Motion Control Strategies for Improved Multi Robot Perception

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***Abstract*—**

This paper describes a strategy to select optimal motions of multi robot systems equipped with cameras in such a way that they can successively improve the observation of the environment. We present a solution designed for omni-directional cameras, although the results can be extended to conventional cameras. The key idea is the selection of a finite set of candidate next positions for every robot within their local landmark-based stochastic maps. In this way, the function cost measuring the perception improvement when a robot moves to a new position can be easily evaluated on the finite set of candidate positions. Then, the robots in the team can coordinate based on these small pieces of information. The proposed strategy is designed to be integrated with a map merging algorithm where robots fuse their maps to get a more precise knowledge of the environment. The interest of the proposed strategy for uncertainty reduction is that it is suitable for visual sensing, allows an efficient information exchange, presents a low computational cost and makes the robot coordination easier.

Keywords: Multi-robot applications. Perception. Robot coordination. Autonomous exploration. Coverage control.

I. INTRODUCTION

The interest of having groups of robots working cooperatively has rapidly increased. There are many advantages in the use of multi-robot systems. Many tasks can not be carried out by robots working alone and the robustness of the global set is higher when working in coordination. One interesting task that can be better achieved using a robotic team is the perception of the environment. Here, the robots in the network are equipped with sensors and they usually communicate their observations to the other team members to acquire a better knowledge of the scene. In these scenarios it would be interesting to make the best motions to optimize the perception of the scene. In this paper, we approach the problem of guiding a robotic team to positions where the environment is better observed for the case that robots are equipped with vision sensors. Robots combine this information and build a local landmark-based stochastic map of the environment. Robots in the team have communication capabilities to exchange their local maps and build a global representation of the environment. However, the construction of this global map may require long communication and computation times. Therefore, it would be interesting that the robots could make decisions based on their local data and small pieces of information received from the other nodes. We will focus on strategies that improve the local maps taking into account the proposed improvements of other robots, giving rise to an improvement of the global map. This problem is close to works on optimization of robot locations, see [1], [2], [3], or *coverage* problems [4], [5], where the objective is to optimally place a group of robots in an environment of interest to achieve maximum coverage. Specifically, it is highly related to *exploration guided by information* and *active sensing*.

Many of the existing solutions for exploration and active sensing are based on occupancy grid maps. Here, frontier cells dividing between already explored and unknown sections can be easily detected. Then, robots can evaluate a cost function on this small subset of destinations and make decisions propagating small pieces of information with the other robots. Some examples of these approach are [6], [7], [8] for the single robot case and [9] for multi robot systems. However, the exploration problem turns out to be more complicated for landmark-based maps, since the number of candidate destinations is infinite.

An alternative to the use of sets of candidate next positions are global optimization methods [10], [11], [12], where robots search for the best position to reduce the whole map uncertainty. Every robot makes decisions based on its current local estimate of the global map and propagates its observations to the other nodes so that they can update their maps. These approaches offer weak robot coordination, which is achieved via an efficient information exchange between the robots. In order to achieve a proper coordination, every robot in the team must have an up to date global map estimate before deciding its motion. Otherwise, different robots may end exploring exactly the same regions. In addition, many of these solutions use gradient methods to find minima on the cost function. Gradient algorithms are computationally expensive since the gradient must be reevaluated at every step. Besides, they may find local minima, and the step size adjustment is complicated.

Exploration and active sensing solutions may also be divided between one step decisions and path planning methods. Most of the previously mentioned works are one step approaches, where robots compute the reduction of the cost function considering exclusively the next robot motion. Approaches based on path planning or trajectory optimization [13], [14] use a larger time horizon and consider the cost function for multiple successive robot motions. Although these strategies may be suitable for a single robot, they present important scaling problems for the multi robot case.

In this paper, we define a method for landmark-based maps which is close to a frontier exploration in the sense that we are able to select a finite set of destinations for every robot.

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Associated to these positions, we are able to compute a set of cost values that can be sent to other robots in the team with the future aim of negotiating their next motions. This solution presents multiple appealing features due to its low computation complexity, and to the fact that the robots do not need to wait for having a good global map estimate. Instead, they can negotiate on small pieces of information, ensuring that the resulting global map will be improved. Besides, it presents the benefit that it is based on well known probabilistic formulation as well as results from the vision literature. Along this paper, we will describe how the *vantage locations* are selected, we will define the cost function, we will provide a general overview of the robotic coordination, and we will show in experiments how this strategy effectively leads to the improvement of the global map.

This paper is organized as follows. Section II explains how the robots can compute a prediction of the expected map resulting of placing the robot into a vantage location in the environment and sensing the scene. In Section III we explain how robots compute these vantage locations for observing features in the map. Then, in Section IV we present the strategy for improved perception, explaining both the global and individual cost function to be minimized. Finally, in Section V we show the performance of the strategy via simulation.

