

# On collective motion in sensor networks: sample problems and distributed algorithms

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**Abstract**—Adopting a tutorial approach, this paper surveys some control and systems theory problems that have recently gained interest in the context of multi-vehicle and sensor networks. By means of illustrative examples, we discuss some challenges in modeling of robotic networks, motion coordination algorithms, sensing and estimation tasks, and complexity of distributed algorithms.

## I. INTRODUCTION

Motion coordination is an extraordinary phenomenon in biological systems, such as schools of fish (see Fig. 1), as well as a remarkable tool for man-made groups of robotic vehicles and active sensors. Even though each individual agent has no global knowledge of the system, complex coordinated behaviors emerge from local interactions.

The objective of this paper is to present, in a tutorial spirit, some sample problems and solutions in the emerging discipline of motion coordination for robotic sensor networks. The key idea is that spatially-distributed sensing tasks, such as surveillance, search and monitoring, can be performed efficiently by robotic networks of sensors.

We begin by discussing models of robotic networks, i.e., groups of agents that can sense, communicate and take local control actions. We present basic notions of coordination tasks and time complexity in an attempt to provide a unifying modeling language for robotic networks. Next, we survey the state of the art on motion coordination by presenting some results on the design of coordination primitives, i.e., basic coordination skills for specific tasks such as deployment or pursuit. Remarkably, the problem of deploying a group of agents to form an arbitrary pattern using distributed decision-making and limited communication is in general an open problem. We emphasize here that the scope of the tools described in this paper is not limited to the problems mentioned within but can be applied to other tasks of similar nature. Indeed such an approach will be useful in the case where the same network of simple mobile agents is required to perform a variety of different motion coordination tasks.

A third focus of this paper is the use of controlled mobility in target and boundary tracking problems. Indeed, interesting and well-motivated coordination problems arise from practical applications where the robotic network is required to track mobile targets or to estimate environmental boundaries, as those exhibited by gas diffusion, heat radiation, or fluid spills. (A third problem is that of estimating environmental

fields such as deterministic functions of the environment, e.g., concentration of a pollutant in a lake, and probabilistic maps representing likelihood of events taking place in the environment, e.g., occupancy maps. We will not talk much about this subject.) By means of some example scenarios, we illustrate how to characterize optimal sensor placement or motion patterns, design distributed sensing schemes, and integrate them with motion coordination algorithms.

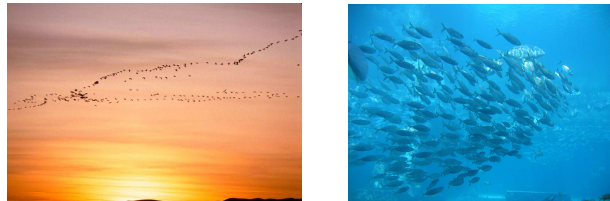


Fig. 1. Geese flying in formation (source: U.S. Fish & Wildlife Service <http://www.fws.gov/midwest/swanlake>) and a school of fish (location: IEEE CDC 2004 at Paradise Island, Bahamas).

Our examples only begin to shed light onto a large set of challenging control problems in which node mobility, communication, computation, and sensing aspects are jointly considered.

The paper is organized as follows. In Section 2, we discuss some models of multi-agent networks. In Section 3, we illustrate some interesting algorithms for basic tasks such as deployment and rendezvous. In Section 4, we focus on the use of mobility in tracking moving targets and boundaries. In all sections we present various sample networks, communication graphs, coordination tasks and algorithms; often we carefully discuss the information flow between agents, i.e., what information each agent is required to possess. Additionally, in each section we highlight some potentially interesting open problems.

## II. ROBOTIC NETWORKS AND COMPLEXITY

The global behavior of a robotic network can be seen as the sum of the local actions taken by its members. Each robot in the network can sense its immediate environment, communicate with its neighbors, process the information gathered and move according to it. The integrated capabilities together determine the behavior of each agent, which in turn impacts the overall collective response. This makes a robotic network a very versatile system and also a very complex one due to the confluence of processing, communication and sensing aspects.

