Grid-Connected Microgrid Participation in Frequency-Regulation Markets via Hierarchical Coordination

Chin-Yao Chang Sonia Martínez Jorge Cortés

Abstract— **The large integration of renewable energy into the power grid requires frequency regulation and ancillary services support. Grid-connected microgrids, which naturally include a portfolio of distributed energy resources and flexible loads, are a promising tool that can provide this utility by adapting their tie-line power to track frequency regulation signals. In this paper, we propose a framework by which grid-connected microgrids can participate in a frequency-regulation market and respond to a frequency-regulation request by leveraging their distributed energy resources and loads. The proposed framework is hierarchical, with a central and lower layers focusing on a microgrid. The central layer solves an energy dispatch problem that aims to match the tie-line power to the reference signal from the Regional Transmission Organization (RTO). The lower layer post-adjusts the optimum of the central layer through distributed optimization with consideration of the power flow constraints and uncertain loads in the microgrid. The framework combines recent progress on distributed algorithms, optimal power flow solvers, and regulation market in a novel way. Simulations demonstrate the effectiveness of the proposed framework.**

I. Introduction

Frequency regulation is a power-grid operation aimed at maintaining the power-grid nominal frequency by compensating the imbalance between generation and demand. Power imbalances and deviation of frequency can negatively affect energy-consuming devices and even lead to blackouts. Traditionally, frequency regulation services have been provided by traditional assets, such as gas turbines or coal generation plants. More recently, with advancements in novel technology such as storage, renewable generation and controllable loads are also being considered for this purpose. A microgrid owns or supervises many distributed energy resources (DERs) and flexible loads (FLs). However, when grid-connected, those resources are typically controlled at a constant power source in the microgrid. A change in operation would allow microgrids to participate in the frequency regulation market, which will bring additional economic and operational benefits. Motivated by this, we present a framework directed toward this vision and leverages on a distributed coordinated algorithm for resource control.

Literature review: Frequency regulation requires the real-time and accurate tracking of the automatic generation control (AGC) signal, which reflects the level of power imbalance. A traditional frequency-regulation market only pays participating resources by their regulation capacity. However, the compensation method does not reimburse resources for a high-ramp rate and accurate tracking capability. The Federal Energy Regulatory Commission (FERC) order 755 addresses this issue by reforming the regulation market into a two-component market-based compensation [1]. One is component compensates resources for their provision of frequency regulation capacity. The other one is known as the *mileage* payment and reflects the true total power adjustment throughout the regulation time. In this paper, we consider such frequency-regulation market.

Electricity storage and thermostatic controlled loads (TCLs) become more valuable in the new regulation market due to their fast-ramp rates, which has been considered in recent work [2], [3]. All the works mentioned above aggregate same types of appliances for frequency regulation. However, coordination of heterogeneous loads can further enhance their financial potential. This aggregation is more challenging and only few works consider such extension; see [4]. Grid-connected microgrid is a natural standalone entity that involves various DERs and FLs. Using the existing infrastructures of microgrid to coordinate those resources for frequency regulation is promising. In such application, the tie-line power of the microgrid to the bulk power grid serves to realize the power injection of the resources, and to track the reference signal specified by the Regional Transmission Organization (RTO). The ideal tracking involves minimization of power adjustment costs and consideration of interconnected electrical network for tracking accuracy. In [5], distributed algorithms for real-time power scheduling of DERs are considered without consideration of power flow constraints. Several works consider distributed optimal power flow (OPF) problems of microgrids that enable near real-time applications [6], [7], but are not suitable for frequency regulation with signals that change in the order of seconds.

Statement of contributions: This paper proposes a framework that enables grid-connected microgrids to support frequency regulation services. We first review the recent reform of the frequency regulation market that emphasizes the tracking accuracy. We next show that the uniform clearing scheme in the regulation market encourages all resources to bid with their marginal cost. With a well-established frequency market, we then focus on developing a framework that coordinates the resources in the microgrid to track the reference signal with its tie-line power. In this hierarchy, the microgrid central layer solves a simple energy dispatch problem to minimize the cost of total power adjustment while matching the reference signal. We then formulate a low-complexity distributed optimization problem that in-

C.-Y. Chang, Sonia Martínez, and Jorge Cortés are with the Department of Mechanical and Aerospace Engineering, University of California, San Diego. Email: {chc433, soniamd, cortes}@ucsd.edu

corporates the optimum of the previous layer, the power flow constraints, and uncertain loads of the microgrid. The optimal solution of the distributed optimization problem encodes the optimum of the OPF problem for the microgrid. Then, the parts that are used for the frequency regulation purpose are extracted. The distributed optimization and the extraction process subtly avoid the highly complex OPF problem. Simulations demonstrate that the two-layer computation framework can find the solution within the time that matches the small sampling time of the reference signal. For reasons of space, proofs for the results are omitted and will appear elsewhere.

