Event-triggered Control for Nonlinear Systems with Time-Varying Input Delay

Erfan Nozari
http://carmenere.ucsd.edu/erfan

University of California, San Diego

55th IEEE Conference on Decision and Control, Las Vegas, USA
December 12, 2016

Joint work with **Pavankumar Tallapragada** and **Jorge Cortés**

UC San Diego
Jacobs School of Engineering

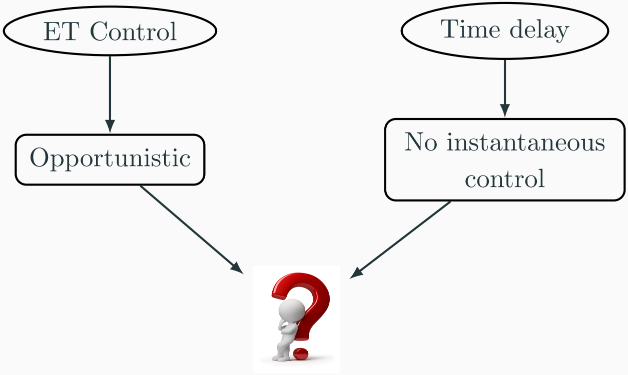
Motivation

Time delay and **bandwidth limitation** are widespread in real-world implementations of networked control systems

Motivation

We address bandwidth limitation using event-triggered (ET) control

? challenging due to **interplay between ET and time delay**



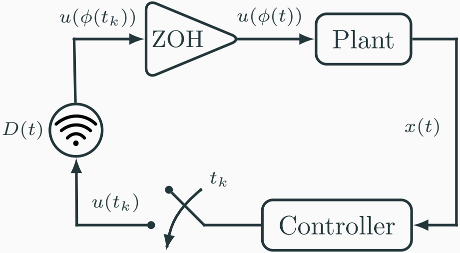
Outline

Outline

1. Problem Statement
2. Event-Triggered Design and Analysis
   - Predictor Feedback
   - Event-Triggered Law
   - Convergence Analysis
3. The Linear Case
   - Communication-Convergence Trade-off
4. Numerical Results
   - Compliant Nonlinear System
   - Non-compliant Nonlinear System

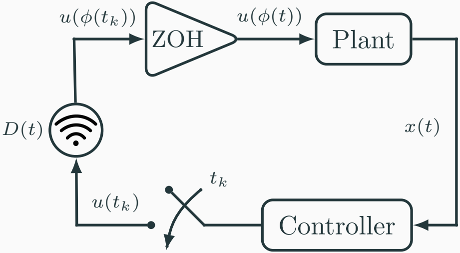
General nonlinear dynamics:

$$\text{Plant:} \begin{cases} \dot{x}(t) = f(x(t), u(\phi(t))) \\ \phi(t) = t - D(t) \end{cases}$$



General nonlinear dynamics:

Plant: $\begin{cases} \dot{x}(t) = f(x(t), u(\phi(t))) \\ \phi(t) = t - D(t) \end{cases}$



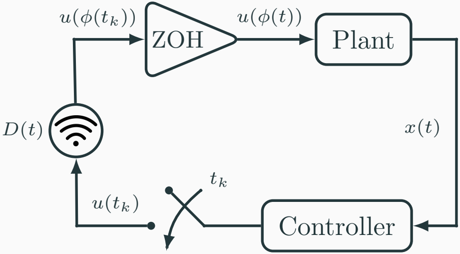
Assumptions

- $\{u(t) \mid \phi(0) \leq t \leq 0\}$ is **given and bounded**
- **No finite escape time**
- **Delay bounds**: $0 < t - \phi(t) \leq M_0$ and $0 < m_2 \leq \dot{\phi}(t) \leq M_1$
- Globally Lipschitz $K : \mathbb{R}^n \to \mathbb{R}$, $K(0) = 0$ exists s.t.