In order to understand the trade-offs between performance, reliability of algorithms and their costs (energy, time, communication, etc), it seems appropriate to propose a common

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modeling framework where the execution of different coordination algorithms can be appropriately formalized, analyzed and compared.

Since this is an important topic, we briefly present in this section some of the concepts that we think should be present in such a model. We do not attempt here to present this model in its full generality, but rather give a flavor of this research avenue. For a more detailed discussion, we refer the reader to [1]. Other models proposed elsewhere in a similar spirit include [2], [3], [4].

We consider *uniform networks of robotic agents* (or *robotic networks*) defined by a tuple  $\mathcal{S} = (I, \mathcal{A}, E_{\text{cmm}})$  consisting of

- (i)  $I = \{1, \dots, N\}$ ; a set of unique identifiers (UIDs);
- (ii)  $\mathcal{A} = \{A^{[i]}\}_{i \in I}$ , with  $A^{[i]} = (X, U, X_0, f)$  (Here,  $X$  is the *state space*,  $U$  is the *input space*,  $X_0$  is the *set of allowable initial states* and  $f$  is a  $C^\infty$  map with domain  $X \times U$ ), a set of identical control systems; this set is called the *set of physical agents*;
- (iii)  $E_{\text{cmm}}$  a map from  $\prod_{i \in I} X$  to the subsets of  $I \times I \setminus \text{diag}(I \times I)$ ; this map is called the *communication edge map*.

The existence of an edge between two nodes in  $E_{\text{cmm}}$  is equivalent to the ability of the corresponding two agents to exchange messages.

Next, a (*synchronous, dynamic*) *control and communication law* for  $\mathcal{S}$  consists of the sets:

- (i)  $\mathbb{T} = \{t_\ell\}_{\ell \in \mathbb{N}_0} \subset \bar{\mathbb{R}}_+$ , an increasing sequence of time instants, called *communication schedule*;
- (ii)  $L$ , a set containing the null element, called the *communication language*; elements of  $L$  are called *messages*;
- (iii)  $W$ , sets of values of some *logic variables*  $w^{[i]} \in W$ ,  $i \in I$ . These sets correspond to the capability of agents to allocate additional variables and store sensor or communication data;
- (iv)  $W_0 \subseteq W$ , subsets of *allowable initial values*;

and the maps:

- (i)  $\text{msg}: \mathbb{T} \times X \times W \times I \rightarrow L$ ,  $i \in I$ , called *message-generation function*;
- (ii)  $\text{stf}: \mathbb{T} \times W \times L^N \rightarrow W$ , called *state-transition function*;
- (iii)  $\text{ctl}: \bar{\mathbb{R}}_+ \times X \times X \times W \times L^N \rightarrow U$ , called *control function*.

By means of a control and communication law, each agent performs the following sequence or cycle of actions. At each instant  $t_\ell \in \mathbb{T}$ , agent  $i$  sends to agent  $j$  a message computed by applying the message-generation function to the current values of  $t_\ell$ ,  $x^{[i]}$  and  $w^{[i]}$ . After a negligible period of time (therefore, still at  $t_\ell \in \mathbb{T}$ ), agent  $i$  resets the value of its logic variables  $w^{[i]}$  by applying the state-transition function to the current value of  $w^{[i]}$ , and to the messages  $y^{[i]}(t_\ell)$  received at  $t_\ell$ . Between communication instants, i.e., for  $t \in [t_\ell, t_{\ell+1})$ , agent  $i$  applies a control action computed by applying the control function to  $x^{[i]}(t_\ell)$ , the current values of  $x^{[i]}$  and  $w^{[i]}$ , and to the messages  $y^{[i]}(t_\ell)$  received at  $t_\ell$ .

Let us present some brief comments. In our present definition, all agents are identical and implement the same algorithm; in this sense the control and communication law is called *uniform* (or anonymous). If  $W = W_0 = \emptyset$ , then the

control and communication law is *static* (or memoryless) and no state-transition function is defined. It is also possible for a law to be *time-independent* if the three relevant maps do not depend on time. In most uniform control and communication laws, the messages interchanged among the network agents are (quantized representations of) the agents' states. In what follows we focus on the static time-independent case.

