Event-triggered Control for Nonlinear Systems with Time-Varying Input Delay

Erfan Nozari   Pavankumar Tallapragada   Jorge Cortés

Abstract— **This paper studies the problem of event-triggered control design for general continuous-time nonlinear systems with time-varying input delay. Our methodology is based on the concept of predictor feedback and is capable of compensating arbitrarily large known time delays. Under mild conditions, we prove that as long as the delay-free system is globally input-to-state stabilizable, it can also be globally asymptotically stabilized via piecewise-constant event-triggered control. We prove that the proposed event-triggering design does not suffer from Zeno behavior as the inter-event times are uniformly lower bounded. We further show that our design achieves exponential stability for a controllable linear system and study the trade-off between convergence speed and communication cost. Various simulations illustrate our results.**

I. Introduction

The increasing adoption of network communications in control system architectures and the growing concern towards lowering the communication costs have led in the recent years to the proliferation of event- and self-triggered control strategies. In these strategies, instead of continuously or regularly updating the control signals, the controller opportunistically decides when to do so in order to drive the closed-loop system toward the desired equilibrium. This leads to a more efficient use of the available resources, at the cost of adding complexity to the design and analysis in the face of uncertainties, time delays, and disturbances. Our work here is devoted to enhance the state-of-the-art in event-triggered control by designing control strategies that stabilize nonlinear systems with time-varying delays in actuation.

Literature review: This work builds upon the areas of opportunistic state-triggered control and stabilization of time-delayed systems. The basic concepts of state-triggered control originally stemmed from event-based and discrete-event systems, see, e.g., [1], [2]. These ideas have since then been extended to consider various control and sensing tasks, see [3]–[5] and references therein. The basic design methodology, which we also employ here, builds on a Lyapunov-based analysis to schedule the triggering times while maintaining a minimum decay in the Lyapunov function.

Among the vast literature of control of time-delay systems, our work is closely related to the design schemes based on the notion of predictor feedback, also called finite spectrum assignment and reduction method, see e.g. [6]–[11]. Roughly speaking, the idea is that the controller first predicts the future state of the system using its mathematical model and then generates the control signal based on this prediction to compensate for the system delay. Our developments here are particularly inspired by the results presented in [11]. Simulation techniques for closed-loop systems using predictor feedback are discussed in [12], [13]. Recent work has explored the event-triggered control of time-delay systems. This problem is particularly challenging due to the interplay between event-triggering and time delays: due to the opportunistic nature of event-triggering, the controller "waits" until the system tends to become unstable and then updates the control accordingly, but if this control takes some time to reach the system, it may no longer be able to prevent the system from instability. Therefore, the controller has to be "sufficiently more conservative" and update the control "sufficiently ahead in time" to ensure closed-loop stability, which makes the design challenging. The work [14] considers linear time-invariant systems subject to a quadratic cost function and designs an event-triggered control scheme that satisfies the feasibility constraints. In a similar setup, [15] considers a general switched linear system subject to unknown time-varying delays with known bounds. Based on these bounds, the paper shows that the uncertainty in the delay has a polytopic nature. Both papers discretize the continuous-time system using a fixed sample time and assume that the time delay is less than the sampling time. This assumption is quite restrictive and leads to a delay-free discrete system for which the event-triggering schemes are designed. Our design and analysis approach does not rely on such assumption and allows us to control a wide class of nonlinear systems subject to arbitrary time delays.

Statement of contributions: Our contributions are threefold. The first contribution is the design of an event-triggering control strategy for a wide class of nonlinear systems with arbitrarily large time delays. Employing the method of predictor feedback to compensate for the system input delay, we co-design the closed-loop control law and the triggering strategy to ensure the monotonic evolution of the Lyapunov function identified in the continuous-time case. Our second contribution presents the convergence analysis of our triggering design. We establish global asymptotic stability of the closed-loop system and prove the existence of a uniform lower bound on the inter-event times, a result which rules out the possibility of Zeno behavior (i.e., the possibility of arbitrarily fast triggering leading to triggering times that accumulate at a finite time instant). We further analyze the particular case of a controllable linear system and give explicit expressions for the design variables. In the linear case, we also show that the design achieves exponential stability. Our final contribution is the analysis of the trade-off between communication cost and convergence speed in event-triggering control for the case of linear systems. We also present various simulations to illustrate our results. For space reasons, proofs are omitted and will appear elsewhere.

The authors are with the Department of Mechanical and Aerospace Engineering, University of California, San Diego, {enozari,ptallapragada,cortes}@ucsd.edu

II. Preliminaries

This section introduces our notational conventions and briefly reviews basic notions on input-to-state stability. We denote by $\mathbb{R}$ and $\mathbb{R}_{\geq 0}$ the sets of reals and nonnegative reals, respectively. Given any vector or matrix, we use $|\cdot|$ to denote the (induced) Euclidean norm. We use the notation $\mathcal{L}_f S$ for the Lie derivative of a function S along the trajectories of a vector field f.