  $\dot{x}(t) = f(x(t), K(x(t)) + w(t))$ is **ISS** with respect to w

General nonlinear dynamics:

$$\text{Plant: } \begin{cases} \dot{x}(t) = f(x(t), u(\phi(t))) \\ \phi(t) = t - D(t) \end{cases}$$



Design Objective

1. **Event-triggered stabilization:** closed-loop GAS using

$$u(t) = u(t_k) \qquad t \in [t_k, t_{k+1}), \; k \in \mathbb{Z}_{\geq 0}$$

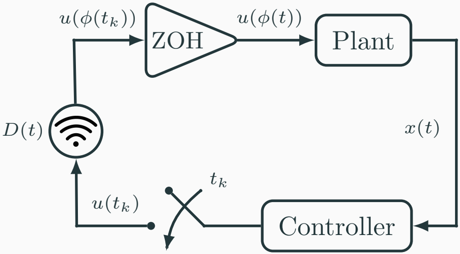
2. **No Zeno behavior:**

$$\lim_{k \to \infty} t_k = \infty$$

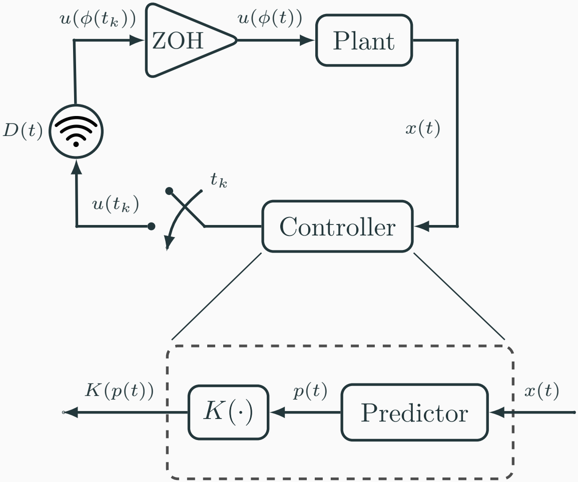
Outline

1. Problem Statement
2. Event-Triggered Design and Analysis
   - Predictor Feedback
   - Event-Triggered Law
   - Convergence Analysis
3. The Linear Case
   - Communication-Convergence Trade-off
4. Numerical Results
   - Compliant Nonlinear System
   - Non-compliant Nonlinear System

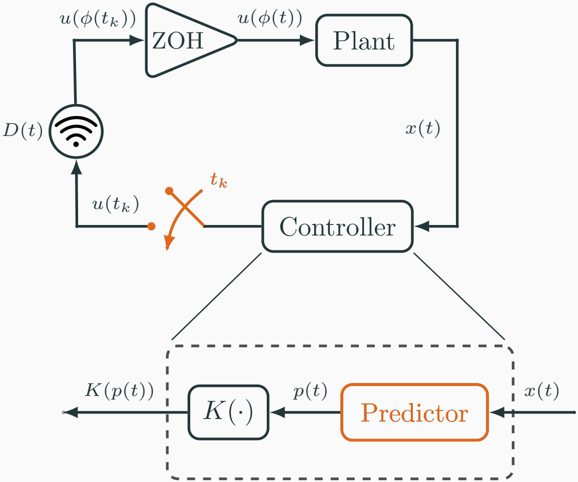
Controller Structure



Controller Structure



Controller Structure



Predictor Feedback [Bekiaris-Liberis and Krstic, 2013]

- $p(t)$ is the **prediction of the future** state of the plant:

$$
\begin{aligned}
p(t) = x(\phi^{-1}(t)) &= x(t) + \int_t^{\phi^{-1}(t)} f\big(p(\phi(\tau)), u(\phi(\tau))\big) d\tau \qquad \to s = \phi(\tau) \\
&= x(t) + \int_{\phi(t)}^{t} f(p(s), u(s)) \frac{d\phi^{-1}(s)}{ds} ds, \qquad t \geq 0
\end{aligned}
$$

Predictor Feedback [Bekiaris-Liberis and Krstic, 2013]

- $p(t)$ is the **prediction of the future** state of the plant:

$$\begin{aligned} p(t) = x(\phi^{-1}(t)) &= x(t) + \int_t^{\phi^{-1}(t)} f\big(p(\phi(\tau)), u(\phi(\tau))\big) d\tau \qquad \to s = \phi(\tau) \\ &= x(t) + \int_{\phi(t)}^{t} f(p(s), u(s)) \frac{d\phi^{-1}(s)}{ds} ds, \qquad t \geq 0 \end{aligned}$$

- Computing $p(t)$ requires:
  1. **State feedback:** $x(t)$
  2. **Control history:** $\{u(s) | \phi(t) \leq s \leq t\}$
  3. **Prediction history:** $\{p(s) | \phi(t) \leq s \leq t\}$

- Either analytical or numerical integration is used

Event-Triggered Law

- $S(x(t)) =$ Lyapunov function for the **delay-free system**:

$$\alpha_1(|x|) \leq S(x) \leq \alpha_2(|x|)$$

$$\frac{\partial S}{\partial x} f(x, K(x) + w) \leq -\gamma(|x|) + \rho(|w|)$$

Event-Triggered Law

- $S(x(t))$ = Lyapunov function for the **delay-free system**:

$$\alpha_1(|x|) \le S(x) \le \alpha_2(|x|)$$
$$\frac{\partial S}{\partial x} f(x, K(x) + w) \le -\gamma(|x|) + \rho(|w|)$$