II. EXPECTED MAPS FOR THE VANTAGE LOCATIONS

Every robot in the team has a local map estimate of the environment. When the robots move to a new location and take new measurements of the environment, its map estimate may be improved, exhibiting a higher precision. Along this section, we will explain how to compute the expected map resulting of moving the robot to a candidate destination. We apply the same algorithm used by the robot to build the current map. It is a SLAM (*Simultaneous Localization and Map Building*) algorithm for bearing only data based on an EKF (*Extended Kalman Filter*). We compute the expected map using the same measurement and odometry models, and assuming that:

- The measurements are exactly the expected bearings from the new robot pose to the features,
- The odometry estimate is exactly the new robot pose.

Let $\hat{\mathbf{x}}_r(k) = (\hat{x}_k, \hat{y}_k, \hat{\theta}_k)^T$ be the robot pose at time k and $\hat{\mathbf{x}}(k) = (\hat{\mathbf{x}}_r(k)^T, \hat{x}_1(k), \hat{y}_1(k), \cdots, \hat{x}_m(k), \hat{y}_m(k))^T$ the local map estimate at time k, with associated covariance $\mathbf{P}(k)$. Let $\mathbf{x}_g = (x_g, y_g, \theta_g)^T$ be the goal vantage location where the robot plans to move to.

A. Relative Motion Computation

The relative translation and rotation $\mathbf{x}_{k+1}^k$ between $\hat{\mathbf{x}}_r(k)$ and $\mathbf{x}_g$ can be computed as:

$$\mathbf{x}_{k+1}^k = (\ominus \hat{\mathbf{x}}_r(k)) \oplus \mathbf{x}_g, \quad (1)$$

where the operator $\ominus$ is the inverse location vector. When applied to $\hat{\mathbf{x}}_r(k)$, it returns a location vector that transforms coordinates from the world frame reference into the robot $\hat{\mathbf{x}}_r(k)$ frame:

$$\ominus \hat{\mathbf{x}}_r(k) = \begin{bmatrix} -\hat{x}_k \cos\hat{\theta}_k - \hat{y}_k \sin\hat{\theta}_k \\ \hat{x}_k \sin\hat{\theta}_k - \hat{y}_k \cos\hat{\theta}_k \\ -\hat{\theta}_k \end{bmatrix}. \quad (2)$$

The operator $\oplus$ is the composition of two location vectors. It returns a location vector that transforms coordinates between the reference frames $\hat{\mathbf{x}}_r(k)$ and $\mathbf{x}_g$. Then, the expression for the relative transformation $\mathbf{x}_{k+1}^k$ between the robot pose at time k and the goal pose at $k+1$ is:

$$\mathbf{x}_{k+1}^k = \begin{bmatrix} (x_g - \hat{x}_k)\cos\hat{\theta}_k + (y_g - \hat{y}_k)\sin\hat{\theta}_k \\ -(x_g - \hat{x}_k)\sin\hat{\theta}_k + (y_g - \hat{y}_k)\cos\hat{\theta}_k \\ \theta_g - \hat{\theta}_k \end{bmatrix}. \quad (3)$$

B. EKF Prediction

The prediction step of the localization and mapping algorithm gives $\bar{\mathbf{x}}(k+1) = \mathbf{x}(k+1|k) = (\bar{\mathbf{x}}_r(k+1)^T, \bar{x}_1(k+1), \bar{y}_1(k+1), \ldots, \bar{x}_m(k+1), \bar{y}_m(k+1))^T$ and covariance $\mathbf{P}(k+1|k)$ based on the previous state $\hat{\mathbf{x}}(k)$ and covariance $\mathbf{P}(k)$ and the odometry measurements $\mathbf{x}_{k+1}^k = (x_{odom}, y_{odom}, \theta_{odom})$ with covariance matrix $\mathbf{P}_{odom}$. The odometry measurements $\mathbf{x}_{k+1}^k$ are given by (3), and the odometry noise is modelled as three independent noises in the perpendicular and parallel translations and rotation, $\mathbf{P}_{odom} = \mathbf{diag}\left(\sigma_x^2, \sigma_y^2, \sigma_\theta^2\right)$, where $\sigma_x = K_x d$ and $\sigma_y = K_y d$ are proportional to the translation distance $d = \sqrt{(x_g - \hat{x}_k)^2 + (y_g - \hat{y}_k)^2}$.