We follow [16] to review the definition of input-to-state stability of nonlinear systems and its Lyapunov characterization. Consider a nonlinear system of the form

$$\dot{x}(t) = f(x(t), u(t)), \tag{1}$$

where $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is continuously differentiable and satisfies $f(0, 0) = 0$. For simplicity, we assume that this system has a unique solution which does not exhibit finite escape time. System (1) is (globally) input-to-state stable (ISS) if there exist $\alpha \in \mathcal{K}$ and $\beta \in \mathcal{KL}$ such that for any measurable locally essentially bounded input $u : \mathbb{R}_{\geq 0} \to \mathbb{R}^m$ and any initial condition $x(0) \in \mathbb{R}^n$, its solution satisfies

$$|x(t)| \leq \beta(|x(0)|, t) + \alpha\big(\sup_{t\geq 0} |u(t)|\big),$$

for all $t \geq 0$. For this system, a continuously differentiable function $S : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is called an ISS-Lyapunov function if there exist $\alpha_1, \alpha_2, \gamma, \rho \in \mathcal{K}_\infty$ such that

$$\begin{aligned} &\forall x \in \mathbb{R}^n && \alpha_1(|x|) \leq S(x) \leq \alpha_2(|x|), \\ &\text{and } \forall (x, u) \in \mathbb{R}^{n+m} && \mathcal{L}_f S(x, u) \leq -\gamma(|x|) + \rho(|u|). \end{aligned} \tag{2}$$

We have the following result.

Proposition 2.1: ([16, Theorem 1]): The system (1) is ISS if and only if it admits an ISS-Lyapunov function.

III. Problem Statement

Here we introduce the problem of interest on event-triggered stabilization of nonlinear systems with input delay. Consider a single-input nonlinear time-invariant system with input delay modeled as

$$\dot{x}(t) = f(x(t), u(\phi(t))), \tag{3}$$

where the vector field $f : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ is continuously differentiable and $f(0, 0) = 0$. The quantity $t - \phi(t) > 0$ is the time delay at time t. The delay might be due to inherent actuator delays or the time that it takes for the controller messages to reach the actuator. We assume that $\{u(t) \,|\, \phi(0) \leq t \leq 0\}$ is given and bounded and that the system (3) does not exhibit finite escape time for any initial condition and any bounded input. For simplicity, we assume that ϕ is continuously differentiable and $\dot{\phi}(t) > 0$ for all $t \geq 0$, so that the time argument of the control does not go back in time. We also assume that the delay and its derivative are bounded, i.e., there exist $M_0, M_1, m_2 > 0$ such that,

$$\forall t \geq 0 \qquad t - \phi(t) \leq M_0 \quad \text{and} \quad m_2 \leq \dot{\phi}(t) \leq M_1. \tag{4}$$

Note that, in the case of a constant time delay D, we have $\phi(t) = t - D$. Furthermore, the conditions (4) are trivially satisfied with $M_0 = D$ and $M_1 = m_2 = 1$.

Regarding stabilization, our starting point is the assumption that the origin is globally asymptotically stabilizable in the absence of delay by a feedback law that makes it ISS with respect to additive input disturbances. Formally, there exists globally Lipschitz $K : \mathbb{R}^n \to \mathbb{R}$, $K(0) = 0$, such that

$$\dot{x}(t) = f(x(t), K(x(t)) + w(t)), \tag{5}$$

is ISS with respect to w. This assumption means that the system is robustly stabilizable when no delays are present, a necessary condition to tackle the more involved case with delay. Here, we are interested in designing opportunistic state-triggered controllers to stabilize the system (3) that do not require the actuator to continuously adjust the forcing input. This is motivated by considerations about the efficient use of the available communication, sensing, or actuation resources. For instance, scenarios where communication between sensor, controller, and actuator is limited (e.g., shared communication network), where applying a continuously changing input to the system is unfeasible. To address these challenges, we seek to design an event-triggered control that only updates the input to the system when necessary. Formally, our objective can be formulated as follows.

Problem 1: (Event-Triggered Stabilization under Input-Delay): Design the sequence of triggering times $\{t_k\}_{k=1}^\infty$ with $t_0 = 0$ and $\lim_{k\to\infty} t_k = \infty$, and the piecewise constant control $\{u(t)\}_{t=0}^\infty$, with

$$u(t) = u(t_k), \qquad t \in [t_k, t_{k+1}), \tag{6}$$

so that the closed-loop system (3) is globally asymptotically stable. •

The requirement $\lim_{k\to\infty} t_k = \infty$ ensures that the resulting design is implementable by avoiding a finite accumulation point for the triggering times.

