Coordinated rendezvous of underwater drifters in ocean internal waves

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Abstract— This paper considers a team of spatially dis-tributed drifters that move underwater under the influence of an ocean internal wave. The team's objective is to estimate the physical parameters that determine the internal wave, use the then-known ocean dynamics to rendezvous underwater, and finally return to the surface as a cluster for easy retrieval. From the structure of the internal wave, the ocean's flowfield is time-varying and spatially dependent on depth and position along the wave propagation direction. The drifters can control their depth by changing their buoyancy and are otherwise subject to the horizontal flowfield at their given depth. We consider two different drifter dynamical models: a first-order Lagrangian model, useful when the drifter's mass is sufficiently small, and a second-order model, where the drag force caused by the water accelerates the drifter. We propose provably correct distributed algorithms that rely on the drifters opportunistically changing their depth so that the ocean flowfield takes them in a desirable direction to perform coordinated motion. The drifters converge to the same depth and position along the wave propagation direction asymptotically. Simulations illustrate our results.

I. INTRODUCTION

Internal waves are waves that propagate within a fluid, rather than on its surface. The type considered here arise when deep oceanic water is disturbed. Normally, water den-sity varies continuously with depth and surfaces of constant density are at a fixed depth. However, when the water receives an energetic disturbance, it leads to time-varying, sinusoidal profiles in the surfaces of constant density. This, in turn, gives rise to a nonlinear, depth- and time-varying dynamical model for the ocean, which can be parame-terized by constants such as amplitude, wavenumber, and frequency. In these scenarios, one can envision using a team of robotic drifters to perform the dual task of estimating the parameters that define the ocean flowfield induced by an internal wave and then using this now-known model to perform motion control. Here we are interested in distributed control laws for rendezvous. Such coordination strategies have important applications because, when oceanographers deploy these robotic sensors, they must retrieve them after the data is collected. However, with long deployment times, the drifters may drift miles apart. For large numbers of drifters, rendezvous strategies can make recovery easier. The problem is challenging because the drifters can only control their depth by changing their buoyancy and are otherwise subject to the horizontal flowfield of the ocean at their given depth. The basic idea of our rendezvous strategy is for drifters to opportunistically change their depth so that the ocean flowfield takes them in a desirable direction to perform coordinated motion.

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Literature review: Internal waves are associated with high concentrations of various types of planktonic organisms and small fishes [1], [2], as well as an agent of larval trans-port [3]. This makes their study important to oceanographers, see e.g., [4], [5] and references therein. Many internal wave models exist in the literature [6], [7]; here, we consider two continuously depth-dependent models proposed in [8]. Scientists widely use drifters drifting passively as monitoring platforms to gather relevant ocean data [9], [10]. The use of autonomous underwater vehicles to detect and characterize internal waves is a relatively new approach. Whereas pre-vious works use ocean measurements such as conductivity, temperature, pressure data [11], [12] or vertical flow veloc-ity [13] to detect and analyze internal waves, our recent work [14] is unique in using inter-vehicles measurements for depth-independent internal wave models. This work is also connected to the increasing literature that deals with cooperative networks of agents estimating spatial natural phenomena, including ocean [15], [16], river [17], and hur-ricane sampling [18]. Furthermore, it is tied to works related to motion planning in oceanic flows. Recent work [19] explores the possibility of actively selecting tidal currents so that drifters can autonomously reach a desired destination. Other researchers have also dealt with marine robots moving through strong flowfields where their actuation is limited and therefore cannot completely compensate the flow [20], [21]. Because the drifters control their vertical velocity which, in turn, affects their depth-dependent horizontal velocity, we use a backstepping framework [22]. We design a virtual control law, based on Laplacian agreement dynamics [23], which causes the drifters to rendezvous and a depth-control law which converges to this virtual control law.