The equations used to predict the new state are

$$\begin{aligned} \bar{\mathbf{x}}(k+1) &= \begin{bmatrix} \hat{\mathbf{x}}_r(k) \oplus \mathbf{x}_{k+1}^k \\ \hat{x}_1(k) \\ \hat{y}_1(k) \\ \vdots \\ \hat{x}_m(k) \\ \hat{y}_m(k) \end{bmatrix}, \\ \mathbf{P}(k+1|k) &= \mathbf{J}_1 \mathbf{P}(k) \mathbf{J}_1^T + \mathbf{J}_2 \mathbf{P}_{odom} \mathbf{J}_2^T, \end{aligned} \quad (4)$$

where the operator $\oplus$ is the composition of the location vectors $\hat{\mathbf{x}}_r(k)$ and $\mathbf{x}_{k+1}^k$

$$\hat{\mathbf{x}}_r(k) \oplus \mathbf{x}_{k+1}^k = \begin{bmatrix} \hat{x}_k + x_{odom}\cos\hat{\theta}_k - y_{odom}\sin\hat{\theta}_k \\ \hat{y}_k + x_{odom}\sin\hat{\theta}_k + y_{odom}\cos\hat{\theta}_k \\ \hat{\theta}_k + \theta_{odom} \end{bmatrix}, \quad (5)$$

and $\mathbf{J}_1$, $\mathbf{J}_2$ are the Jacobians of the prediction operation relative to, respectively, the map and the odometry measurement:

$$\begin{aligned}
\mathbf{J}_1 &= \begin{bmatrix} \mathbf{j}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \\
\mathbf{J}_2 &= \begin{bmatrix} \mathbf{j}_2 \\ \mathbf{0} \end{bmatrix}, \\
\mathbf{j}_1 &= \begin{bmatrix} 1 & 0 & -x_{odom}\sin\hat{\theta}_k - y_{odom}\cos\hat{\theta}_k \\ 0 & 1 & x_{odom}\cos\hat{\theta}_k - y_{odom}\sin\hat{\theta}_k \\ 0 & 0 & 1 \end{bmatrix}, \\
\mathbf{j}_2 &= \begin{bmatrix} \cos\hat{\theta}_k & -\sin\hat{\theta}_k & 0 \\ \sin\hat{\theta}_k & \cos\hat{\theta}_k & 0 \\ 0 & 0 & 1 \end{bmatrix}.
\end{aligned} \tag{6}$$

C. Measurement Prediction

For every feature with coordinates $\bar{\mathbf{x}}_i(k+1) = (x_i, y_i)$ in the map, the expected measurement of the angle of the feature with respect to the robot that should be obtained from the current robot pose $\bar{\mathbf{x}}_r(k+1) = (\bar{x}_{k+1}, \bar{y}_{k+1}, \bar{\theta}_{k+1})^T$ is computed as:

$$h_i(\bar{\mathbf{x}}_r(k+1), \bar{\mathbf{x}}_i(k+1)) = \arctan\left(\frac{-(x_i-\bar{x}_{k+1})\sin\bar{\theta}_{k+1}+(y_i-\bar{y}_{k+1})\cos\bar{\theta}_{k+1}}{(x_i-\bar{x}_{k+1})\cos\bar{\theta}_{k+1}+(y_i-\bar{y}_{k+1})\sin\bar{\theta}_{k+1}}\right) \tag{7}$$

The Jacobian of this observation model relative to the map is:

$$\begin{aligned}
\mathbf{H}_i &= \begin{bmatrix} \frac{\partial \mathbf{h}_i}{\partial \bar{\mathbf{x}}_r(k+1)} & 0\cdots 0 & \frac{\partial \mathbf{h}_i}{\partial \bar{\mathbf{x}}_i(k+1)} & 0\cdots 0 \end{bmatrix}, \\
\frac{\partial \mathbf{h}_i}{\partial \bar{\mathbf{x}}_r(k+1)} &= \begin{bmatrix} \frac{\partial \mathbf{h}_i}{\partial \bar{x}_{k+1}} & \frac{\partial \mathbf{h}_i}{\partial \bar{y}_{k+1}} & \frac{\partial \mathbf{h}_i}{\partial \bar{\theta}_{k+1}} \end{bmatrix}, \\
\frac{\partial \mathbf{h}_i}{\partial \bar{\mathbf{x}}_i(k+1)} &= \begin{bmatrix} \frac{\partial \mathbf{h}_i}{\partial x_i} & \frac{\partial \mathbf{h}_i}{\partial y_i} \end{bmatrix},
\end{aligned} \tag{8}$$

where

$$\begin{aligned}
\frac{\partial \mathbf{h}_i}{\partial \bar{x}_{k+1}} &= -\frac{\bar{y}_{k+1}-y_i}{(\bar{x}_{k+1}-x_i)^2+(\bar{y}_{k+1}-y_i)^2}, \\
\frac{\partial \mathbf{h}_i}{\partial \bar{y}_{k+1}} &= \frac{\bar{x}_{k+1}-x_i}{(\bar{x}_{k+1}-x_i)^2+(\bar{y}_{k+1}-y_i)^2}, \\
\frac{\partial \mathbf{h}_i}{\partial \bar{\theta}_{k+1}} &= -1, \\
\frac{\partial \mathbf{h}_i}{\partial x_i} &= \frac{\bar{y}_{k+1}-y_i}{(\bar{x}_{k+1}-x_i)^2+(\bar{y}_{k+1}-y_i)^2}, \\
\frac{\partial \mathbf{h}_i}{\partial y_i} &= -\frac{\bar{x}_{k+1}-x_i}{(\bar{x}_{k+1}-x_i)^2+(\bar{y}_{k+1}-y_i)^2}.
\end{aligned} \tag{9}$$