IV. Event-Triggered Design and Analysis

In this section, we propose an event-triggered control policy to solve Problem 1. Our design is based on the predictor-based feedback control solution for stabilization [11], which we review in Section IV-A. We present our event-triggered control design in Section IV-B and analyze its convergence properties in Section IV-C.

A. Predictor Feedback Control for Time-Delay Systems

Here we review the continuous-time stabilization of the dynamics (3) by means of a predictor-based feedback control [11]. For convenience, we denote the inverse of ϕ by

$$\sigma(t) = \phi^{-1}(t),$$

for all $t \geq 0$. The inverse exists since ϕ is strictly monotonically increasing. From (4), we have, for all $t \geq \phi(0)$,

$$\frac{1}{\sigma(t) - t} \geq m_0 \quad \text{and} \quad m_1 \leq \dot{\sigma}(t) \leq M_2,$$

for $m_0 = \frac{1}{M_0}$, $m_1 = \frac{1}{M_1}$, and $M_2 = \frac{1}{m_2}$. To compensate for the delay, at any time $t \geq \phi(0)$, the controller makes the following prediction of the future state of the plant,

$$p(t) = x(\sigma(t)) = x(t^+) + \int_{\phi(t^+)}^{t} \dot{\sigma}(s) f(p(s), u(s)) ds, \tag{7}$$

where $t^+ = \max\{t, 0\}$. This integral is computable by the controller since it only requires knowledge of the initial or current state of the plant (gathered from the sensors) and the history of $u(t)$ and $p(t)$, both of which are available to the controller. Nevertheless, for general nonlinear vector fields f, (7) may not have a closed-form solution and it has to be computed using numerical integration methods. The controller applies the control law K to the prediction p in order to compensate for the delay, i.e.,

$$u(t) = K(p(t)), \qquad t \geq 0. \tag{8}$$

The next result establishes the convergence of the closed-loop system.

Proposition 4.1: (Asymptotic Stabilization by Predictor Feedback [11]): Under the aforementioned assumptions, the closed-loop system (3) under the controller (8) is globally asymptotically stable, i.e., there exists $\beta \in \mathcal{KL}$ such that for any $x(0) \in \mathbb{R}^n$ and bounded $\{u(t)\}_{t=\phi(0)}^{0}$, for all $t \geq 0$,

$$|x(t)| + \sup_{\phi(t)\leq\tau\leq t} |u(\tau)| \leq \beta\Big(|x(0)| + \sup_{\phi(0)\leq\tau\leq 0} |u(\tau)|, t\Big).$$

B. Design of Event-triggered Control Law

Following the exposition of Section IV-A, we let the controller make the prediction $p(t)$ according to (7) for all $t \geq \phi(0)$. Since the controller can only update $u(t)$ at discrete event times $\{t_k\}_{k=0}^{\infty}$, it uses the piecewise-constant control (6) and assigns the control

$$u(t_k) = K(p(t_k)), \tag{9}$$

for all $k \geq 0$. In order to design the triggering times $\{t_k\}_{k=1}^{\infty}$, we use Lyapunov stability tools to determine when the controller has to update $u(t)$ to prevent instability. We define the triggering error for all $t \geq \phi(0)$ as,

$$e(t) = \begin{cases} p(t_k) - p(t) & \text{if} \quad t \in [t_k, t_{k+1}) \text{ for some } k \geq 0, \\ 0 & \text{if} \quad t \in [\phi(0), 0), \end{cases} \tag{10}$$

so that $u(t) = K(p(t) + e(t))$, for $t \geq 0$. Let

$$w(t) = u(t) - K(p(t) + e(t)),$$

for all $t \geq \phi(0)$. Clearly, $w(t) = 0$ for $t \geq 0$ but $w(t)$ may not be zero when $t \in [\phi(0), 0)$. The closed-loop system can then be written as

$$\dot{x}(t) = f\big(x(t), K\big(x(t) + e(\phi(t))\big) + w(\phi(t))\big),$$

for all $t \geq 0$. Let $g(x, w) = f(x, K(x) + w)$ for all x, w. By the assumption that $\dot{x} = g(x, w)$ is ISS with respect to w, there exists a continuously differentiable function $S : \mathbb{R}^n \to \mathbb{R}$ and class $\mathcal{K}_\infty$ functions $\alpha_1, \alpha_2, \gamma$, and ρ such that

$$\alpha_1(|x(t)|) \leq S(x(t)) \leq \alpha_2(|x(t)|), \tag{11}$$

and $(\mathcal{L}_g S)(x, w) \leq \gamma(|x|) + \rho(|w|)$. Therefore, we have

$$\begin{aligned} &(\mathcal{L}_f S)\big(x(t), K\big(x(t) + e(\phi(t))\big) + w(\phi(t))\big) \qquad (12) \\ &= (\mathcal{L}_g S)\big(x(t), K\big(x(t){+}e(\phi(t))\big){+}w(\phi(t)){-}K(x(t))\big) \\ &\leq -\gamma(|x(t)|) + \rho\big(\big|K\big(x(t){+}e(\phi(t))\big){+}w(\phi(t)){-}K(x(t))\big|\big). \end{aligned}$$