II. Preliminaries

This section introduces notation, graph-theoretic and optimization concepts used throughout the paper.

Notation: We denote the set of natural, real, and complex numbers by $\mathbb{N}$, $\mathbb{R}$ and $\mathbb{C}$, respectively. Let $\mathbb{R}^n$, $\mathbb{R}^n_+$, and $\mathbb{C}^n$ be the sets of n-dimensional real, non-negative real, and complex vectors, respectively. Let $\mathbb{S}_+$ and $\mathcal{H}^n$ be the set of positive semidefinite matrices and n-dimensional Hermitian matrices, respectively. The cardinality of a set $\mathcal{N}$ is denoted by $|\mathcal{N}|$. For a complex number $a \in \mathbb{C}$, we let $|a|$ and $\angle a$ denote its complex modulus and angle. We denote by $\|v\|$ the 2-norm of a complex vector $v \in \mathbb{C}^n$. For a complex matrix $A \in \mathbb{C}^{n_1 \times n_2}$, we let A^* be its conjugate transpose. Finally, $\mathbf{Tr}\{A\}$ denotes the trace of A. We next review basic concepts from graph theory in the following [8]. We denote an undirected graph as $\mathcal{G} = (\mathcal{N}, \mathcal{E})$, where $\mathcal{N} \subseteq \mathbb{N}$ is the set of vertices and $\mathcal{E}$ is a set of undirected pairs of vertices. An unordered pair of vertices or edge is denoted as $\{i,k\} = \{k,i\} \in \mathcal{E}$. The local neighborhood of a node $k \in \mathcal{N}$ in the undirected graph is denoted as $\mathcal{N}_k := \{l \in \mathcal{N} \mid \{l,k\} \in \mathcal{E}\} \cup \{k\}$. The degree of node k in $\mathcal{G}$ is the number $|\mathcal{N}_k| - 1$ of edges connected to k.

Convex Optimization: Following [9], consider a (primal) convex optimization problem of the form

$$\min_{x \in \mathbb{R}^n} f_0(x), \quad \text{s.t. } Ax = b,\ f_i(x) \leq 0,\ i = 1, \dots, m, \quad (1)$$

where $f_0, \dots, f_m : \mathbb{R}^n \to \mathbb{R}$ are convex functions, $A \in \mathbb{R}^{l \times n}$, $b \in \mathbb{R}^l$, and $Ax = b$ defines affine equality constraints. The dual problem of (1) is given as

$$\max_{\lambda \geq 0, \mu} \left(\min_x f_0(x) + \sum_{i=1}^m \lambda_i f_i(x) + \mu^\top (Ax - b) \right), \quad (2)$$

where λ and μ are known as Lagrange Multipliers. Let $p^\star$ and $d^\star$ be the optimal values of the primal and dual problems, respectively. Strong duality holds if $p^\star = d^\star$. Under strong duality, the following Karush-Kuhn-Tucker (KKT) conditions are a necessary and sufficient characterization of the optimality of the primal-dual solution $(x^\star, \lambda^\star, \mu^\star)$,

$$\begin{cases} 0 \in \bigtriangledown f_0(x^\star) + \sum_{i=1}^m \lambda_i^\star \bigtriangledown f_i(x^\star) + (\mu^\star)^\top A x^\star, \\ \lambda_i^\star f_i(x^\star) = 0, \quad \forall i = 1, \dots, m, \\ (\mu^\star)^\top (Ax^\star - b) = 0, \\ f_i(x^\star) \leq 0, \quad Ax^\star = b, \\ \lambda_i^\star \geq 0, \quad \forall i = 1, \dots, m. \end{cases}$$

These conditions correspond to stationarity, complementary slackness, and primal and dual feasibility, respectively. A refined version of Slater's condition holds if there exist $x \in \mathbb{R}^n$ such that

$$Ax = b \text{ and } f_i(x) < 0, \quad \forall i = 1, \dots, m.$$

Slater's condition implies that strong duality holds.