Statement of contributions: We develop a solution to a two-part problem for a group of robotic drifters: determine the parameters of a oceanic internal wave flowfield and then use this knowledge to autonomously rendezvous for easy recovery. In the first part, we introduce a new method to estimate the parameters of a depth-dependent internal wave which specifically harnesses the added structure of the depth-dependency. The bulk of the paper is concerned with developing a distributed rendezvous method for the team of drifters. We develop distributed control law allowing all drifters to asymptotically rendezvous under two different drifter dynamical models. In the first (Lagrangian) model, we assume that the drifters are sufficiently small, and so, their velocity is equal to that of the ocean's velocity at their current location. In the second (drag-based) model, we assume that drifters are accelerated by a drag force proportional to the difference in their velocity and the ocean's. The rendezvous

problem is difficult for two reasons. First, since the drifters may only directly change their vertical depth in the water column, they must then rely on the horizontal current at that depth to move them towards rendezvous. Second, the flowfields are time-varying, creating situations where periodically the drifters have not enough and even no control authority. Our technical approach is broken into two stages. First, assuming drifters have direct control in the horizontal direction, we design a provably correct law, that we term 'virtual', that allows all drifters to rendezvous despite the time-varying control authority. Second, we design a depth-control law for the true system and establish its convergence to the virtual control law in finite time. The desired property of asymptotic rendezvous for the full dynamics then follows. Several simulations illustrate our results. For reasons of space, we omit the proofs, which will appear elsewhere.

II. Preliminaries

This section contains some preliminary notation. Let $\mathbb{R}$, and $\mathbb{R}_{>0}$ denote the set of all and positive real numbers, respectively. Given a set of points $P \subset \mathbb{R}^d$, let the convex hull $\mathrm{co}(P)$ of P be the smallest convex set which contains P. We let $\mathfrak{B}(\mathbb{R}^d)$ be the collection of all subsets of $\mathbb{R}^d$. The saturation function $\mathrm{sat} : \mathbb{R} \times \mathbb{R}_{>0} \to \mathbb{R}$ is defined as $\mathrm{sat}(x, M) = M$ if $x > M$, $\mathrm{sat}(x, M) = -M$ if $x < M$, and $\mathrm{sat}(x, M) = x$ if $-M \leq x \leq M$. The sign function $\mathrm{sgn} : \mathbb{R} \to \mathbb{R}$ is defined as $\mathrm{sgn}(x) = 1$ if $x > 0$, $\mathrm{sgn}(x) = -1$ if $x < 0$, and $\mathrm{sgn}(0) = 0$. We let L denote the Laplacian matrix of a line graph

$$
L = \begin{bmatrix}
1 & -1 & 0 & \dots & 0 \\
-1 & 2 & -1 & \ddots & \vdots \\
0 & \ddots & \ddots & \ddots & 0 \\
\vdots & \ddots & -1 & 2 & -1 \\
0 & \dots & 0 & -1 & 1
\end{bmatrix}.
$$

III. Problem Statement

This section presents the internal wave and drifter models and a formal problem statement.

A. Internal wave model

The internal wave models are specified in a global reference frame $\Sigma = (\mathbf{p}, \{\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z\})$. The origin $\mathbf{p}$ is an arbitrary point at the ocean surface; the vector $\mathbf{e}_x$ corresponds to the direction of wave propagation, which we assume parallel to the ocean bottom, and $\mathbf{e}_z$ is perpendicular to the ocean bottom, pointing from bottom to surface. The coordinates induced by Σ are $\{x, y, z\}$. Following [8], we consider a continuously stratified density profile and the mode-1 internal waves produced in it. Here, a fluid has a finite depth H and the density increases linearly from ocean surface to ocean bottom. This leads to the following horizontal and vertical flowfields, cf. Figure 1, induced by

the presence of the internal wave,

$$
f_x(x, z, t) = \frac{\alpha \pi}{H} \cos\left(\frac{\pi z}{H}\right) \sin(kx - \omega t + \phi), \qquad (1a)
$$

$$
f_z(x, z, t) = -\alpha k \sin\left(\frac{\pi z}{H}\right) \cos(kx - \omega t + \phi). \qquad (1b)
$$