- $V(t)$ = Lyapunov function of the **delayed system** $(b > 0)$

$$V(t) = S(x(t)) + \frac{2}{b} \int_0^{2L(t)} \frac{\rho(r)}{r} dr, \quad L(t) = \sup_{t \le \tau \le \sigma(t)} |e^{b(\tau - t)} w(\phi(\tau))|$$
$$w(t) = u(t) - K(p(t_k))$$

Event-Triggered Law

- $S(x(t))$ = Lyapunov function for the **delay-free system**:

$$\alpha_1(|x|) \leq S(x) \leq \alpha_2(|x|)$$
$$\frac{\partial S}{\partial x} f(x, K(x) + w) \leq -\gamma(|x|) + \rho(|w|)$$

- $V(t)$ = Lyapunov function of the **delayed system** $(b > 0)$

$$V(t) = S(x(t)) + \frac{2}{b} \int_0^{2L(t)} \frac{\rho(r)}{r} dr, \quad L(t) = \sup_{t \leq \tau \leq \sigma(t)} |e^{b(\tau - t)} w(\phi(\tau))|$$
$$w(t) = u(t) - K(p(t_k))$$

Proposition: Bound on $\dot{V}$

If $e(t) = p(t_k) - p(t)$ is the **prediction error**,

$$\dot{V}(t) \leq -\gamma(|x(t)|) - \rho(2L(t)) + \rho(2L_K|e(\phi(t))|)$$

Proposition: Bound on $\dot{V}$

If $e(t) = p(t_k) - p(t)$ is the **prediction error**,

$$\dot{V}(t) \leq -\gamma(|x(t)|) - \rho(2L(t)) + \rho(2L_K|e(\phi(t))|)$$

Proposition: Bound on $\dot{V}$

If $e(t) = p(t_k) - p(t)$ is the **prediction error**,

$$\dot{V}(t) \leq \boxed{-\gamma(|x(t)|)} - \rho(2L(t)) + \boxed{\rho(2L_K|e(\phi(t))|)}$$

$\times\theta \in (0, 1)$

Event-Triggered Law

Proposition: Bound on $\dot{V}$

If $e(t) = p(t_k) - p(t)$ is the **prediction error**,

$$\dot{V}(t) \leq \boxed{-\gamma(|x(t)|)} - \rho(2L(t)) + \boxed{\rho(2L_K|e(\phi(t))|)}$$

$\times\theta \in (0,1)$

Triggering Condition

$$\rho(2L_K|e(\phi(t))|) \leq \theta\gamma(|x(t)|) \Leftrightarrow |e(t)| \leq \frac{\rho^{-1}(\theta\gamma(|p(t)|))}{2L_K}, \qquad \theta \in (0,1)$$

Event-Triggered Law

Proposition: Bound on $\dot{V}$

If $e(t) = p(t_k) - p(t)$ is the **prediction error**,

$$\dot{V}(t) \leq \boxed{-\gamma(|x(t)|)} - \rho(2L(t)) + \boxed{\rho(2L_K|e(\phi(t))|)}$$

$\times\theta \in (0,1)$

Triggering Condition

$$\rho(2L_K|e(\phi(t))|) \leq \theta\gamma(|x(t)|) \Leftrightarrow |e(t)| \leq \frac{\rho^{-1}(\theta\gamma(|p(t)|))}{2L_K}, \qquad \theta \in (0,1)$$

$$\dot{V}(t) \leq -(1-\theta)\gamma(|x(t)|) - \rho(2L(t))$$

1. **Event-triggered stabilization:**

Corollary

There exists $\beta \in \mathcal{KL}$ s.t. for any $x(0) \in \mathbb{R}^n$ and bounded $\{u(t)\}_{t=\phi(0)}^{0}$,

$$|x(t)| + \sup_{\phi(t) \le \tau \le t} |u(\tau)| \le \beta\Big(|x(0)| + \sup_{\phi(0) \le \tau \le 0} |u(\tau)|, t\Big), \qquad t \ge 0$$