Combining the information from all the features in the map, matrices $\mathbf{h}$ and $\mathbf{H}$ are constructed as

$$\mathbf{h} = \begin{bmatrix} h_1 \\ \vdots \\ h_n \end{bmatrix}, \quad \mathbf{H} = \begin{bmatrix} \mathbf{H}_1 \\ \vdots \\ \mathbf{H}_n \end{bmatrix}. \tag{10}$$

D. EKF Update

The final map estimate $\hat{\mathbf{x}}(k+1) = \mathbf{x}(k+1|k+1)$ and covariance $\mathbf{P}(k+1|k+1)$ are obtained as

$$\begin{aligned}
\hat{\mathbf{x}}(k+1) &= \bar{\mathbf{x}}(k+1) + \mathbf{K}\nu, \\
\mathbf{P}(k+1|k+1) &= (I - \mathbf{K}\mathbf{H})\mathbf{P}(k+1|k), \\
\nu &= \mathbf{z} - \mathbf{h}, \\
\mathbf{K} &= \mathbf{P}(k+1|k)\mathbf{H}^T\mathbf{S}^{-1}, \\
\mathbf{S} &= \mathbf{H}\mathbf{P}(k+1|k)\mathbf{H}^T + \mathbf{R},
\end{aligned} \tag{11}$$

where ν is the *innovation*, $\mathbf{S}$ in the *innovation covariance* and $\mathbf{K}$ is the *Kalman gain*. $\mathbf{z}$ is the vector with all the measurements of the features. Since the observations are taken equal to the predicted measurements, then $\mathbf{z} = \mathbf{h}$ and the innovation ν is zero. Therefore the state vector does not change: $\hat{\mathbf{x}}(k+1) = \bar{\mathbf{x}}(k+1)$. The matrix $\mathbf{R}$ is the covariance of the observation noise and is equal to $\sigma_z^2\mathbf{I}$, where σ_z is the standard deviation of the sensor noise.

E. Remarks

Once we have computed the final map estimate, the matrix $\mathbf{P}(k+1|k+1)$ will be used to compute the cost value described in Section IV. From equations (1)-(11) we can conclude that the robot orientation has no effect on the matrix $\mathbf{P}(k+1|k+1)$. The value of this matrix only depends on the new robot position (x_g, y_g), the previous map $\hat{\mathbf{x}}(k)$, its covariance $\mathbf{P}(k)$, and the odometry and observation noise models $\mathbf{P}_{odom}$ and $\mathbf{R}$.

III. SELECTION OF VANTAGE LOCATIONS

In order to manage a finite set of candidate next positions for every robot, we adopt a *one feature* improvement strategy. Every robot in the team will attempt to improve, at least, the observation of one of the landmarks. As a side effect, the observations of other landmarks are also improved. We define every vantage location as the optimal position for observing a landmark, which is computed as the robot position where the observation of the landmark produces a higher uncertainty reduction. In [15] authors discuss uncertainty minimization within the EKF framework. They show that the minimization of the map uncertainty $\mathbf{P}$ is highly related to the maximization of the covariance of the innovation $\mathbf{S}$. In addition, they show that if the system is driven to the optimal position to obtain maximum information gain, it results in numerically unstable update steps for the EKF. In particular, they show that the system becomes more unstable as the robot moves closer to a landmark. To derive the vantage locations to observe the landmarks, we will analyze the robot locations that lead to the maximization of $\mathbf{S}$.

For a single landmark i, the covariance of the innovation S_i is computed as, see (11):

$$S_i = \mathbf{H}_i\mathbf{P}(k+1|k)\mathbf{H}_i^T + \sigma_z^2. \tag{12}$$

If we take $\mathbf{P}(k+1|k) = I$ and apply (8), (9), we can express S_i as:

$$S_i = 1 + \sigma_z^2 + \frac{2}{r^2}, \tag{13}$$

where $r = \sqrt{(x_{k+1} - x_i)^2 + (y_{k+1} - y_i)^2}$ is the distance between the robot pose and the landmark. As it can be observed, the maximization of S_i is equivalent to the minimization of r. Now we introduce into the study the landmark covariance. We take

$$\mathbf{P}(k+1|k) = \left[\begin{array}{c|cc|c} \mathbf{I} & \multicolumn{2}{c|}{\mathbf{0}} & \mathbf{0} \\ \hline \mathbf{0} & P_{xx} & 0 & \mathbf{0} \\ & 0 & P_{yy} & \\ \hline \mathbf{0} & \multicolumn{2}{c|}{\mathbf{0}} & \mathbf{I} \end{array}\right], \tag{14}$$

and express $P_{yy} = kP_{xx}$, with $k > 1$. This models an uncertainty ellipse with its mayor axis perpendicular to the

y-axis. For robot poses at a constant distance r, $x_{k+1} = r\cos\alpha$, $y_{k+1} = r\sin\alpha$, the value of S_i is:

$$S_i = 1 + \sigma_z^2 + \frac{1+P_{xx}}{r^2} + (k-1)\frac{P_{xx}}{r^2}\cos^2(\alpha), \quad (15)$$