Here, α is the ordering parameter proportional to the wave amplitude, H is the water column depth, k is the horizontal wavenumber, ω is the frequency, and ϕ is the initial phase.
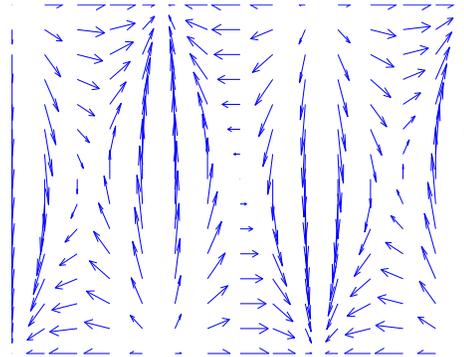
Fig. 1. The plot illustrates the dependency of internal wave flowfield cf. (1) as a function of depth ($-z$-direction) and wave propagation direction ($+x$-direction) at one time instant. As the wave propagates to the right, the flowfield translates to the right at a fixed speed.

This wave model does not produce motion in the y-direction and depends continuously on depth as a result of the dependency of density on depth. This adds an additional layer of complexity with respect to simpler, two-layer fluid models where the flowfield changes discontinuously at the wave interface, such as the ones considered in [24], [4]. In fact, our developments here are based on the observation that the added complexity can be leveraged to allow for coordinated motion of the drifters. There exist other continuously stratified models for internal waves, see [8], such as the hyperbolic tangent model capturing the density depth profile of deep ocean water. Although we do not consider them for reasons of space, our proposed design could be adapted to produce similar results to the ones presented here.

B. Drifter model

A drifter is a submersible buoy which can drift in the ocean, unattached to the ocean floor or a boat, and is able to change its depth in the water by controlling its buoyancy. A drifter can measure its own absolute position and relative distance to nearby drifters. Although GPS is not available underwater, oceanographers can deploy a grid of acoustic pingers to provide absolute position to the drifters. The drifters, equipped with hydrophones, can determine their location based on the travel time of the pingers' pulses [25], [26]. Consider a group of N drifters, each with a reference frame $\Sigma_i = (\mathbf{p}_i, \{\mathbf{e}_{x_i}, \mathbf{e}_{y_i}, \mathbf{e}_{z_i}\})$, $i \in \{1, \ldots, N\}$, attached to it. The origin $\mathbf{p}_i$ corresponds to the location of the drifter. In general, this reference frame might not be aligned with the global wave frame Σ. However, for simplicity, we assume the drifters know the rotation between frames. Our previous work [14] shows how each drifter can determine the rotation

between its local frame and the global frame by using relative distance and distance derivative measurements. The drifters can change its vertical velocity via buoyancy control.

Different models exist to describe an object's dynamics within an ocean flowfield. The classical drag-based model is a second-order dynamics where the object feels a drag force that is a function of the difference in velocity between the flowfield and the object [27]. Instead, objects that are very small or are in laminar flows are driven by a second-order dynamics where acceleration is governed by Stokes drag,

$$\dot{x}_i = v_i, \tag{2a}$$

$$\dot{v}_i = -\frac{c_d}{m}(v_i - f_x(x_i, z_i, t)) \tag{2b}$$

$$\dot{z}_i = f_z(x_i, z_i, t) + u_i. \tag{2c}$$

Here, c_d is the drag parameter and m is the combined drifter mass and inertial added mass [27]. We also consider first-order Lagrangian dynamics, where the drifter's velocity is equal to ocean's velocity at its current location,

$$\dot{x}_i = f_x(x_i, z_i, t), \tag{3a}$$

$$\dot{z}_i = f_z(x_i, z_i, t) + u_i. \tag{3b}$$

Note that as the mass m tends to zero, the second-order dynamics models become the first-order Lagrangian model.

C. Problem statement

Consider a group of N spatially distributed drifters that move underwater under the influence of an ocean internal wave. The overall objective is to estimate the physical parameters that determine the internal wave, use the then-known ocean dynamics to rendezvous underwater, and finally return to the surface as a cluster for easy retrieval. Formally, let the N drifters be at initial states $x(0), v(0), z(0)$. All drifters i in $\{1, \ldots, N\}$ have continuous access to x_j, v_j, $\dot{v}_j$, and z_j, for j in $\{i-1, i, i+1\}$. Under the influence of the flowfield (1), the drifters must first use their local measurements to determine the parameters which define their motion: α, k, ω, and ϕ. After that, they should enact an algorithm so that they may surface near each other. Our objective is to design a distributed feedback control law that makes all drifters rendezvous at a common location in the wave propagation direction, i.e.,

$$\lim_{t \to \infty} x_i(t) - x_j(t) = 0, \qquad \forall i, j \in \{1, \ldots, N\}.$$

In practical implementation, once rendezvous is achieved to a desired level of accuracy, drifters can surface synchronously near each other for easy retrieval.