III. Frequency Regulation Market

The frequency regulation market is operated by a nonprofit corporation known as RTO that coordinates the response of the participating energy resources (e.g., fast-ramping generators, electricity storage, aggregated TCLs, pool pumps, or appliances participating in demand response). The goal of the frequency regulation market is to assign the regulation signal (or reference signal) to the resources with the purpose of restoring power balance of the power network. The consecutive phases of the market are as follows, cf. [10],

- *Qualification:* Every potential resource that aims to participate in the frequency regulation market should meet certain minimum threshold on their ramp rate and capacity. The resource should also pass a regulation test from the RTO to demonstrate its capability of tracking.
- *Off-line bidding:* Every resource can either bid in the day-ahead or real-time regulation markets. The bid includes the resource's maximum capacity $\overline{c}_i \in \mathbb{R}_+$, the capacity unit price $p_i^c \in \mathbb{R}_+$, and the price per mileage $p_i^m \in \mathbb{R}_+$.
- *Market clearing:* The RTO clears the market with a capacity and mileage clearing price. The RTO also assigns capacity and mileage to the resources.
- *Real-time signal tracking:* During the regulation period, every resource's power tracks the regulation signal.
- *Post settlement:* The RTO makes capacity and mileage payment to each resource based on their assigned capacity and actual mileage during the tracking phase.

The concepts of capacity and mileage of a resource are explained in the following. For simplicity, we only consider regulation up, where the regulation signal is always positive. The capacity $c_i \in \mathbb{R}_+$ of resource i is the absolute value of the maximum power that bounds the assigned regulation signal during the entire duration of regulation time. Referring to Figure 1, the capacity upper bounds all of the regulation way points. The mileage m_i is defined as the sum of the absolute changes of the regulation way points, which corresponds to the sum of the lengths of the solid lines in Figure 1. The tracking error is the difference between the actual telemetry versus the regulation signals, illustrated in the right of Figure 1. A smaller tracking error means better actual mileage service. Since the RTO's payment to the resources is based on their actual mileage, they have an incentive to track the regulation way points accurately.

A. Regulation Market Clearance

Here, we formalize the market clearance mechanism for capacity and mileage employed by the RTO and examine the soundedness of employing honest bidding when dealing

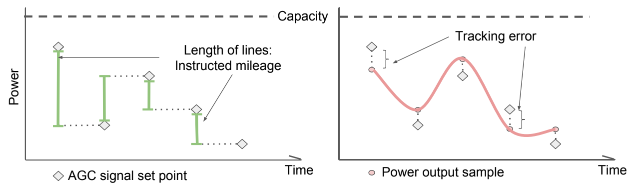

Fig. 1. Illustration of computation of mileage

with two products. Let $\mathcal{V}$ be the set of participating resources. Given the amount of total regulation capacity R^c and mileage R^m needed, the RTO solves the following optimization problem for the optimal capacity $c^\star \in \mathbb{R}^n_+$ and mileage $m^\star \in \mathbb{R}^n_+$,

$$\textbf{(P1)} \quad \min_{c,m} \sum_{i\in\mathcal{V}} c_i p_i^c + m_i p_i^m$$

$$\textbf{s.t.} \sum_{i\in\mathcal{V}} c_i \geq R^c, \quad \sum_{i\in\mathcal{V}} m_i \geq R^m, \tag{3a}$$

$$0 \leq c_i \leq \bar{c}_i, \quad c_i \leq m_i \leq \beta_i c_i \qquad \forall i \in \mathcal{V},$$

where $n = |\mathcal{V}|$ and $\beta_i > 1$ is a mileage multiplier for resource $i \in \mathcal{V}$. The RTO computes the mileage multiplier based on the ramp rate and historical tracking performance of each resource, cf. [11]. Resources with high historical performance and ramp rate are likely to be rewarded high mileage multipliers, which potentially increases the income. Solving **(P1)** gives an optimal capacity and mileage dispatch that satisfies the regional need (R^c, R^m) and the capacity and mileage limits of the resources.