We assume that ρ is such that $\int_0^1 \frac{\rho(r)}{r} < \infty$. This assumption is not restrictive and is satisfied by most well-known class $\mathcal{K}$ functions. Then, define

$$V(t) = S(x(t)) + \frac{2}{b}\int_0^{2L(t)} \frac{\rho(r)}{r} dr,$$

where

$$L(t) = \sup_{t\leq\tau\leq\sigma(t)} |e^{b(\tau-t)} w(\phi(\tau))|, \tag{13}$$

and $b > 0$ is a design parameter. The next result establishes an upper bound on the time derivative of V.

Proposition 4.2: (Upper-bounding $\dot{V}(t)$): For the system (3) under the control defined by (6) and (9) and the predictor (7), we have

$$\dot{V}(t) \leq -\gamma(|x(t)|) - \rho(2L(t)) + \rho(2L_K|e(\phi(t))|),$$

for all $t \geq 0$, where L_K is the Lipschitz constant of K. •

Proposition 4.2 is the basis for our event-trigger design. Formally, we select $\theta \in (0, 1)$ and require

$$\rho(2L_K|e(\phi(t))|) \leq \theta\gamma(|x(t)|), \qquad t \geq 0,$$

which can be equivalently written as

$$|e(t)| \leq \frac{\rho^{-1}(\theta\gamma(|p(t)|))}{2L_K}, \qquad t \geq \phi(0). \tag{14}$$

From (10), we see that (14) is trivially satisfied on $[\phi(0), 0)$. As mentioned earlier, the first triggering happens at time $t_0 = 0$. Therefore, $e(0) = 0$ and (14) is again trivially satisfied. After time 0, the controller (which has access to both $p(t)$ and $e(t)$) keeps evaluating the inequality (14). As long as it is satisfied, no event is triggered. The moment when (14) holds with equality, the controller triggers an event (determining t_1), so $e(t_1) = 0$ and (14) is maintained. This process is repeated and the triggering times are determined online. Notice that from (14), "larger" function γ and "smaller" ρ are desirable as they result in less communication cost. This conforms with the intuition that larger γ and smaller ρ characterize systems with "stronger" stability, c.f. (2), so the control action can be updated less frequently. In the next section, we show that this design achieves global asymptotic stability and guarantees that the inter-event times are uniformly lower bounded.

C. Convergence Analysis under Event-triggered Law

We start by establishing the asymptotic stability of the closed-loop system under the event-triggered law (9).

Corollary 4.3: (Closed-loop Asymptotic Stability): Under the assumptions of Proposition 4.2, if (14) is satisfied, then there exists $\beta \in \mathcal{KL}$ such that for any $x(0) \in \mathbb{R}^n$ and bounded $\{u(t)\}_{t=\phi(0)}^{0}$, we have,

$$|x(t)| + \sup_{\phi(t)\leq\tau\leq t} |u(\tau)| \leq \beta\Big(|x(0)| + \sup_{\phi(0)\leq\tau\leq 0} |u(\tau)|, t\Big), \tag{15}$$

for all $t \geq 0$. •

Corollary 4.3 guarantees that the closed-loop system is globally asymptotically stable under the event-triggered design as long as (14) holds. To address Problem 1 completely, we need to ensure that the event-triggered law gives rise to executions that are feasible, meaning that triggering times do not have a finite accumulation point. The next result establishes a stronger fact, that is, that the inter-event times are uniformly lower bounded.

Proposition 4.4: (Uniform Lower Bound for the Inter-Event Times): For the system (3) under the control (6)-(9) and the triggering condition (14), $t_{k+1}-t_k \geq \delta$ for all $k \geq 1$ where δ is the time that it takes for the solution of

$$\dot{r} = M_2(1+r)(L_f(1+L_K)+L_f L_K r), \qquad (16)$$

to go from 0 to $\frac{1}{2L_{\gamma^{-1}\rho/\theta}L_K}$. •

Proposition 4.4 is of both theoretical and practical importance. From a theoretical perspective, the result guarantees that the solution of the hybrid system (3), (7), (9) exists for all time $t \geq 0$. From a practical point of view, since (16) fully determines the lower bound on the inter-event times and can be computed a priori, a designer can determine the requirements on the hardware that are necessary to make the design implementable.

