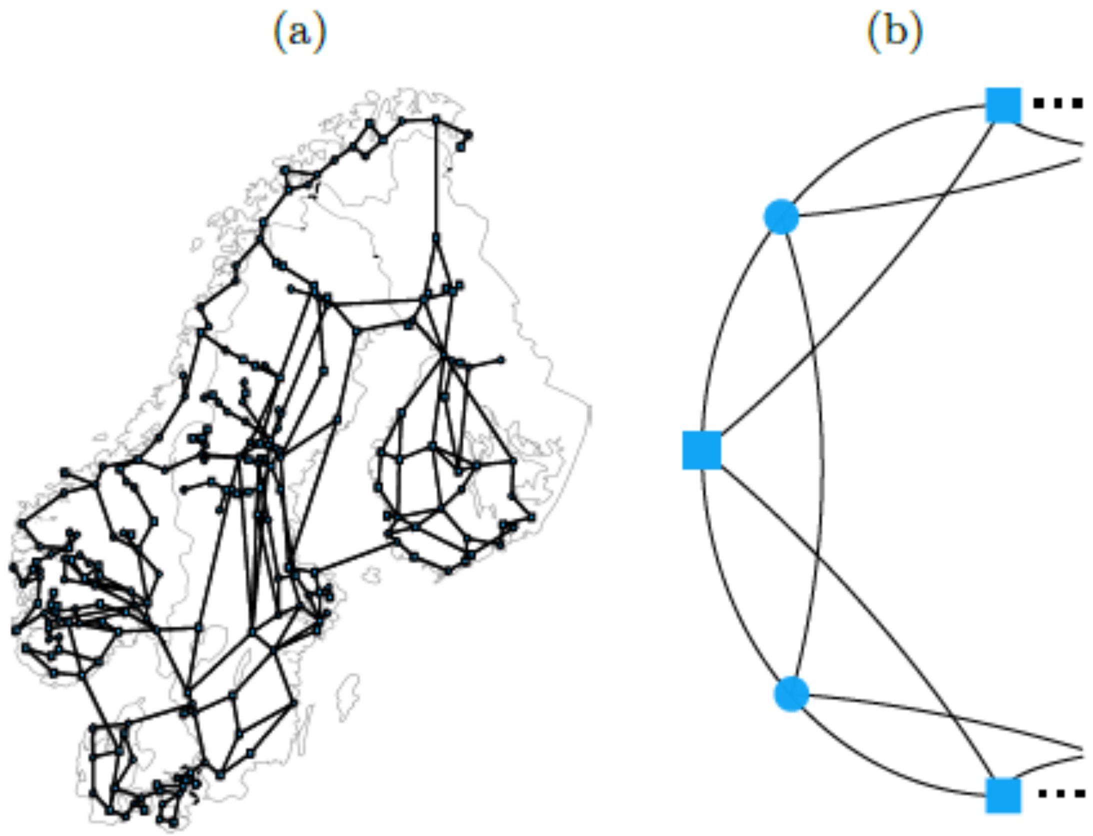


FIG. 1: **Network models. a** The Scandinavian (extra-)high voltage transmission grid. **b** A circle topology with coupling to next-nearest neighbours, i.e. a coupling radius of $R = 2$. In both cases, squares denote net consumers and circles net producers.

New multistability occurs

- Shift of limit cycle:

$$\omega'_{\mathrm{lc}} = \omega_{lc} - \frac{K}{D}\sin(\alpha)$$

- Strongly change the basin structure of the solitory (periodic) solution
- Even flips signs of rotation ➔ **exotic** oscillates in opposite dir. **solitary state**
- Further solutions appear