Computing the first and the second derivative of S_i, the critical points of S_i are $\alpha = 0 + n\pi, n \in \mathbb{Z}$ and $\alpha = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$. Besides we have that

$$\frac{\partial^2 S_i}{\partial \alpha^2} = 2(k-1)\frac{P_{xx}}{r^2}(1 - 2\cos^2(\alpha)). \quad (16)$$

Since $k > 1$, S_i reaches a maximum for $\alpha = 0 + n\pi, n \in \mathbb{Z}$ and a minimum for $\alpha = \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$. Then we can conclude that S_i is maximized when the distance between the robot and the feature is minimized, and that for constant distances, S_i is maximized when the landmark is observed from a position in a line perpendicular to its mayor axis. However, minimizing r may lead to a situation where the robot lays within the landmark covariance region. These situations are problematic since the predicted measurement cannot be correctly modelled by a gaussian, and instead, as robot approaches the landmark position, this distribution is more similar to a uniform distribution.

To avoid this situation, we compute the optimal position for the feature observation as a point in a line perpendicular to the mayor axis of its covariance ellipse which passes through its center, and at a distance from the center so that the angle α that passes from the extreme points of the ellipse is less than $\frac{\pi}{2}$. Notice that using this restriction, we prevent the robot from lying inside the ellipse even in the worst case where the ellipse is a circle, see Fig.1.

IV. STRATEGY FOR IMPROVED PERCEPTION

Our approach is based on the idea that, if a landmark is observed by one of the robots with high precision, the whole team will benefit of this information. As we mentioned before, this strategy for improved perception is designed to be integrated with a map merging algorithm [16] where robots fuse their local maps into a global map. This process may be executed at every time instant, not necessarily while they are deciding their motions.

Given n independent local maps characterized by a mean $\mathbf{x}_i$ and a covariance matrix $\mathbf{P}_i = \mathbf{P}_i(k+1|k+1)$ for $i \in \{1,\ldots,n\}$, the maximum-likelihood estimate $\mathbf{x}_{global}$ for the global map and its associated covariance matrix $\mathbf{P}_{global}$ are

$$\begin{aligned} \mathbf{x}_{global} &= \left(\sum_{i=1}^n A_i^T\, P_i^{-1}\, A_i\right)^{-1} \sum_{i=1}^n A_i^T\, P_i^{-1}\, x_i, \\ \mathbf{P}_{global} &= \left(\sum_{i=1}^n A_i^T\, P_i^{-1}\, A_i\right)^{-1}, \end{aligned} \quad (17)$$

where A_i are some observations matrices to allow the robots to observe only a portion of the total amount of features. From this expression, we can see that feature estimates with smaller covariances greatly influence their estimates in the global map. Therefore, a precise estimate of a feature can be obtained if, at least, one robot has observed it with high precision.

A. Aggregate Objective Function

We define our global cost function F to measure the best contributions for the estimate of every feature. We compile the individual costs associated the most precise feature estimates among the robots:

$$F(\mathbf{x}_1,\cdots,\mathbf{x}_n) = \sum_{j=1}^m \min_i f_{ij}(\mathbf{x}_i)), \quad (18)$$

where $\mathbf{x}_i$ is the next position of the robot i at time $k+1$, for $i \in \{1,\ldots,n\}$ and f_{ij} is the individual cost for the feature j in the local map of the robot i when the next position of the robot i is $\mathbf{x}_i$.

Every individual cost function f_{ij} measures the uncertainty of the sub-matrix within $\mathbf{P}_i$ associated to the feature j. There exist many metrics for measuring the uncertainty in a covariance matrix $\mathbf{P} = [p_{ij}]$, $i,j \in 1\{1,\ldots,m\}$ matrix. In [17] authors compare metrics based on its determinant, eigenvalues and trace, concluding that both of them performed properly. Here, we select the Trace (the sum of its diagonal elements) $\mathrm{Tr}(\mathbf{P}) = \sum_{i=1}^m p_{ii}$ due to the interesting property that it allows the decomposition of the cost function into the individual contributions of the features. Notice that if we name $\mathbf{P}$ the covariance matrix of a map (referring to only the feature estimates) and $\mathbf{P}_{jj}$ the submatrix with the covariance of feature j, then

$$\mathrm{Tr}(\mathbf{P}) = \sum_{j=1}^m \mathrm{Tr}(\mathbf{P}_{jj}). \quad (19)$$

