Event-Triggered Control Design with Performance Barrier

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Motivating Example

Example from [P. Tabuada 2007], a well-cited paper in ET control

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

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Event-triggered: Update u above when $\|e\| = \sigma \|x\|$, $e = x - x_k$

σ is a design parameter.
Lower $\rightarrow$ better **performance**
Higher $\rightarrow$ less trigger, **conserving resources**

Key Idea to Take Away from My Talk

- **Motivation:**
  1. Unclear on how to tune the design parameter to create a balance between trigger frequency and performance
  2. Standard ET design scheme can be inefficient in achieving desired performance
- **Assumption:**
  1. Desired performance can be achieved in continuous time
- **Approach:**
  1. Throw away the Lyapunov's criterion for stability, i.e. $\dot{V} \leq 0$
  2. Allow $\dot{V} > 0$
  3. Incorporate performance requirement into the trigger condition

Outline

Event-triggered control design overview:

- Linear system example
- Identify inefficiencies in satisfying a given desired performance

Our design:

- Incorporating performance requirement
  - Use barrier concept
- Advantages
- Apply our new design idea to distributed cases

Wrapping Up My Talk

- Simulations
- Conclusion and future ideas

Design Parameter

How might σ be picked?

From earlier example: Lyapunov function

$$V(x) = x^T \begin{bmatrix} 1 & 1/4 \\ 1/4 & 1 \end{bmatrix} x \implies \dot{V} \leq -0.44\|x\|^2 + 8\|e\|\|x\|$$

using ET control, $\dot{V} \leq -(0.44 - 8\sigma)\|x^2\|$, $\sigma = 0.05$ was picked, but why?

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We can reverse the process.
Given performance specification $S(t) \leq V(x_0)\exp(-rt)$, one can find σ

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  - $\dot{S} = -h(S), \; S(x_0, 0) \geq V(x_0)$ where h locally Lipschitz, class $\mathcal{K}$
  - Note: earlier, special case $\dot{S} = -rS$

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  - Note: earlier, special case $\dot{S} = -rS$
- Performance achievable in continuous time
  - $-\alpha(\|x\|) < -h(\bar{V}(x))$

Derivative based Trigger

Design idea: **forget σ**, make $\dot{V} \leq -h(V)$, then $V \leq S$ (Comparison Lemma)

Derivative-based ET

$$t_{k+1} = \min_{t} \left\{ t > t_k \mid g(x(t), e(t)) + h(\bar{V}(x(t))) = 0 \right\}$$

where $\mathcal{L}_f V(x) \leq g(x, e) \leq -\alpha(\|x\|) + \gamma(\|e\|)$

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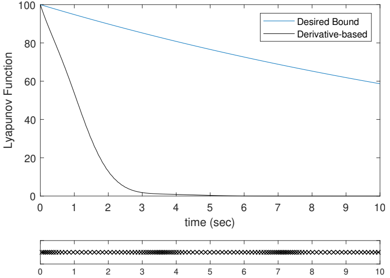

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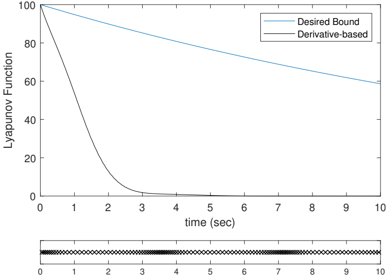

Problem? The trigger is too early. There is room for improvements.

Lyapunov Function Trigger

Design idea: just make $V \leq S$

Function-based ET

$$t_{k+1} = \left\{ t > t_k \mid S(x_0, t) - V(x(t)) = 0 \right\}$$

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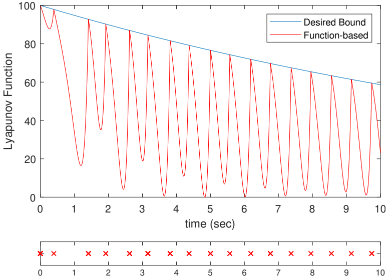
Straightforward. Performance immediately satisfied



Efficient, less triggers, but there is no **robustness** to time delay

Performance Barrier Design

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$$t_{k+1} = \min_t \left\{ t > t_k \mid \underbrace{g(x(t), e(t)) + h(\bar{V}(x(t)))}_{derivative-based} = \beta\big(S(x_0, t) - \bar{V}(x(t))\big) \right\}$$