IV. INTERNAL WAVE PARAMETER ESTIMATION

Our previous work [14] has provided one solution to the problem of estimating the parameters of linear interfacial internal waves for first-order drifters. Though the ocean model considered there is simpler, the proposed algorithm may be readily adapted to determine the parameters of depth-dependent internal waves when the drifters maintain a prescribed depth. Here, we introduce a new parameter

identification strategy that specifically utilizes the added structure of depth-dependent wave models and the knowledge of absolute position. The motion for a depth-keeping particle in an interfacial internal wave is given by [24]

$$\dot{x} = \frac{a\omega}{z_u k} \sin(kx - \omega t + \phi) \tag{4}$$

One can solve (4) to find the analytic expression for a depth-keeping particle's trajectory influenced by an internal wave.

Lemma 4.1: (Depth-keeping drifter motion [14]) The solution of (4) starting from $x(0)$ is

$$x(t) = \frac{\omega}{k}\left(1 - \sqrt{1-(a/z_u)^2}\right)t + \Xi(t) - \frac{\phi}{k},$$

$$\Xi(t) = \frac{2}{k}\tan^{-1}\left(\frac{a}{z_u} - \sqrt{1-\left(\frac{a}{z_u}\right)^2}\tan\left(\frac{\pi t}{T} + \Lambda_0\right)\right)$$

$$- \frac{2\pi}{k}\left\lfloor \frac{t}{T} + \frac{\Lambda_0}{\pi} - \left\lfloor \frac{kx(0) + \phi + \pi}{2\pi}\right\rfloor + \frac{1}{2}\right\rfloor + \frac{2\pi}{kT}t,$$

with period T and initial condition Λ_0 as

$$T = \frac{2\pi}{\omega\sqrt{1-(a/z_u)^2}},$$

$$\Lambda_0 = \tan^{-1}\left(\frac{1}{\sqrt{1-(a/z_u)^2}}\left(\frac{a}{z_u} - \tan\left(\frac{kx(0)+\phi}{2}\right)\right)\right).$$

Now, we adapt Lemma 4.1 to depth-keeping particles in (1). The dynamics of the depth-keeping particle is

$$\dot{x} = \frac{\alpha\pi}{H}\cos(\frac{\pi z}{H})\sin(kx - \omega t + \phi) \tag{5}$$

Using Lemma 4.1, one finds that the period T of the particle's motion at depth z is

$$T(z) = \frac{2\pi}{\omega\sqrt{1 - \left(\frac{\alpha\pi}{\omega H}\cos(\frac{\pi z}{H})\right)^2}} \tag{6}$$

and the net displacement Δx_T of the particle over one period T while at depth z is

$$\Delta x_T(z) = \frac{2\pi}{k}\frac{\left(1 - \sqrt{1 - \left(\frac{\alpha\pi}{\omega H}\cos(\frac{\pi z}{H})\right)^2}\right)}{\sqrt{1 - \left(\frac{\alpha\pi}{\omega H}\cos(\frac{\pi z}{H})\right)^2}}. \tag{7}$$

We detail next the procedure to determine the parameters.

Lemma 4.2: (Determination of wave parameters) If a drifter waits at two different depths z_1 and z_2 for one period at each depth, recording the periods T_1 and T_2 and the displacement $\Delta x_T(z_1)$, the parameters ω, α, and k can be uniquely determined by solving (6) and (7). Finally, the other parameter ϕ can be found by solving (5) and using knowledge of $v(t)$, $x(t)$, and ω, α, and k.