Therefore, if we name $\mathbf{P}_i(\mathbf{x}_i)$ the covariance matrix associated to the map in robot i when it moves to the location $\mathbf{x}_i$, computed as explained in Section II, and we name $[\mathbf{P}_i(\mathbf{x}_i)]_{jj}$ the 2×2 sub-matrix associated to the feature j, the individual cost f_{ij} is

$$f_{ij}(\mathbf{x}_i)) = \mathrm{Tr}([\mathbf{P}_i(\mathbf{x}_i)]_{jj}). \quad (20)$$

B. Strategy

In the previous sections we have described how a robot can compute the optimal position for observing a landmark, and the procedure for predicting the resulting map when we place the robot in this position. We have also presented the global cost function to be minimized together with the individual cost functions associated to the features. In this section, we will explain how this information can be used to coordinate the robot team in order to improve the global knowledge of the environment.

The proposed strategy for motion coordination consists of an iterative algorithm where, at every time step, the team of robot performs next actions:

Optimal position for feature observation: Every robot i computes the best position for observing the K features with higher covariances in its local map, where $K \leq m_i$ for $i \in \{1,\ldots,n\}$ is adjusted depending on the performance requirements. As a result, robot $i \in \{1,\ldots,n\}$, obtains K next position candidates which we express as $\mathbf{x}_i^1,\cdots,\mathbf{x}_i^K$.

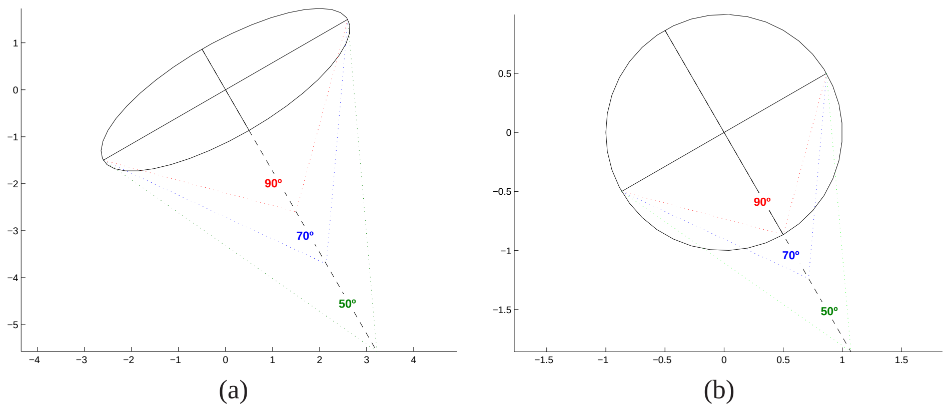

Fig. 1. **Optimal positions for feature observation**. We display the optimal positions for observing a feature for $\alpha = 90, 70, 50$ degrees, and for **(a)** an ellipse, and **(b)** a circle. As it can be seen, the optimal position remains outside the uncertainty ellipse associated to the feature as long as $\alpha < \frac{\pi}{2}$.

TABLE I
PREDICTED COSTS FOR THE CANDIDATE NEXT MOTIONS FOR ROBOT i

	feat 1	$\cdots$	feat m
$\mathbf{x}_i^1$	$f_{i1}(\mathbf{x}_i^1)$	$\cdots$	$f_{im}(\mathbf{x}_i^1)$
$\vdots$	$\vdots$		$\vdots$
$\mathbf{x}_i^K$	$f_{i1}(\mathbf{x}_i^K)$	$\cdots$	$f_{im}(\mathbf{x}_i^K)$

TABLE II
GLOBAL COST FOR A SPECIFIC SELECTION OF NEXT ROBOT POSES

	feat 1	$\cdots$	feat m
$\mathbf{x}_1^{l_1}$	$f_{11}(\mathbf{x}_1^{l_1})$	$\cdots$	$f_{1m}(\mathbf{x}_1^{l_1})$
$\vdots$	$\vdots$		$\vdots$
$\mathbf{x}_n^{l_n}$	$f_{n1}(\mathbf{x}_n^{l_n})$	$\cdots$	$f_{nm}(\mathbf{x}_n^{l_n})$
min	$\min_i f_{i1}(\mathbf{x}_i^{l_i})$	$\cdots$	$\min_i f_{im}(\mathbf{x}_i^{l_i})$

Map prediction: For all the candidate next positions $\mathbf{x}_i^1, \cdots, \mathbf{x}_i^K$, robot i computes the predicted map and evaluates the local cost function $f_{ij}(\mathbf{x}_i^l)$ for all the features, $l \in \{1, \ldots, K\}$, $j \in \{1, \ldots, m\}$, $i \in \{1, \ldots, n\}$. If an estimate of the feature j cannot be found in the local map of robot i, then we set $f_{ij}(\mathbf{x}_i^l) = \infty$ for all $l \in \{1, \ldots, K\}$. Every robot i can construct a table with the values of f_{ij} (see Table I).