In order to analyze the performance of a motion coordination algorithm, we need to establish the notion of coordination task, and of task achievement by a robotic network. A (*static*) *coordination task* for a network  $\mathcal{S}$  will be a map  $\mathcal{T}: \prod_{i \in I} X^{[i]} \rightarrow \{\text{true}, \text{false}\}$ . Additionally, let  $\mathcal{CC}$  be a motion coordination algorithm for  $\mathcal{S}$ . We say that  $\mathcal{CC}$  achieves the task  $\mathcal{T}$  if for all initial conditions  $x_0^{[i]} \in X_0$ , the corresponding network evolution  $t \mapsto x(t)$  has the property that there exists  $T \in \mathbb{R}_+$  such that  $\mathcal{T}(x(t)) = \text{true}$  for all  $t \geq T$ .

In some situations achieving a task efficiently means stabilizing the system. In other situations efficiency might be measured by required communication/control energy or by speed of completion. For the latter, we can establish the following notions of time complexity.

- (i) The *time complexity to achieve  $\mathcal{T}$  with  $\mathcal{CC}$  from  $x_0 \in \prod_{i \in I} X_0^{[i]}$*  is

$$\text{TC}(\mathcal{T}, \mathcal{CC}, x_0) = \inf \{ \ell \mid \mathcal{T}(x(t_k)) = \text{true}, \forall k \geq \ell \},$$

where  $t \mapsto (x(t))$  is the evolution of  $(\mathcal{S}, \mathcal{CC})$  from  $x_0$ .

- (ii) The *time complexity to achieve  $\mathcal{T}$  with  $\mathcal{CC}$*  is

$$\text{TC}(\mathcal{T}, \mathcal{CC}) = \sup \left\{ \text{TC}(\mathcal{T}, \mathcal{CC}, x_0) \mid x_0 \in \prod_{i \in I} X_0^{[i]} \right\}.$$

- (iii) The *time complexity of  $\mathcal{T}$*  is

$$\text{TC}(\mathcal{T}) = \inf \{ \text{TC}(\mathcal{T}, \mathcal{CC}) \mid \mathcal{CC} \text{ achieves } \mathcal{T} \}.$$

Another important notion is that of communication complexity, that, roughly speaking represents the overall number of messages exchanged to complete a coordination task. In the following sections, we will describe certain coordination algorithms, some of which have been cast into this modeling framework and their complexity properties analyzed; see [1]. In the interest of space and to preserve the tutorial flavor of the paper, we will not model the algorithms in this framework here but will only provide an informal description of them. We will, however, state their complexity properties whenever possible.

### III. MOTION COORDINATION

Loosely speaking, by a motion coordination problem we mean any task where the network objective can be captured by the final spatial configuration of its agents and/or of their velocity vectors. Key problems include flocking [5], foraging [6], [7], rendezvous [8], [9], cyclic pursuit [10], coverage [11], [12], cooperative search [13], and formation control [14], [15]. Heuristic approaches to the design of emerging behaviors have been investigated within the literature on behavior-based robotics; see [16], [17], [18], [19].

Lately there been a systematic effort to design scalable and efficient algorithms; see [5], [11], [9].

Our method of approaching motion coordination problems exploits their inherent geometric [20], [21], graph-theoretical [22], and optimization [23] structure. The sensing capabilities of the agents are captured through geometric models; the information flow/neighborhood relationship of the agents is represented by appropriate graphs; and the network objective is characterized via appropriate utility functions. Algorithms are then designed via gradient/greedy methods. We illustrate our approach by discussing two basic types of problems: deployment and rendezvous.