V. RENDEZVOUS OF FIRST-ORDER DRIFTERS

This section details the rendezvous control law for a group of first-order drifters. We assume that all drifters know the wave parameters. Since the drifters do not have direct control in the x-direction, we break the design in two steps,

[Informal description]: We employ a two-part, backstepping strategy. First, we assume we can actuate in the x-direction and design a virtual control law that causes the drifters to rendezvous. Second, we design the depth-control law that converges to and tracks the virtual control law in finite time.

A. Virtual rendezvous control law

Here we design a control law that achieves rendezvous when direct control authority is available in the x-direction. For each $i \in \{1, \ldots, N\}$, we consider the dynamics

$$\dot{x}_i = \frac{\alpha\pi}{H} w_i \sin(kx_i - \omega t + \phi), \tag{8}$$

where w_i is the 'virtual' control. Specifically, we design the distributed controller with design parameter $A \in \mathbb{R}_{>0}$,

$$w_i(x, t) = -A \tanh(Lx)_i \sin(kx_i - \omega t + \phi). \tag{9}$$

The closed-loop dynamics is

$$\dot{x}_i = -A \frac{\alpha\pi}{H} \tanh(Lx)_i \sin(kx_i - \omega t + \phi)^2. \tag{10}$$

This controller has two terms. The first, $-A \tanh(Lx)_i$, pushes agents in the direction of the average of their neighbors and could be implemented with relative distance measurements. The second, $\sin(kx_i - \omega t + \phi)$, ensures that the ocean flow always pushes agents in the right direction, as seen in (10). This second term requires absolute position information to know which direction the wave is currently The next result shows that the state of the drifters following (10) remains bounded.

Lemma 5.1: (Boundedness of the trajectories) Given any $x(0) \in \mathbb{R}^N$ for the group of drifters, the network trajectory under (10) satisfies $x(t) \in [x(0)_{\min}, x(0)_{\max}]^N$ for all $t \geq 0$.

We combine the boundedness of the drifters' evolution stated in Lemma 5.1 with Barbalat's Lemma [22], to deduce that the closed-loop dynamics (9) makes the drifters asymptotically rendezvous in the wave propagation direction.

Proposition 5.2: (Asymptotic network rendezvous) Given any $x(0) \in \mathbb{R}^N$ for the group of drifters, the dynamics (10) makes the network rendezvous in the x-direction,

$$\lim_{t \to \infty} x_i(t) - x_j(t) = 0, \quad \forall i, j \in \{1, \ldots, N\}.$$

B. Backstepping depth-control law

Here, we build on the developments of Section V-A to design a depth-control law that achieves the desired network objective for the drifter dynamics (3). We make the assumption that each drifter has enough control authority in the vertical direction to cancel the vertical motion induced by the wave, and that the magnitude of the remaining control is bounded by M. For simplicity, we abuse notation to re-define u_i to be the control after canceling the vertical motion f_z. Thus, for each $i \in \{1, \ldots, N\}$, we rewrite (3) as

$$\dot{x}_i = \frac{\alpha\pi}{H} \cos\left(\frac{\pi z_i}{H}\right) \sin(kx_i - \omega t + \phi), \qquad \dot{z}_i = u_i,$$

where $|u_i| \leq M$. Defining $d_i = \cos(\frac{\pi z_i}{H})$,

$$\dot{x}_i = \frac{\alpha\pi}{H} d_i \sin(kx_i - \omega t + \phi), \quad \dot{d}_i = -\frac{\pi}{H} \sqrt{1 - d_i^2} u_i,$$

where d_i is constrained to $[-1, 1]$. We refer to it as the 'transformed depth'. Given the discussion of Section V-A, our basic idea is to synthesize a design that makes the drifter's transformed depth track the virtual control law (9). Consequently, we define the error variables $e_i = d_i - w_i$ and rewrite the dynamics in terms of x_i and e_i as

$$\dot{x}_i = \frac{\alpha\pi}{H}\big(w_i \sin(kx_i - \omega t + \phi) + e_i \sin(kx_i - \omega t + \phi)\big), \tag{11a}$$