Minimization of the global cost: Given a selected combination of next robot poses $\mathbf{x}_1^{l_1}, \cdots, \mathbf{x}_n^{l_n}$, its associated global cost is computed as $\sum_{j=1}^m \min_i f_{ij}(\mathbf{x}_i^{l_i}))$. This is equivalent to sum the values in the last row (*min*) in Table II. The best robot-vantage location assignment is the one minimizing the global cost F (see equation (18)). Every robot can compute this value, based on the information received from the other robots, and on its own data. This is a classical task-assignment problem where there are K^n possible combinations, and one of them produces the best cost. This kind of problems have been long studied and there exist multiple efficient suboptimal methods. We plan to study these methods in the future in order to select one that performs efficiently for this situation.

Motion and observation Once we have decided the best motions for the team, the robots move to these new positions, they observe the environment and update their local maps.

The reader may notice that we have not considered the map merging within the strategy. This is due to the fact that, as we mentioned before, it can be executed in parallel, or even after the robots finish the exploration.

V. RESULTS

In order to show the performance of the algorithm, a simulation has been carried out where a team composed by three robots explore an obstacle-free environment. They estimate their motions based on odometry information and sense the environment using an omnidirectional camera that provides bearing to the landmarks. In the experiments we use an observation noise $\sigma_z = 1$ degree and an odometry noise $\sigma_x = 0.01d$, $\sigma_y = 0.01d$, $\sigma_\theta = 2.5$ degrees. The translation noise is proportional to the travelled distance d, and the rotation noise is not really used in the algorithm since, as we mentioned before, for omnidirectional devices, the robot orientation does not affect the perception improvement. In this simulation, the robots process the odometry data and the measurements to construct their local maps using a SLAM algorithm for bearing-only data, see [18] for a detailed description. We initialize every local map with two robot poses to recover the position of some of the landmarks in the map. Their initial maps can be seen in Fig. 2(a). We display in black the ground-truth information, using points for the landmark positions, lines for the robot motions, and triangles for the robot poses. The maps and trajectories estimated by the robots are shown in different colors.

At every step, the robots compute their candidate next positions, and evaluate the cost function at these vantage points. In the experiments, we simulate a perfect robot coordination and we provide each robot with its best robot-candidate destination assignment. In Fig. 2(a,c,d) we display these selected motions and the resulting local maps for successive steps. In Fig. 2(b,d,e) we show their associated global map computed by (17).

In Fig. 2(a) we show the initial local maps of the robots. Since they have been constructed using two close robot poses, their landmark estimates present high uncertainties. Its associated global map can be found at Fig. 2(b). In this example, we can see the behavior we explained along this paper. If a feature has been observed by multiple robots, and at least one of them has estimated it with high precision, its estimate in the global map presents a small uncertainty. See for instance $F16$ which has been observed by the three robots; Robot 1 (blue) possess a very uncertain estimate, Robot 3 (red) has a better estimate, although still uncertain, and the estimate at Robot 2 (green) is very precise. After merging their maps, the final estimate for $F16$ is very precise. Therefore, it is desirable that the robots move to reduce the uncertainties of features which have not been precisely estimated by none of the robots, instead of reducing their local uncertainties.

In Fig. 2(c) we can observe the decisions made by the proposed algorithm and their effects on the global map Fig. 2(d). The robots in the team move to positions that optimize the global knowledge, and as a result, the global map (h) presents a high improvement. They do not exclusively try to reduce their local uncertainties, but instead take care of features uncovered by the team members. See for instance Robot 1 (blue). Its worse feature estimates in its previous local map (Fig. 2(a)), blue, are $F6$ and $F16$. He will consider moving to positions where both of them present a reduction on their uncertainties. However, exchanging the information with the other robots, it realizes that one of the team members already possess a precise estimate for $F16$, and therefore it moves to a extremely bad conditioned pose for the observation of $F16$, but well conditioned to observe other features. Besides, we can see that the robots tend to move to different regions in the environment, improving the coverage of all the features. In Fig. 2(e-f) we show the next step of the algorithm, where again the robots move to improve the scene perception. Since this global map has reached a high precision, next iterations of the algorithm add no significant improvements.

VI. CONCLUSIONS

Along this paper we have proposed a motion control strategy for improved perception of a scene capable of efficiently managing landmark-based maps. This strategy is designed to be integrated in a multi robot system, where robots use a map merging algorithm to fuse their maps and get a more precise knowledge of the environment. The described strategy is capable of selecting a finite set of candidate motions to the robots, and computing its associated cost in the form of the individual contributions of every feature. Therefore, this cost presents a space complexity linear on the map size. This information can be used by the team members to negotiate their next motions, presenting the benefit that robots do not need to wait for having a good global map estimate when they coordinate. In the experiments, we have seen that this approach offers good results in terms of the reduction of both the uncertainties in the local and global maps. Future extensions of this work are in the line of designing distributed coordination strategies for the robots, so that they can negotiate on the information provided by the described algorithm. Other interesting extensions are in the line of considering restricted robot motions, and environments with obstacles.