Satisfaction of Design Objectives

1. **Event-triggered stabilization:**

Corollary

There exists $\beta \in \mathcal{KL}$ s.t. for any $x(0) \in \mathbb{R}^n$ and bounded $\{u(t)\}_{t=\phi(0)}^0$,

$$|x(t)| + \sup_{\phi(t) \le \tau \le t} |u(\tau)| \le \beta\Big(|x(0)| + \sup_{\phi(0) \le \tau \le 0} |u(\tau)|, t\Big), \qquad t \ge 0$$

2. **No Zeno behavior:**

Proposition

- Solve $\dot{r} = M_2(1+r)(L_f(1+L_K) + L_f L_K r), \; r(0) = 0$
- Define $\delta = r^{-1}\left(\frac{1}{2L_{\gamma^{-1}\rho/\theta} L_K}\right)$

Then:

$$t_{k+1} - t_k \ge \delta, \qquad k \ge 1$$

Outline

$$\dot{x}(t) = f(x(t), u(\phi(t))) = Ax(t) + Bu(\phi(t))$$

$$\dot{x}(t) = f(x(t), u(\phi(t))) = Ax(t) + Bu(\phi(t))$$

✓ $K(x) = Kx$, globally Lipschitz
  - $L_K = |K|$

$$\dot{x}(t) = f(x(t), u(\phi(t))) = Ax(t) + Bu(\phi(t))$$

- ✓ $K(x) = Kx$, globally Lipschitz
  - $L_K = |K|$
- ✓ $S(x) = x^T P x$
  - $(A + BK)^T P + P(A + BK) = -Q, \qquad Q > 0$

$$\dot{x}(t) = f(x(t), u(\phi(t))) = Ax(t) + Bu(\phi(t))$$

✓ $K(x) = Kx$, globally Lipschitz
- $L_K = |K|$

✓ $S(x) = x^T Px$
- $(A + BK)^T P + P(A + BK) = -Q, \qquad Q > 0$

✓ Triggering condition: $|e(t)| \leq \frac{\lambda_{\min}(Q)\sqrt{\theta}}{4|PB||K|}|p(t)|, \qquad \theta > 0$

$$\dot{x}(t) = f(x(t), u(\phi(t))) = Ax(t) + Bu(\phi(t))$$

✓ $K(x) = Kx$, globally Lipschitz
- $L_K = |K|$

✓ $S(x) = x^T P x$
- $(A + BK)^T P + P(A + BK) = -Q, \qquad Q > 0$

✓ Triggering condition: $|e(t)| \leq \frac{\lambda_{\min}(Q)\sqrt{\theta}}{4|PB||K|}|p(t)|, \qquad \theta > 0$

✓ Closed-loop GES with rate $\mu = \frac{(2-\theta)\lambda_{\min}(Q)}{4\lambda_{\max}(P)}$

$$\dot{x}(t) = f(x(t), u(\phi(t))) = Ax(t) + Bu(\phi(t))$$

✓ $K(x) = Kx$, globally Lipschitz
- $L_K = |K|$

✓ $S(x) = x^T P x$
- $(A + BK)^T P + P(A + BK) = -Q, \qquad Q > 0$

✓ Triggering condition: $|e(t)| \leq \frac{\lambda_{\min}(Q)\sqrt{\theta}}{4|PB||K|}|p(t)|, \qquad \theta > 0$

✓ Closed-loop GES with rate $\mu = \frac{(2-\theta)\lambda_{\min}(Q)}{4\lambda_{\max}(P)}$

$$\dot{x}(t) = f(x(t), u(\phi(t))) = Ax(t) + Bu(\phi(t))$$

✓ $K(x) = Kx$, globally Lipschitz
- $L_K = |K|$

✓ $S(x) = x^T P x$
- $(A + BK)^T P + P(A + BK) = -Q, \qquad Q > 0$

✓ Triggering condition: $|e(t)| \leq \frac{\lambda_{\min}(Q)\sqrt{\boldsymbol{\theta}}}{4|PB||K|}|p(t)|, \qquad \theta > 0$

✓ Closed-loop GES with rate $\mu = \frac{(2-\boldsymbol{\theta})\lambda_{\min}(Q)}{4\lambda_{\max}(P)}$

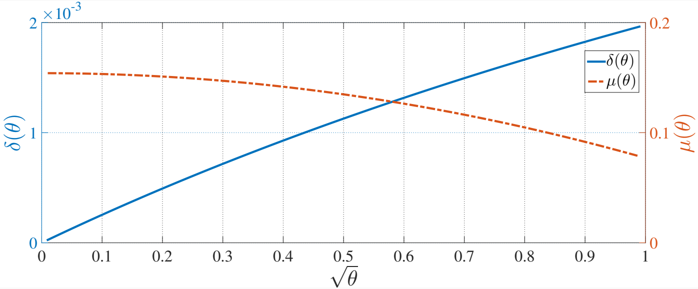
Question: How to balance communication cost ($\sim \delta$)
and convergence speed ($\sim \mu$)?