$$\dot{e}_i = -\frac{\pi}{H} \sqrt{1 - d_i^2} u_i - \dot{w}_i. \tag{11b}$$

Our proposed design is the depth-control law,

$$u_i(d, x, t) = \tag{12}$$
$$\operatorname{sat}_M\left(\frac{H}{\pi\sqrt{1 - d_i^2}}\left(C \operatorname{sgn}(d_i - w_i(x, t)) - \dot{w}_i(x(t), t)\right)\right),$$

where $\dot{w}_i(x(t), t)$ is the total derivative and $C \in \mathbb{R}_{>0}$ is a design parameter. We next show that this controller makes the transformed depth converge to a generic virtual control law in finite time if the magnitude of the control law and its time derivative are small enough.

Lemma 5.3: (Finite-time convergence to virtual control law) Let $M \in \mathbb{R}_{>0}$ and $d(0) \in (-1, 1)^N$. Suppose there exist $\epsilon, \beta \in \mathbb{R}_{>0}$ and a continuous and increasing function $h : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ with $h(0) = 0$ such that the magnitude of $g : \mathbb{R}_{\geq 0} \to \mathbb{R}$ and its derivative are uniformly bounded by β and $h(\beta)$, respectively, and

$$\frac{Hh(\beta)(1 + \epsilon)}{\pi(1 - (1 + \epsilon)^2 \beta^2)} \leq M. \tag{13}$$

Then, for any $C \geq h(\beta)(1 + \epsilon)$, the error variables e_i, for all $i \in \{1, \ldots, N\}$, evolving the dynamics (11b) with control law (12) converges to 0 in finite time.

Because the result does not consider the exact form of the virtual control law, we are able to re-use the designed depth-control law for the second-order case in Section VI. The next result states that under (11) with distributed control given by (9) and (12), all drifters rendezvous in the x-direction.

Theorem 5.4: (Asymptotic network rendezvous) For any $x(0) \in \mathbb{R}^N$, depth $z(0) \in (0, H)^N$, $\epsilon \in \mathbb{R}_{>0}$, and bound $M \in \mathbb{R}_{>0}$ on the magnitude of depth control actuation, let the magnitude A of the virtual control law (9) satisfy

$$\frac{H((4 + k)\frac{\alpha\pi}{H} A^2 + \omega A)(1 + \epsilon)}{\pi(1 - (1 + \epsilon)^2 A^2)} \leq M,$$

and the gain $C \geq ((4 + k)\frac{\alpha\pi}{H} A^2 + \omega A)(1 + \epsilon)$ in (12). Then, all drifters asymptotically rendezvous under (11) with the controllers (9) and (12), i.e., for all $i, j \in \{1, \ldots, N\}$,

$$\lim_{t \to \infty} x_i(t) - x_j(t) = 0, \quad \lim_{t \to \infty} z_i(t) = \frac{-H}{2}.$$

Figure 2 depicts the evolution of 4 drifters performing rendezvous. Because the drifters start near the top/bottom of the ocean, their depth-control authority is saturated and some

of them are therefore pushed in the wrong direction. As they move towards the middle, their depth-control is no longer saturated and then they are successfully able to rendezvous.
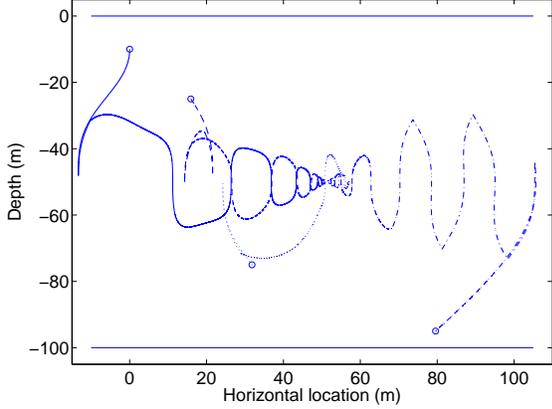
Fig. 2. This plot illustrates an evolution of the proposed distributed rendezvous algorithm ((9)-(12)) for the Lagrangian dynamics (11) for a group of 4 drifters. The parameters are $k = \frac{2\pi}{50}\frac{1}{m}$, $\omega = \frac{2\pi}{50}\frac{1}{s}$, $\alpha = 10$, and $H = 100m$. The drifters are initially at $(0, -10)$, $(16, -25)$, $(32, -75)$ and $(80, -95)$, marked by the 'o's. Their neighbors are the drifters on either side of them initially. They converge to around $(52, -50)$.