### A. Deployment problems

First, we consider the *area-coverage deployment* problem in a convex polygonal environment. The objective is to maximize the area within close range of the mobile nodes. This models a scenario in which the nodes take local measurements. Assume that certain regions in the environment are more important than others and describe this by a density function  $\phi$ . Our recent work [11], [24] shows how this problem leads to the coverage performance metric  $\mathcal{H}(p_1, \dots, p_N) = \sum_{i=1}^N \int_{V_i} f(\|q - p_i\|) \phi(q) dq$ . Here  $p_i$  is the position of the  $i$ th node,  $f$  measures the performance of an individual sensor, and  $\{V_1, \dots, V_N\}$  is the Voronoi partition of the nodes  $\{p_1, \dots, p_N\}$ . If we assume that each node obeys a first order dynamical behavior, then a simple gradient scheme can be easily implemented in a spatially-distributed manner. Because the closed-loop system is a gradient flow for the cost function  $\mathcal{H}$ , performance is locally, continuously optimized. Fig. 2 illustrates the performance of this coordination algorithm. As a special case, when the environment is a segment and  $\phi = 1$ , the time complexity of the algorithm can be shown to be  $O(N^3 \log(N\epsilon^{-1}))$  where  $\epsilon$  is a threshold value below which we consider the task accomplished; see [1].

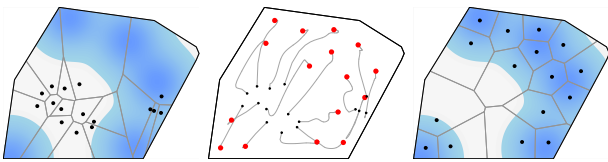


Fig. 2. Area-coverage deployment for 16 agents on a convex polygonal environment. The shaded region represents the density function  $\phi$ . The left (respectively, right) figure illustrates the initial (respectively, final) locations and Voronoi partition. The central figure illustrates the gradient ascent flow.

Second, we consider the problem of deploying to *maximize the likelihood of detecting a source*. For example, consider devices equipped with acoustic sensors attempting to detect a sound-source (or similarly, antennas detecting RF signals, or chemical sensors localizing a pollutant source). For a variety of criteria, when the source emits a known signal and the noise is Gaussian, we know that (1) the optimal detection algorithm involves a matched filter, (2) detection performance is a function of signal-to-noise-ratio, and, in turn, (3) signal-to-noise ratio is inversely proportional to the sensor-source distance. How do we deploy the nodes and maximize the detection probability? We design a motion coordination

algorithm to maximize detection likelihood as follows: each node moves toward the circumcenter<sup>1</sup> of its Voronoi cell. Our work [25] shows that (1) the detection likelihood is inversely proportional to the circumradius of each node's Voronoi cell, and (2) if the nodes follow this algorithm, then the detection likelihood increases monotonically as a function of time; see Fig. 3. (This algorithm is designed for the detection problem; source localization/tracking is discussed in the next section.)

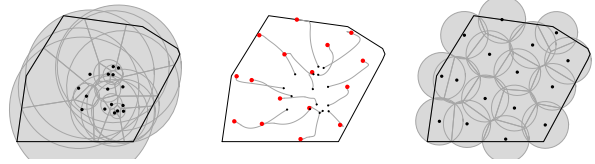


Fig. 3. The nodes solve a maximum-likelihood-detection deployment; the figure depicts the nodes final position and the circumcircles for each node.

Third, we consider a *visibility-based deployment* of nodes in a planar non-convex polygonal environment. Here, the coverage objective is to deploy the ad hoc network in such a way as to obtain complete visibility of the environment. This coverage problem is a distributed feedback version of the so-called “art gallery problem” which is a classic topic in computational geometry [26], [27]. Let us now describe an algorithm for this type of deployment. At every time instant, each node  $p_i$  computes a dominance region as the set of points for which  $p_i$  is either the only visible node or the closest visible node;  $p_i$  then moves toward the furthest vertex in its dominance region. The performance of this algorithm is, at this time, known only via simulations on a class of floor plan environments; e.g., see Fig. 4.