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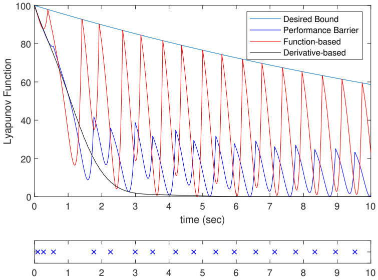
- It's like we set a **barrier** on V with S

Example

For the earlier example, update u when

$$\underbrace{-0.44\|x\|^2 + 8\|e\|\|x\|}_{g(x,e)} + \underbrace{0.032V(x)}_{h(V(x))} = \underbrace{10(V(x_0)\exp(-0.032t) - V(x))}_{\beta(S(x_0,t)-V(x))}$$



Maintain some level of **robustness** to time delay, not too inefficient in triggering

Advantages

Advantages of **performance barrier** design include:

- compared to **derivative-based**, guarantee higher minimum **interevent time**
  - because in each interval, derivative-based has to happen first
  - we provide the bound for the interevent time for linear case
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- flexibility for distributed implementation
  - can extend performance barrier design to distributed scenarios
  - some interesting things can happen... (future work)

Distributed Systems

For the distributed system

$$\dot{x}_i = f_i(x_{\mathcal{N}_i^2}, e_{\mathcal{N}_i}^{(i)})$$

Under the following **assumptions**:

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Distributed Performance Barrier Design

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Simulations

Consider a full-state controlled system

$$\dot{x} = \begin{bmatrix} 2 & 2 & 1 & 0 & 0 \\ -3 & -3 & 0 & 2 & 0 \\ 1 & 0 & -2 & 3 & 1 \\ 0 & 1 & 3 & -4 & 5 \\ 0 & 0 & 1 & -2 & 1 \end{bmatrix} x + u, \;\; x, u \in \mathbb{R}^5$$

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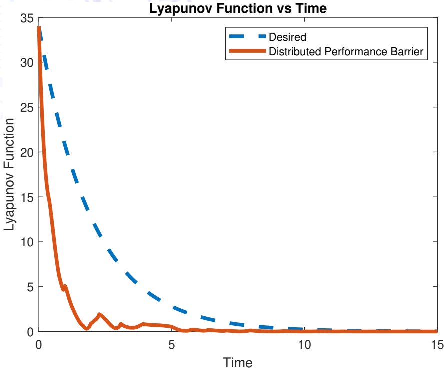
For continuous signal, cancel the off-diagonal

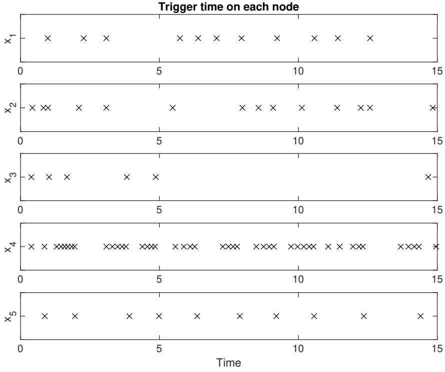
$$u = -\begin{bmatrix} 3 & 2 & 1 & 0 & 0 \\ -3 & 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 3 & 1 \\ 0 & 1 & 3 & 0 & 5 \\ 0 & 0 & 1 & -2 & 2 \end{bmatrix} x \implies \dot{x} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 \\ 0 & -3 & 0 & 0 & 0 \\ 0 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & -4 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} x$$

This satisfy $S(x_0, t) = V(x_0)\exp(-0.5t)$ with some margin.
But, using **derivative-based** design $\rightarrow$ **Zeno!**

Simulations (cont.)

Using **performance barrier** design,





No Zeno Behavior!

Conclusions

New event-triggered design schemes - **Performance Barrier**

- Increase minimum interevent time by relaxing condition on $\dot{V}$
- Maintain some level of robustness
- Interesting application possibility in distributed scenarios

Future Work

- Characterize increase in interevent time and tradeoff with robustness
- Explore the benefits in distributed settings
  1. When can performance barrier fix Zeno behavior?
  2. Can distributed system communicate the residual and collaborate?

Question?

Questions and feedback are welcome!

Extra Slide

Possible future work: **Rebalancing residuals** between nodes