VI. Rendezvous of second-order drifters

Here we adapt the method developed in Section V to the linear, second-order model (2) for the drifter dynamics. As before, our starting point is the assumption that all drifters know the wave parameters (in fact, one can determine them using measurements collected by the drifters and fitting them (2) but we do not present this discussion here).

A. Virtual rendezvous control law

We begin by designing a control law w_i, $i \in \{1, \dots, N\}$, assuming that the drifters can directly actuate their own x-acceleration through their depth, i.e.,

$$\dot{x}_i = v_i, \quad \dot{v}_i = -\frac{c_d}{m}(v_i - \frac{\alpha\pi}{H}w_i \sin(kx_i - \omega t + \phi)),$$

Our proposed distributed design is

$$w_i(x, v, t) = -A\sin(kx_i - \omega t + \phi)\tanh(L(\frac{c_d}{m}x + v)_i) \quad (14)$$

so that the closed-loop dynamics takes the form

$$\dot{x}_i = v_i, \quad (15a)$$
$$\dot{v}_i = -\frac{c_d}{m}\big(v_i + \frac{\alpha\pi}{H}\sin^2(kx_i - \omega t + \phi)\cdot$$
$$A\tanh(L(\frac{c_d}{m}x + v)_i). \quad (15b)$$

The next result shows that each drifters' sum of position and velocity remains bounded.

Lemma 6.1: (Boundedness of the sum of position and velocity) Given any initial condition $(x(0), v(0)) \in \mathbb{R}^{2N}$ for the group of drifters, the network trajectory under (15) satisfies $\frac{c_d}{m}x(t) + v(t) \in [(\frac{c_d}{m}x(0) + v(0))_{\min}, (\frac{c_d}{m}x(0) + v(0))_{\max}]^N$ for all $t \geq 0$.

Building on Lemma 6.1, we next show that, in fact, the position of the drifters remains bounded as well.

Lemma 6.2: (Boundedness of position) Given any initial condition $(x(0), v(0)) \in \mathbb{R}^{2N}$ for the group of drifters, the network trajectory under (15) satisfies $x(t) \in [x(0)_{\min}, x(0)_{\max}]^N$, for all $t \geq 0$.

We are now ready to show that the drifters all converge to rendezvous in the x-direction under the control law (15).

Proposition 6.3: (Asymptotic network rendezvous) Given any initial condition $(x(0), v(0)) \in \mathbb{R}^{2N}$ for the group of drifters, the dynamics (15) makes the network rendezvous in the x-direction: for all i, j in $\{1, \dots, N\}$

$$\lim_{t\to\infty} x_i(t) - x_j(t) = 0, \qquad \lim_{t\to\infty} v_i(t) = 0.$$

B. Backstepping depth-control law

As before, we re-define the control authority u_i to be the remaining control after canceling the vertical dynamics, so that the dynamics looks,

$$\dot{x}_i = v_i, \qquad \dot{z}_i = u_i.$$
$$\dot{v}_i = -\frac{c_d}{m}(v_i - \frac{\alpha\pi}{H}\cos(\frac{\pi z_i}{H})\sin(kx - \omega t + \phi)),$$

Then, going through the same variable changes as for the Lagrangian case, we arrive at

$$\dot{x}_i = v_i \quad (16a)$$
$$\dot{v}_i = -\frac{c_d}{m}(v_i - \frac{\alpha\pi}{H}w_i \sin(kx_i - \omega t + \phi)) + \quad (16b)$$
$$\frac{c_d}{m}\frac{\alpha\pi}{H}\sin(kx_i - \omega t + \phi)e_i,$$
$$\dot{e}_i = -\frac{\pi}{H}\sqrt{1 - d_i^2}u_i - \dot{w}_i. \quad (16c)$$

We select the depth-control law with the same structure as (12) but now with the virtual control law given by (14). The next result characterizes the asymptotic convergence properties of the closed-loop system.