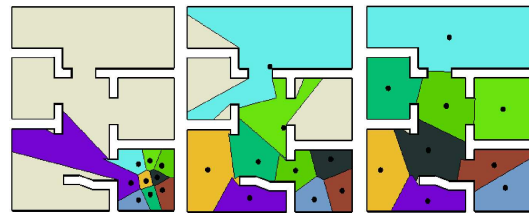


Fig. 4. The nodes solve the visibility deployment problem for a nonconvex polygonal environment shaped as a typical floor-plan.

### B. Rendezvous problems

In the context of motion coordination, the rendezvous objective is to achieve agreement over the location of the agents, that is, to steer each agent to a common location. We consider two scenarios which differ in the agents' sensing/communication capabilities and the environment to which the agents belong. Let  $P = \{p_1, \dots, p_N\}$  represent the set of locations of the agents.

Let us first consider the problem of rendezvous for agents equipped with *range-limited sensors*. In this case, each agent

<sup>1</sup>The circumcircle of a polygon is the smallest circle enclosing the polygon; circumradius and circumcenter are radius and center of the circumcircle, respectively.

is capable of sensing in a closed disk of bounded radius and belongs to the unbounded space  $\mathbb{R}^d$  of arbitrary dimension  $d$ . This is described by the  $r$ -disk graph,  $\mathcal{G}_{\text{disk}}(r)$ , in which two agents are neighbors if and only if the Euclidean distance between them is less than or equal to  $r$ . For a complete discussion of this problem, see [28].

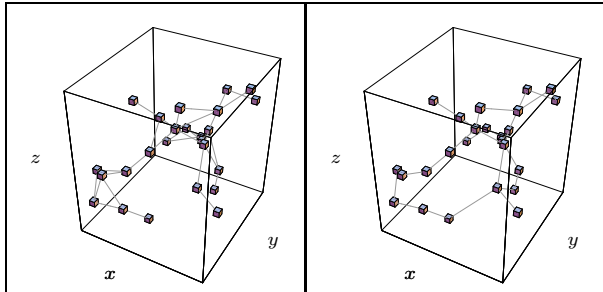


Fig. 5. The  $r$ -disk and Relative Neighborhood ( $\mathcal{G}_{\text{RN}}(r)$ ) graphs in  $\mathbb{R}^3$  for a configuration of 25 agents with coordinates uniformly randomly generated within the cube  $[-7, 7] \times [-7, 7] \times [-7, 7]$ . The parameter  $r$  is taken equal to 4. See [28] for a discussion on proximity graphs.

Second, we consider *visually-guided agents*. Here the agents are assumed to belong to a nonconvex simple polygonal environment  $Q$ . Each agent can sense within line-of sight any other agent as well as sense the distance to the boundary of the environment. The relationship between the agents can be characterized by the visibility graph,  $\mathcal{G}_{\text{vis},Q}$ . Two agents are neighbors if they are mutually visible to each other. In this case, we also assume that the evolution of the network occurs in a compact subset of the environment not containing the reflex vertices; see [29] for a complete discussion.

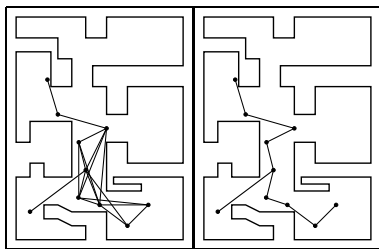


Fig. 6. From left to right, visibility graph and the locally-cliqueless visibility graph. The locally-cliqueless visibility graph is spatially distributed over the visibility graph (see [29]).

In both scenarios, it is impossible to solve the rendezvous problem with distributed information if the agents are placed in such a way that they do not form a connected communication or sensing graph. Arguably, a good property of any algorithm for rendezvous is that of maintaining some form of connectivity between agents, which in turn imposes constraints on the motion of the agents. Motion constraints that maintain connectivity are designed in [8], [29] and exploit the geometric properties of proximity graphs. For example, it is understood now that motion constraints need not be imposed between any pair of neighbors and that instead it is sufficient to impose constraints according to certain sparse proximity graphs. For the disk graph scenario, an appropriate

graph is the so-called Relative Neighborhood graph depicted in Fig. 5.