V. The Linear Case

In this section, we show how the general treatment of Section IV is specialized and simplified if the dynamics (3) is linear, i.e, when we have

$$\dot{x}(t) = Ax(t) + Bu(\phi(t)), \qquad t \geq 0, \qquad (17)$$

subject to initial conditions $x(0) \in \mathbb{R}^n$ and bounded $\{u(t)\}_{t=\phi(0)}^0$. Assuming that the pair (A, B) is controllable, we can use pole placement to find a linear feedback law $K : \mathbb{R}^n \to \mathbb{R}$ that makes (5) ISS. Moreover, $p(t)$ can be explicitly solved from (7) to obtain

$$p(t) = e^{A(\sigma(t)-t^+)}x(t^+) + \int_{\phi(t^+)}^{t} \dot{\sigma}(s)e^{A(\sigma(t)-\sigma(s))}Bu(s)ds, \qquad (18)$$

for all $t \geq \phi(0)$ and the closed-loop system takes the form

$$\dot{x}(t) = (A+BK)x(t) + Bw(\phi(t)) + BKe(\phi(t)).$$

Furthermore, given an arbitrary $Q = Q^T > 0$, the continuously differentiable function $S : \mathbb{R}^n \to \mathbb{R}$ is given by

$$S(x) = x^T P x,$$

where $P = P^T > 0$ is the unique solution to the Lyapunov equation

$$(A+BK)^T P + P(A+BK) = -Q.$$

It is clear that (11) holds with $\alpha_1(r) = \lambda_{\min}(P)r^2$ and $\alpha_2(r) = \lambda_{\max}(P)r^2$. To show (12), notice that using Young's inequality [17],

$$\begin{aligned}\mathcal{L}_f S(x(t)) =& -x(t)^T Q x(t) \\ &+ 2x(t)^T PB(w(\phi(t)) + Ke(\phi(t))),\end{aligned}$$

so (12) holds with $\gamma(r) = \frac{1}{2}\lambda_{\min}(Q)r^2$ and $\rho(r) = \frac{2|PB|^2}{\lambda_{\min}(Q)}r^2$. In this case, the trigger (14) takes the simpler form

$$|e(t)| \leq \frac{\lambda_{\min}(Q)\sqrt{\theta}}{4|PB||K|}|p(t)|. \qquad (19)$$

In addition to the simplifications, we show in the next section that the closed-loop system is globally exponentially stable in the linear case.

A. Exponential Stabilization under Event-triggered Control

In the next result we show that in the linear case we obtain the stronger feature of global exponential stability of the closed-loop system, though this requires a slightly different Lyapunov-Krasovskii functional.

Theorem 5.1: (Exponential Stability of the Linear Case): The system (17) subject to the piecewise-constant closed-loop control

$$u(t) = Kp(t_k), \qquad t \in [t_k, t_{k+1}),$$

with $p(t)$ given in (18) and $\{t_k\}_{k=1}^{\infty}$ determined according to (19) satisfies

$$|x(t)|^2 + \int_{\phi(t)}^{t} u(\tau)^2 d\tau \leq Ce^{-\mu t}\left(|x(0)|^2 + \int_{\phi(0)}^{0} u(\tau)^2 d\tau\right),$$

for some $C > 0$, $\mu = \frac{(2-\theta)\lambda_{\min}(Q)}{4\lambda_{\max}(P)}$, and all $t \geq 0$. •

As given by Theorem 5.1, the exponential rate of convergence μ depends on θ and the matrices Q and P. The ratio $\frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)}$ depends on the closed-loop matrix $A+BK$ and can be increased by placing the eigenvalues of $A+BK$ at larger negative values, though care should be taken as this also amplifies the noise in practical implementations. We can also increase μ by decreasing θ which comes at the cost of faster event triggering and more communications. We analyze this trade-off in detail in the next section.

B. Optimizing the Communication-Convergence Trade-off

In this section we discuss the role of the free parameters of our design in optimizing the performance of the closed-loop system. As it can be seen from the Lyapunov analysis of Section IV, in an even-triggering scenario, the controller keeps the control unchanged until an appropriate Lyapunov function tends to increase, when the controller updates the control signal to maintain a desired decay of the Lyapunov function. Clearly, this will potentially come at the cost of slowing down the convergence speed of the closed-loop system. This trade-off is more clear in the linear case, where increasing the parameter $\theta \in (0, 1)$ allows for less communications through (19) but decreases the convergence rate μ given by Theorem 5.1.