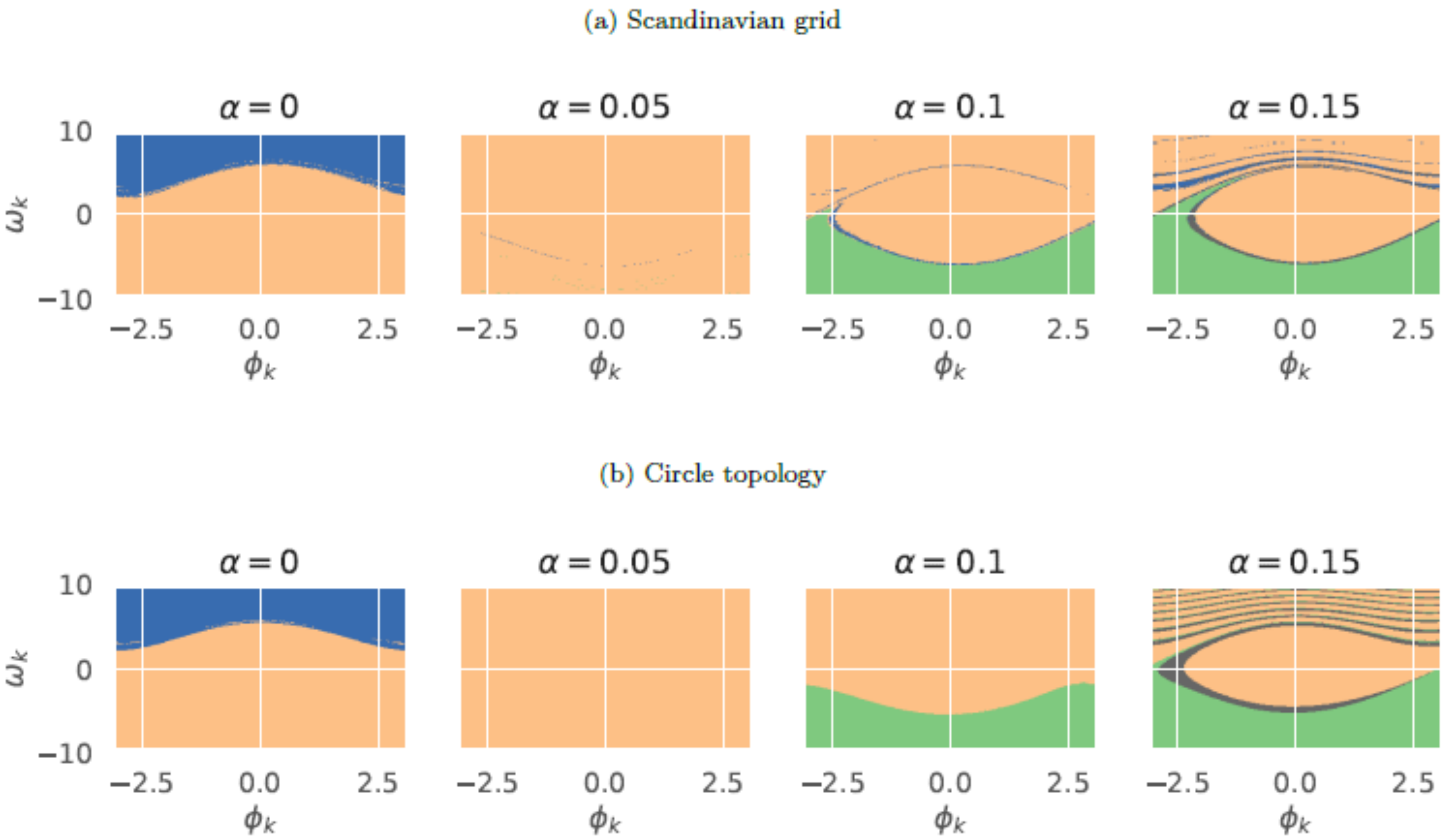


FIG. 2: **Phase space cross sections.** Cross section of the phase space corresponding to phase ϕ_k and phase velocity ω_k of a randomly chosen node of **a** the Scandinavian power grid and **b** the circle topology (both with standard parametrisation and control, see Methods). Each point belongs to the sync basin (■), the basin of a solitary state rotating naturally (■) or in the basin of an exotic solitary state (■). Other asymptotic states are marked in grey (■). Further parametrisations are given in SI.