We are now ready to outline an algorithm that solves the problems for both communication scenarios. The agents execute what we shall refer to as the *Circumcenter Algorithm*; here is an informal description. Each agent iteratively performs the following tasks:

- 1: detects its neighbors according to  $\mathcal{G}$
- 2: computes the circumcenter of the point set comprised of its neighbors and of itself
- 3: moves toward this circumcenter while maintaining connectivity with its neighbors.

Fig. 7 and 8 illustrate the performance of the Circumcenter Algorithm for the first and second scenario, respectively. One can prove that, under technical conditions, the algorithm does achieve the rendezvous task in both scenarios. Additionally, when  $d = 1$ , it can be shown that the time complexity of the rendezvous task using the Circumcenter Algorithm is  $\Theta(N)$ ; see [1].

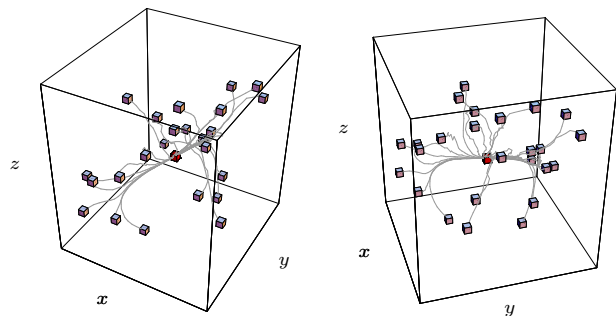


Fig. 7. Evolution (in light gray) of the Circumcenter Algorithm from two viewpoints. The initial configuration of the network is as in Fig. 5.

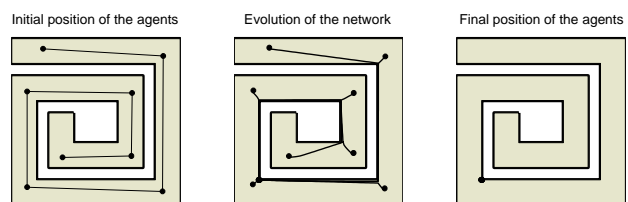


Fig. 8. Simulation results of the Circumcenter Algorithm on a network of agents distributed in a spiral polygon. The algorithm is run over  $\mathcal{G}_{\text{vis},Q}$ .

In this section, we have provided examples of certain motion coordination tasks and outlined approaches to solving the problems. However, many open research questions still remain unanswered. One such problem is that of achieving arbitrary patterns. The problem of deploying and controlling visually-guided agents is another problem where a deeper understanding is needed. Apart from motion coordination, another class of interesting problems for sensor networks is that of localizing and estimating moving targets and fields. We shall try to illustrate some of these problems in the following section.



#### IV. TARGET AND BOUNDARY TRACKING

The subject of this section is the design of algorithms that exploit controlled mobility to efficiently localize moving targets (or sources) and boundaries, and to efficiently estimate environmental fields (here we mean both functions of the environment, e.g., concentration of a pollutant in a lake, and probabilistic maps representing likelihood of events taking place in the environment, e.g., occupancy maps). Practical solutions to these estimation problems would play an important role in many scientific and public safety applications.

A possible approach to exploiting controlled mobility is based on a *next-best-view* paradigm. The key idea is to design greedy policies that move the network nodes in such a way as to maximize the information that the nodes will gather with subsequent measurements. Put into a broader perspective, an integrated algorithm entails an estimation filter and a motion coordination algorithm that takes the network agents to optimal sensor positions. Accordingly, a fundamental objective of this approach is to characterize optimal sensor placements or optimal sensor motion patterns for various estimation problems.

The literature on (static) sensor networks performing various estimation tasks is vast and we only mention the two survey papers [30], [31] that are somehow related to our approach. From a robotic viewpoint, an incomplete list of works on active target tracking for controlled-mobility networks includes [32] and [33]. Related to our next-best-view and optimal sensor placement approach is the literature on optimum experimental design. Here the references [34], [35] show how to define appropriate “sensitivity performance measure” for optimal sensor placement; see also [36]. Boundary estimation has been recently studied in the context of static sensor fields; e.g., see [37], [38], [39], [40]. Researchers in mobile robotics have explored alternative approaches for boundary estimation. In [41] Bertozzi et al. present a set of collective motion mechanisms based on energy-minimizing curves or “snakes” from image processing. Other related references include the gradient climbing algorithms in [42].