To quantify this trade-off, we define two objective functions and formulate the trade-off as a multi-objective optimization. Let δ be the time that it takes for the solution of (16) to go from 0 to $\frac{1}{2L_{\gamma^{-1}\rho/\theta}L_K}$. As shown in Section IV-C, the inter-event times are lower bounded by δ, so it can be used to roughly measure the communication cost of the control scheme. For ease of notation, let

$$a = M_2 L_f L_K, \quad c = M_2 L_f(1+L_K), \quad R = \frac{1}{2L_{\gamma^{-1}\rho/\theta} L_K}$$

where $L_f = \sqrt{2}(|A|+|B|)$, $L_K = |K|$, and $L_{\gamma^{-1}\rho/\theta} = \frac{2|PB|}{\lambda_{\min}(Q)\sqrt{\theta}}$. Then, the solution of (16) with initial condition $r(0) = 0$ is given by $r(t) = \frac{ce^{at} - ce^{ct}}{ae^{ct} - ce^{at}}$. Solving $r(\delta) = R$ for δ gives

$$\delta = \frac{\ln \frac{c+Ra}{c+Rc}}{a-c}.$$

The objective is to maximize δ and μ by tuning the optimization variables θ and Q. For the sake of simplicity, let $\theta = \nu^2$ and $Q = qI_n$ where $\nu, q > 0$. Then, our objective functions take the explicit form

$$\delta(\nu) = \frac{1}{a-c} \ln \frac{c + \frac{\nu}{|P_1 B||K|} a}{c + \frac{\nu}{|P_1 B||K|} c}, \quad \mu(\nu) = \frac{2-\nu^2}{4\lambda_{\max}(P_1)},$$

where $P_1 = q^{-1}P$ is the solution of the Lyapunov equation $(A+BK)^T P_1 + P_1(A+BK) = -I_n$. Figure 1 depicts δ and μ as functions of ν and illustrates the communication-convergence trade-off.

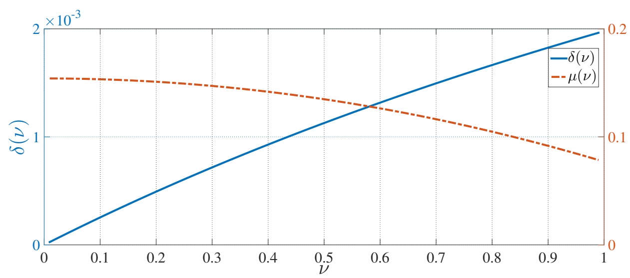


Fig. 1: The values of the lower bound of the inter-event times (δ) and exponential rate of convergence (μ) for different values of the optimization parameter ν for a third-order unstable linear system with $M_2 = 1$. This graph clearly illustrates the communication-convergence trade-off.

To balance these two objectives, we define the aggregate objective function as a convex combination of δ and μ, i.e.,

$$J(\nu) = \lambda\delta(\nu) + (1-\lambda)\mu(\nu),$$

where $\lambda \in [0,1]$ determines the (subjective) relative importance of convergence rate and communication cost. Notice that due to the difference between the (physical) units of δ and μ, one might multiply either one by a unifying constant, but we are not doing this as it leads to an equivalent optimization problem with a different λ. It is straightforward to verify that J is strongly convex and its unique maximizer is given by the positive real solution of $c_3\nu^3 + c_2\nu^2 + c_1\nu + c_0 = 0$ where $c_3 = a(1-\lambda)$, $c_2 = (a+c)|P_1B||K|(1-\lambda)$, $c_1 = c|P_1B|^2|K|^2(1-\lambda)$, and $c_0 = -2\lambda_{\max}(P_1)|P_1B||K|\lambda$. Figure 2 illustrates the optimizer of the aggregate objective function $J(\nu)$ for different values of the weighting factor λ.

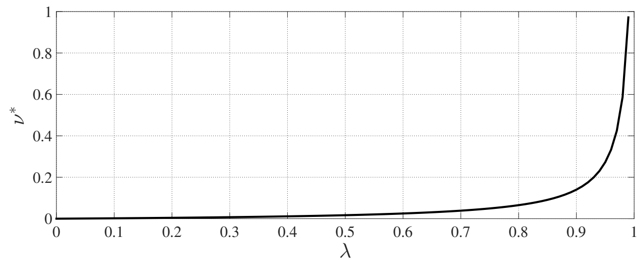


Fig. 2: The unique maximizer ν^* of the aggregate objective function $J(\nu)$ for different values of the weighting factor λ. It can be seen that as λ goes from 0 to 1, more weight is given to the maximization of δ which increases ν^*.

VI. Simulations

Here we illustrate the performance of our event-triggered predictor-based design. Example 6.1 is a two-dimensional nonlinear system that satisfies all the hypotheses required to ensure global asymptotic convergence of the closed-loop system. Example 6.2 is a different two-dimensional nonlinear system which instead does not, but for which we observe convergence in simulation.