Using $Q = qI_n, P_1 = q^{-1}P$, we have a multi-objective optimization:

$$J_1(\theta) = \delta = \frac{1}{a-c} \ln \frac{c + \frac{\sqrt{\theta}}{|P_1 B||K|} a}{c + \frac{\sqrt{\theta}}{|P_1 B||K|} c}, \qquad J_2(\theta) = \mu = \frac{2-\theta}{4\lambda_{\max}(P_1)}$$

Using $Q = qI_n, P_1 = q^{-1}P$, we have a multi-objective optimization:

$$J_1(\theta) = \delta = \frac{1}{a-c} \ln \frac{c + \frac{\sqrt{\theta}}{|P_1 B||K|} a}{c + \frac{\sqrt{\theta}}{|P_1 B||K|} c}, \qquad J_2(\theta) = \mu = \frac{2-\theta}{4\lambda_{\max}(P_1)}$$

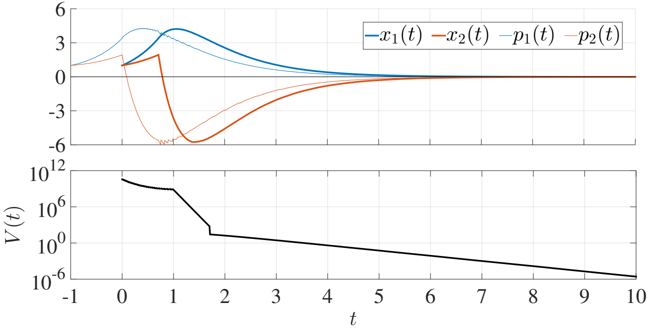


✓ The Pareto front is the entire domain $\theta \in [0, 1]$

Outline

1. Problem Statement
2. Event-Triggered Design and Analysis
   - Predictor Feedback
   - Event-Triggered Law
   - Convergence Analysis
3. The Linear Case
   - Communication-Convergence Trade-off
4. Numerical Results
   - Compliant Nonlinear System
   - Non-compliant Nonlinear System

$$f(x,u) = \begin{bmatrix} x_1 + x_2 \\ \tanh(x_1) + x_2 + u \end{bmatrix}, \qquad t - \phi(t) = D(t) = \frac{1+t}{1+2t}$$

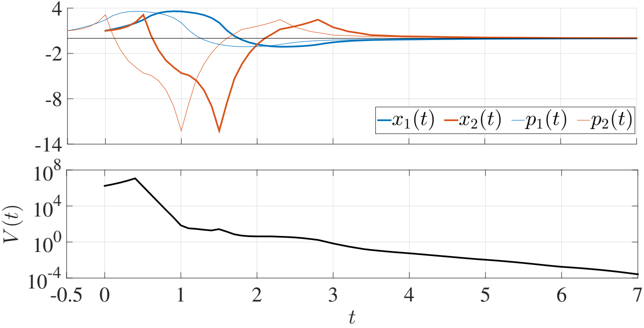


Triggering condition: $|e(t)| \leq \overline{\rho}|p(t)|$

✓ Analytically $\overline{\rho} \simeq 0.015$, but stability remains until $\overline{\rho} \simeq 0.9$

$$f(x,u) = \begin{bmatrix} x_1 + x_2 \\ x_2 + x_1^3 + u \end{bmatrix}, \qquad t - \phi(t) = D + a\sin(t)$$

✓ $D = 0.5$ is known but the perturbation magnitude $a = 0.05$ is not



Conclusions and Future Work

In this talk, we

- ✓ **designed** a predictor-based event-triggered GAS control law for **arbitrary, known time-varying** delays
- ✓ **uniformly lower bounded** the inter-event times
- ✓ **proved** GES in the linear case
- ✓ **analyzed** the communication-convergence trade-off for linear systems

Conclusions and Future Work

In this talk, we

- ✓ **designed** a predictor-based event-triggered GAS control law for **arbitrary, known time-varying** delays
- ✓ **uniformly lower bounded** the inter-event times
- ✓ **proved** GES in the linear case
- ✓ **analyzed** the communication-convergence trade-off for linear systems

Future work includes the extension of this approach to

- ? systems with **disturbances**
- ? systems with **unknown time delays**
- ? networked control scenarios with **multiple agents**

Questions and Comments