##### A. Target tracking

In this section we present an example approach to target tracking. For this problem, an appropriate sensitivity performance measure in 2D and 3D environments is the determinant of the Fisher Information Matrix (FIM). The determinant measures [43] the information produced by a set of measurements in estimating a set of unknown parameters; its inverse, called the Cramer-Rao-Lower-Bound, characterizes the best achievable estimation error covariance. Under the assumptions of Gaussian independent noise, a 2D environment, and a stationary sound-source, the global maxima of the FIM determinant correspond to an optimal pattern in which the sensors are uniformly placed in circular fashion around the target. We use this information to improve the performance of a Kalman filter-based algorithm for target localization. In short, we implement a motion coordination algorithm that steers the mobile sensor network to an optimal deployment; we do not detail this algorithm here, but note that it is related to the ones presented in the

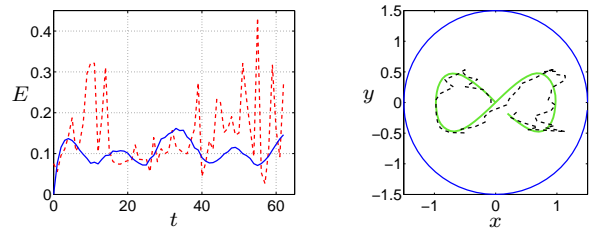


Fig. 9. Comparison of target localization algorithm for static and controlled mobile sensors. The left figure compares the estimation errors and the right figure shows the estimated target trajectories. The lower error and the more accurate “figure-eight” trajectory are achieved by the controlled mobile sensors. The mobile sensors move along the boundary of the environment which in this case is a circle as seen in the figure on the right.

previous section. A schematic description of the algorithm is as follows. Each agent iteratively performs the following tasks:

- 1: measures target location and shares new measurement with neighbors
- 2: computes new estimate of target location
- 3: moves according to motion algorithm (based on target estimate and neighbors’ positions).

Fig. 9, taken from [33], illustrates how this integrated motion/sensing/estimation algorithms lead to improved performance of an extended Kalman filter in a target tracking scenario where the target moves along a “figure-eight” pattern.

##### B. Boundary estimation

Here we consider a boundary estimation problem. The objective is to select a boundary interpolation technique and to deploy the sensors in such a way as to construct the best boundary estimate. In other words, we define a cost function quantifying an estimation error and then design a motion coordination algorithm that minimizes it. The details are as follows. Assume that the unknown set  $Q$  is the planar subset where a certain environmental quantity, e.g., heat or chemical concentration, is above a given threshold. The objective is to estimate the boundary  $\partial Q$  by means of an array of sensors able to locally detect  $\partial Q$  and to move towards and along it. Let us consider the following basic task: how to place the robots along  $\partial Q$  in such a way that the polygon, whose vertices are the robots’ positions, is a good approximation of  $Q$ . To simplify the following discussion, we assume that  $Q$  is convex. Therefore, our optimal estimation problem is equivalent to finding the “best”  $N$ -vertices polytope inscribed inside  $Q$  that best approximates  $Q$  according to some metric. This setup is interesting also because polygonal approximations of planar convex bodies is a well-studied subject, e.g., see the surveys [44], [45], [46]. It is known, for example, that the distance between the convex body  $Q$  and its best (as measured according to various metrics) inner polygonal approximation belongs to  $O(\frac{1}{\sqrt{N}})$ .

Let us formalize one of these error formulations. Once the robots reach the boundary we order them in counterclockwise order  $\{p_1, \dots, p_N\}$ ; for convenience, we set  $p_0 = p_N$  and

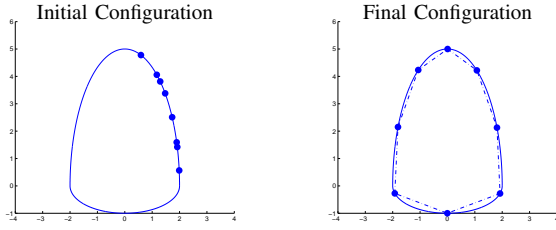


Fig. 10. Gradient flow maximizing inner approximating polygon for  $N = 8$ .