Example 6.1: (Compliant Nonlinear System): Consider the 2-dimensional system given by

$$f(x,u) = \begin{bmatrix} x_1 + x_2 \\ \tanh(x_1) + x_2 + u \end{bmatrix}, \quad t - \phi(t) = \frac{1+t}{1+2t}.$$

This system satisfies all the assumptions of our design with the feedback law $K(x) = -6x_1 - 5x_2 - \tanh(x_1)$. In particular, we have

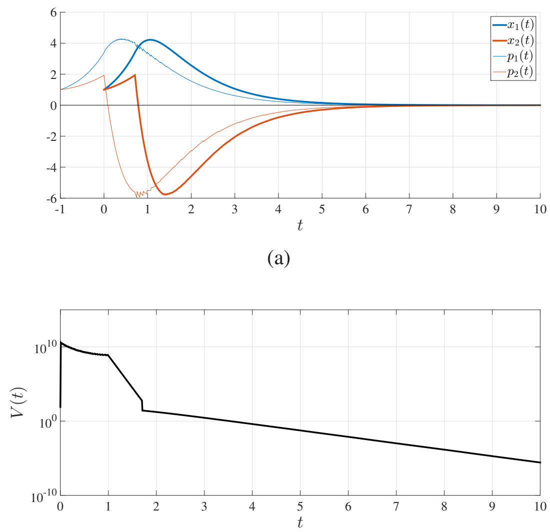
$$L_f = 2\sqrt{3}, \quad L_K = 7\sqrt{2}, \quad M_0 = 1, \quad m_2 = 1, \quad M_1 = 2,$$
$$S(x) = x^T P x, \quad \gamma(r) = \frac{\lambda_{\min}(Q)}{2} r^2, \quad \rho(r) = \frac{2|PB|^2}{\lambda_{\min}(Q)} r^2,$$

where $P = P^T > 0$ is the solution of $(A+Bk)^T P + P(A+Bk) = -Q$ for $A = [1\ 1;\ 0\ 1]$, $B = [0;\ 1]$, $k = [-6\ \ -5]$, and arbitrary $Q = Q^T > 0$. The simulation results of this example are depicted in Figure 3 for $\theta = 0.5$ and $b = 10$. In practice, b should be chosen sufficiently large to make V monotonically decreasing but the choice of θ is subjective. It is to be noted that for this example, (14) simplifies to $|e(t)| \le \overline{\rho}|p(t)|$ with $\overline{\rho} = 0.015$, but the closed-loop system remains stable when increasing $\overline{\rho}$ until 0.9. •

Example 6.2: (Non-compliant Nonlinear System): Here, we consider an example that violates several of the hypotheses required by Proposition 4.2 and Corollary 4.3 to guarantee asymptotic stability and study the performance of the proposed algorithm. Let

$$f(x,u) = \begin{bmatrix} x_1 + x_2 \\ x_2 + x_1^3 + u \end{bmatrix}, \quad t - \phi(t) = D + a\sin(t),$$

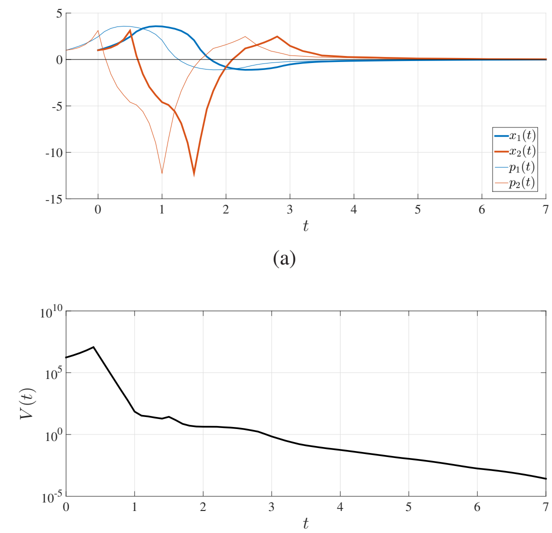
where the nominal delay $D = 0.5$ is known but the perturbation magnitude $a = 0.05$ is not. The control law $K(x) = -6x_1 - 5x_2 - x_1^3$ makes the closed-loop system ISS, but is not globally Lipschitz. Furthermore, the zero-input system exhibits finite escape time. The controller is



(a)

(b)

Fig. 3: Simulation of (a) state and predictor trajectories and (b) Lyapunov-Krasovskii functional V for Example 6.1. Note the logarithmic scaling of the vertical axis in (b). The initial large value of V is due to the contribution of L, which increases exponentially with b.