Theorem 6.4: (Asymptotic network rendezvous) For any initial position $(x(0), v(0)) \in \mathbb{R}^{2N}$, depth $z(0) \in (0, H)^N$, $\epsilon \in \mathbb{R}_{>0}$, and bound $M \in \mathbb{R}_{>0}$ on the magnitude of depth control actuation, let the magnitude A of the law (14) satisfy

$$\frac{(A^2\alpha\pi(4\frac{c_d}{m} + k) + \omega AH)(1 + \epsilon)}{\pi(1 - (1 + \epsilon)^2 A^2)} \leq M,$$

and the gain $C \geq (\frac{\alpha\pi}{H}A^2(4\frac{c_d}{m} + k) + \omega A)(1 + \epsilon)$ in the depth-control law (12). Then, all drifters asymptotically rendezvous under the dynamics of (16) with the controllers (14) and (12), i.e., for all $i, j \in \{1, \dots, N\}$,

$$\lim_{t\to\infty} x_i(t) - x_j(t) = 0, \quad \lim_{t\to\infty} v_i(t) = 0, \quad \lim_{t\to\infty} z_i(t) = \frac{-H}{2}.$$

Figure 3 depicts the evolution of 5 drifters performing rendezvous with second-order dynamics. Note that even though drifters start near the internal wave mean depth where their depth-control authority is not saturated, for some of the drifters, the flow was going in the wrong direction. Thus, they change depth until the flow is aligned with their desired direction of motion. By repeatedly switching between being

above the wave and below it depending on the flow, the drifters asymptotically rendezvous.
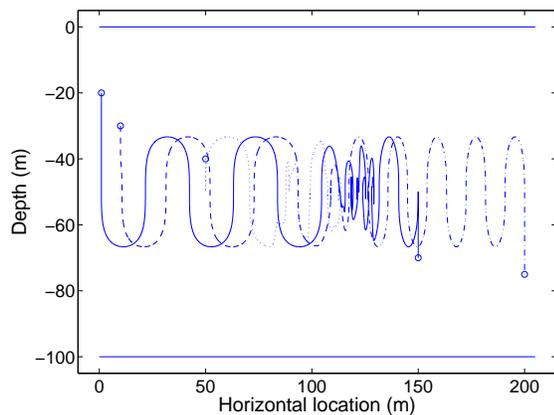
Fig. 3. This plot illustrates an evolution of the proposed distributed rendezvous algorithm ((14)-(12)) for the Lagrangian dynamics (11). The parameters are $k = \frac{2\pi}{100}\frac{1}{m}$, $\omega = \frac{2\pi}{1000}\frac{1}{s}$, $\alpha = 10$, $\frac{c_d}{m} = 4$, and $H = 100m$. The drifters, initially located at $(1, -20)$, $(10, -30)$, $(50, -40)$, $(150, -70)$ and $(200, -75)$ with no horizontal velocity, are marked by the 'o's. Their neighbors are the drifters on either side of them initially. They converge to around $(113, -50)$.

VII. Conclusions

We have developed distributed algorithmic solutions for a group of robotic drifters that allow them to estimate the parameters of a oceanic internal wave flowfield and use this knowledge to perform coordinated rendezvous for easy recovery. Regarding the first task, we have introduced a novel distributed estimation method that relies on drifters having absolute position information and inter-drifter distance measurements. Regarding the second task, we have developed distributed control laws for two different dynamical (Lagrangian and drag-based) models that allow all drifters to asymptotically rendezvous in the wave direction. Our design employs a backstepping methodology, where we first synthesize a rendezvousing controller assuming direct control in the horizontal direction is available and then design a depth-control law that is guaranteed to converge to this controller in finite time. Future work will be devoted to extend our results to increasingly realistic scenarios, including the consideration of other wave models [8], the presence of multiple internal waves, the ability to estimate wave parameters for second-order drifter dynamics, the consideration of nonlinear second-order drifter dynamics, and the synthesis of strategies that only rely on relative distance inter-drifter measurements.

Acknowledgments

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