$p_{N+1} = p_1$ . Among the possible choices of metric we consider  $\mathcal{H}(g, g_I) := \int_a^b \|g(t) - g_I(t)\| dt$ , where  $g$  and  $g_I$  are parametric representations of the boundary of  $Q$  and of the interpolating lines between any two nodes, respectively. We regard  $\mathcal{H}$  as a cost function that we minimize through a motion coordination algorithm. It turns out that  $\mathcal{H}$  is the area of the convex set  $Q$  minus its inner approximating polygon. Thus

$$\begin{aligned} \min_{p_1, \dots, p_N \in \partial Q} \mathcal{H}(g, g_I) \\ = A(Q) - \max_{p_1, \dots, p_N \in \partial Q} A(\text{co}(p_1, \dots, p_N)), \end{aligned} \quad (1)$$

where  $A$  is area function, and  $\text{co}$  is the convex hull of its arguments. Since  $\text{co}(p_1, \dots, p_N)$  is a subset of  $Q$ , we know that  $\mathcal{H}$  is always non-negative. The area of the polygon  $\text{co}(p_1, \dots, p_N)$  is easily expressed as a function of position of the vertices, that is,  $A(\text{co}(p_1, \dots, p_N)) = \frac{1}{2} \sum_{k=1}^N (x_k y_{k+1} - x_{k+1} y_k)$ , where  $p_k = (x_k, y_k)$ . To maximize  $\mathcal{H}$  we consider the following gradient flow:

$$\begin{aligned} \dot{p}_i &= \text{proj}_{T\partial Q} \left( \frac{\partial A(\text{co}(p_1, \dots, p_N))}{\partial p_i} \right) \\ &= \frac{1}{2} \text{proj}_{T\partial Q} \begin{pmatrix} y_{i+1} - y_{i-1} \\ x_{i-1} - x_{i+1} \end{pmatrix}, \end{aligned}$$

where  $\text{proj}_{T\partial Q}$  is the orthogonal projection onto the tangent contour  $T\partial Q$ . (A nonsmooth gradient flow can be designed to handle nonsmooth contours.) Note that, in order to implement this gradient flow, every agent has only knowledge of the positions of its immediate clockwise and counter-clockwise neighbors and of the gradient of the contour at its position; as for the target tracking problem, this information requirements can be formalized using proximity graph models as in the previous sections. By design, the gradient flow is guaranteed to lead the robots to the set of critical configurations of  $\mathcal{H}$ ; it turns out that  $\mathcal{H}$  is not a strictly concave and it possesses multiple critical points.<sup>2</sup> Fig. 10 illustrates the gradient flow performance.

In this section, we have discussed two types of problems related to target tracking and localization for mobile sensor networks. However, numerous open questions still remain unanswered. For example, there is a need for appropriate "sensitivity performance measures" for next-best-view algorithms in target, boundary and function estimation. Also, the

<sup>2</sup>If  $Q$  is a regular  $N$ -polygon, then two equilibrium configurations of the area-maximization gradient flow consist of the  $N$  nodes placed either at the vertices or at the edges' midpoints of  $Q$ .

study of different sensor models, e.g., sensors with limited footprint, is another interesting area of research. Finally, in the problem of localization of a target that we considered in this paper, the agents require global knowledge of the target estimate. Solving the problem with local estimates of the target position is another challenging problem.

## V. CONCLUSIONS

This paper attempts to survey some control problems related to collective motion and estimation for sensor networks. Specifically, we have talked about deployment and rendezvous as examples of motion coordination tasks and target tracking and boundary estimation as examples of localization/estimation tasks. We have outlined possible ways to approach these problems and also mentioned some new directions of work in this area.

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