(a)

(b)

Fig. 4: Simulation of (a) state and predictor trajectories and (b) the Lyapunov-Krasovskii functional V for Example 6.2. We have set $\theta = 0.5$ and $b = 10$. (14) is replaced with $|e| \leq \overline{\rho}|p|$, and $\overline{\rho} = 0.5$. Note the logarithmic scaling of the vertical axis in (b). The initial large value of V is due to the contribution of L, which increases exponentially with b.

designed by *assuming* that $\phi(t) = t - D$. The simulation results of this example are illustrated in Figure 4. It can be seen that although V is not monotonically decreasing, the event-triggered control law is able to stabilize the system, showing that the control law is applicable to a wider class of systems than those satisfying the assumptions. $\bullet$

VII. Conclusions and Future Work

We have considered the problem of event-triggered control design for nonlinear systems with time-varying input delay. Based on the notion of predictor feedback, we have designed an event-triggered control law that can globally asymptotically stabilize the closed-loop system for arbitrary time delays. We further proved that the inter-event times for our triggering condition are uniformly lower bounded. We have studied the particular case of a controllable linear system and showed that the closed-loop system is exponentially stable. We also analyzed the trade-off between communication cost and convergence speed for the case of linear systems. Future work will include the extension of this approach to systems with disturbances and systems with unknown time delays, as well as networked control scenarios with multiple agents.

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References

[1] H. Kopetz, *Operating Systems of the 90s and Beyond: International Workshop Proceedings*. Springer Berlin Heidelberg, 1991, ch. Event-triggered versus time-triggered real-time systems, pp. 86–101.

[2] K. J. Åström and B. M. Bernhardsson., "Comparison of Riemann and Lebesgue sampling for first-order stochastic systems," in *IEEE Conf. on Decision and Control*, Las Vegas, NV, Dec. 2002, pp. 2011–2016.

[3] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks," *IEEE Transactions on Automatic Control*, vol. 52, no. 9, pp. 1680–1685, 2007.

[4] X. Wang and M. D. Lemmon, "Event-triggering in distributed networked control systems," *IEEE Transactions on Automatic Control*, vol. 56, no. 3, pp. 586–601, 2011.

[5] W. P. M. H. Heemels, K. H. Johansson, and P. Tabuada, "An introduction to event-triggered and self-triggered control," in *IEEE Conf. on Decision and Control*, Maui, HI, 2012, pp. 3270–3285.

[6] O. J. M. Smith, "A controller to overcome dead time," *ISA Transactions*, vol. 6, no. 2, pp. 28–33, 1959.

[7] D. Q. Mayne, "Control of linear systems with time delay," *Electronics Letters*, vol. 4, no. 20, pp. 439–440, October 1968.

[8] A. Manitius and A. Olbrot, "Finite spectrum assignment problem for systems with delays," *IEEE Transactions on Automatic Control*, vol. 24, no. 4, pp. 541–552, Aug 1979.

[9] M. T. Nihtila, "Finite pole assignment for systems with time-varying input delays," in *Decision and Control, Proceedings of the 30th IEEE Conference on*, vol. 1, Dec 1991, pp. 927–928.

[10] M. Krstic, *Delay Compensation for Nonlinear, Adaptive, and PDE Systems*, 1st ed., ser. Systems & Control: Foundations & Applications. Birkhäuser, 2009.

[11] N. Bekiaris-Liberis and M. Krstic, *Nonlinear Control Under Nonconstant Delays*, ser. Advances in Design and Control. SIAM, 2013.

[12] L. Mirkin, "On the approximation of distributed-delay control laws," *Systems & Control Letters*, vol. 51, no. 5, pp. 331 – 342, 2004.

[13] Q. C. Zhong, "On distributed delay in linear control laws-part i: discrete-delay implementations," *IEEE Transactions on Automatic Control*, vol. 49, no. 11, pp. 2074–2080, Nov 2004.

[14] W. Wu, S. Reimann, D. Grges, and S. Liu, "Suboptimal event-triggered control for time-delayed linear systems," *IEEE Transactions on Automatic Control*, vol. 60, no. 5, pp. 1386–1391, May 2015.

[15] L. Hetel, J. Daafouz, and C. Iung, "Stabilization of arbitrary switched linear systems with unknown time-varying delays," *IEEE Transactions on Automatic Control*, vol. 51, no. 10, pp. 1668–1674, Oct 2006.

[16] E. D. Sontag and Y. Wang, "On characterizations of the input-to-state stability property," *Systems & Control Letters*, vol. 24, no. 5, pp. 351–359, 1995.

[17] W. H. Young, "On classes of summable functions and their Fourier series," *Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character*, vol. 87, no. 594, pp. 225–229, 1912.