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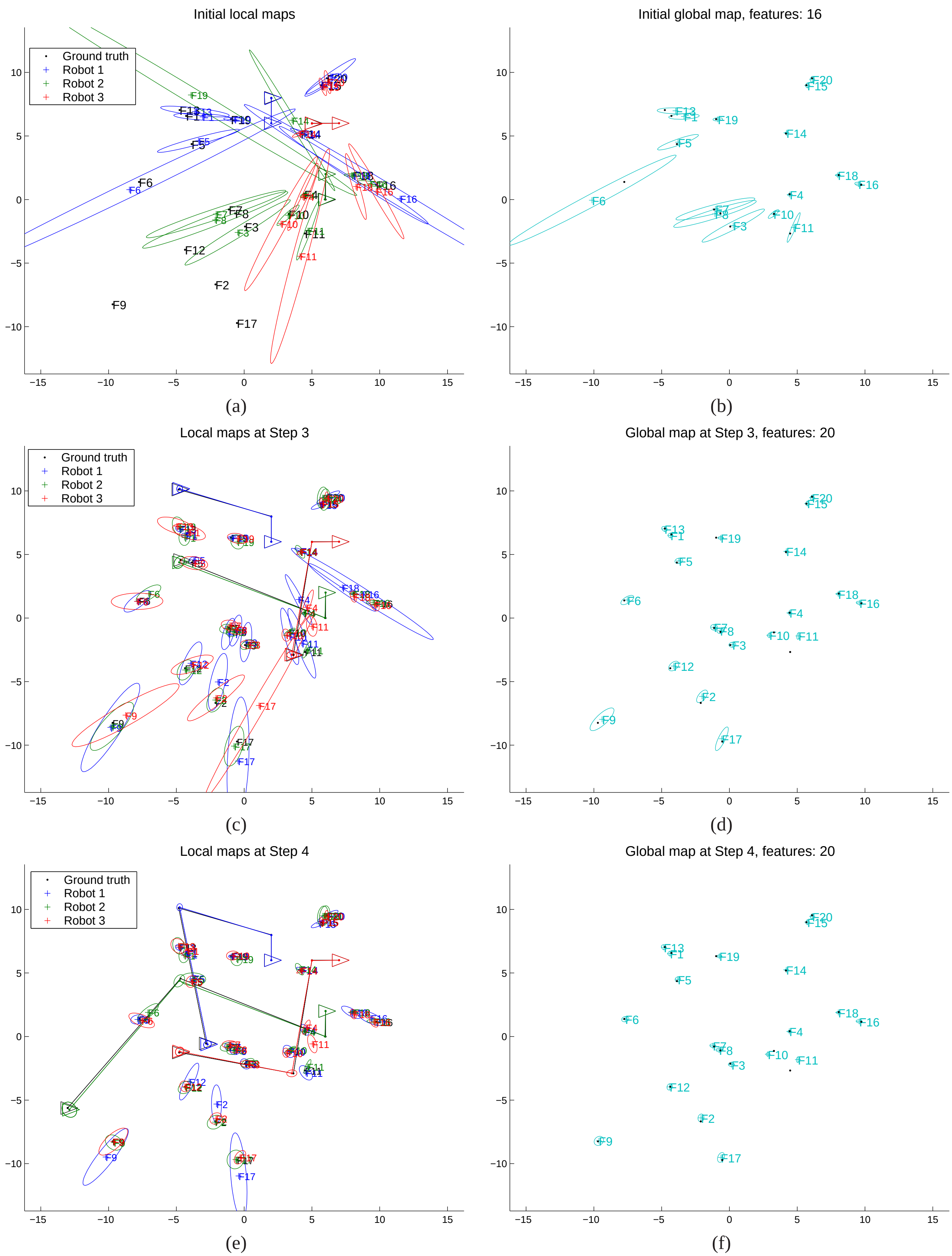

Fig. 2. **Strategy for improved perception**. We display some steps of the strategy for improved perception. Black dots are the ground-truth landmark positions. Local map estimates from different robots are showed in a different color. Figures at the right show the maximum-likelihood associated to the local maps. Notice that although the robots do not know this global map, their actions reduce its uncertainty. **(a)** Initial maps for robots 1,2,3. **(b)** Initial global map. **(c)** Maps for robots 1,2,3 after the first execution of the algorithm. **(d)** Global map for (c). **(e)** Maps for robots 1,2,3 after the second execution of the algorithm. **(f)** Global map for (e).

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