Average Singel Node Basin Stability (ABSB) in dependence on the loss α

(realistic perturbations)

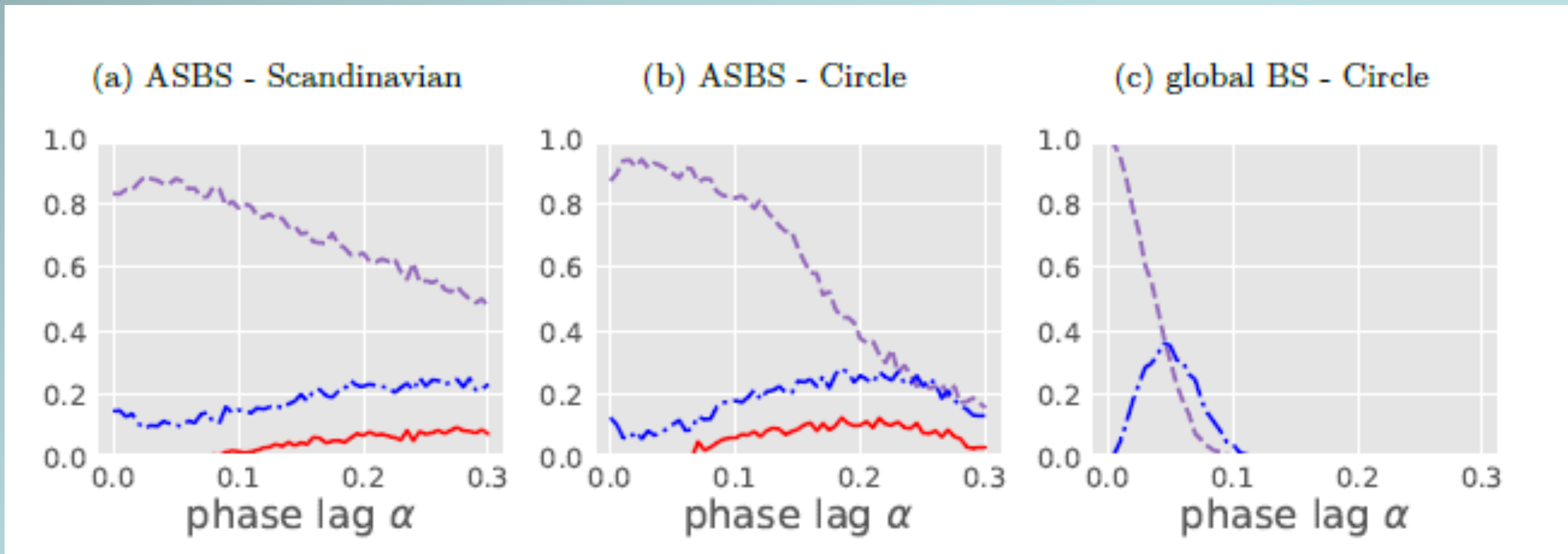


FIG. 3: **Basin stability.** The top row shows the average single node basin stability ASBS (**a**, **b**) and the global basin stability BS (**c**) of three types of asymptotic regimes: synchronisation (- - -), exotic solitaries (——) and the union of normal and exotic solitaries (-·-·-). Simulations were performed with standard parametrisation and control

- Exotic solitary solutions found analytically via a mean field approach
- Appear through a homoclinic bifurcation

- Losses are realistic and have to be included (even in high voltage power grids)
- Losses induce new exotic solitary waves
- They pose a challenge for control

Nat Comm 2020

Outlook

- Sampling-based approaches as basin stability, survivability, threshold stability, stochastic basin stability are promising
- Many open problems:
  - Prove the techniques - mathematical foundation
  - include time-varying price feedback
  - include renewable components (wind, sun)
  - design for islands (not only diesel)
  - transient dynamics

Our Papers

- Nature Physics 9, 89 (2013)
- Nature Communication 5, 3969 (2014)
- Comm. Comp. Inform. Sc. 438, 211 (2014)
- EPJ ST 223, 2593 (2014)
- New J. Phys. 16, 125001 (2014)
- Phys. Rev. E 90, 022812 (2014)
- Phys. Rev. Lett. 112, 114102 (2014)
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- Scient Rep. 5, 16196 (2015) Meccanica (2016)
- New J. Phys. 18, 013004 (2016) CHAOS 27, 127003 (2017)
- Scient. Rep. 6, 21449 (2016) New J. Phys. 19, 023005 (2017)
- Scient. Rep. 6, 29654 (2016) New J. Phys. 19, 033029 (2017)
- Physics Rep. 610, 1 (2016) Scient. Rep. 7, 9336 (2017)
- CHAOS 26, 073117 (2016) CHAOS 28, 043102 (2018)
- CHAOS 30, 013110 (2020) Phys Rev Res 2, 023409 (2020)
- Nature Communications 11, 592 (2020)