The optimal Lagrange multipliers associated with constraints (3a), $\lambda_c^\star$ and $\lambda_m^\star$, define the market clearance prices for the capacity and mileage. The regulation market adopts a uniform market clearing policy, and hence every resource is paid the same unit prices, given by $\lambda_c^\star$ and $\lambda_m^\star$. We make the following assumption, commonly used in perfect competition market scenarios.

Assumption III.1. ***(Price-taker resources).*** *All resources are price takers; that is, $\lambda_c^\star$ and $\lambda_m^\star$ are set by the RTO.*

Assumption III.1 holds if there is no resource that has a dominating share of the total capacity or mileage of all resources. The marginal cost of a resource $i \in \mathcal{V}$ is the cost for it to provide unit additional capacity. We denote it $\partial p_i^c \in \mathbb{R}_+$. For simplicity, we assume that every resource has a constant marginal cost, which is fixed for any $0 \leq c_i \leq \bar{c}_i$. We define $\partial p_i^m \in \mathbb{R}_+$ in a similar way. It is well understood that in a perfect competition market scenario, the best strategy to maximize the revenue for a seller is bidding its marginal price [12]. However, the result only holds for the single product case, while the regulation market above involves two distinguished products: capacity and mileage. In principle, the resources may bid untruthfully with more than one products [13]. Proposition III.2 shows that, under Assumption III.1, every resource $i \in \mathcal{V}$ maximizes its revenue by bidding p_i^c and p_i^m with ∂p_i^c and ∂p_i^m, respectively.

Proposition III.2. ***(Honest bidding).*** *Under Assumption III.1, if the Slater's condition holds for* ***(P1)****, then every resource i maximizes its income by bidding*

$$p_i^c = \partial p_i^c \text{ and } p_i^m = \partial p_i^m. \tag{4}$$

After the market has been cleared, the actual regulation phase takes place, where the RTO assigns the regional regulation signal to the procured resources. Currently, this is done proportionally to the procured mileage of each resource. In case that this method violates the capacity limit of several resources, the RTO redistributes the overshoot power to other resources again proportionally to their procured mileage. The simple distribution method of the regulation signal above matches the clearing capacity and mileage well. In what follows, we do not get into the specifics of this, and instead assume that the scaled regulation signal given to the microgrid does not violate its maximum capacity and mileage.

IV. Microgrid Coordination Framework for Frequency Regulation

A microgrid owns or supervises various sets of DERs and FLs. The value of those resources for a standalone microgrid operation (islanded from the bulk power grid) is well recognized. Less customary, but equally appropriate, is the usage of these resources in providing frequency regulation services for the bulk power grid when the microgrid is in grid-connected mode. In this section, we consider a microgrid operating under this scenario. From the point of view of the RTO, the entire microgrid represents as single resource. The tie-line power of the microgrid to the bulk power grid serves to realize the power injection of this resource, and to track the regulation signal specified by the RTO. Our goal here is to develop a framework that describes how this can be done.

Consider a microgrid whose topology is given by $\mathcal{G}(\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is the set of buses and $\mathcal{E}$ is the set of lines. The microgrid interacts its nodes via a microgrid central controller (MGCC) system. The active and reactive power injections at bus i are given as P_i and Q_i, respectively. Here, we assume that some of the nodes $\mathcal{D} \subseteq \mathcal{N}$ have the flexibility to adjust their active power. Each agent $i \in \mathcal{D}$ provides its upper and lower bounds on active power injection ($\overline{P}_i$ and $\underline{P}_i$), cost of the active power adjustment p_i^d, and ramp rate r_i^d, to the microgrid central controller. Here, we propose that the MGCC employs a uniform clearing strategy so that p_i^d is the marginal cost of capacity for every $i \in \mathcal{D}$ if the market is perfect. With this information, the MGCC can estimate the aggregated capacity and price to bid with the RTO, and its capability to track a regulation signal. Both the estimation of the aggregated capacity/price for a microgrid and the development of a control framework that enables accurate real-time signal tracking are problems that need specific attention. Here, we focus on the latter question of how to perform signal tracking. Our proposed framework for the real-time regulation signal tracking consists of two layers: a centralized power dispatch and the implementation of a distributed algorithm that post-modifies the optimum of the centralized layer. We discuss both next.

A. Centralized Power Dispatch of Resources in Microgrid

For every given time stamp t, microgrid receives a regulation signal that it needs to track in the following time stamp, P_{tot}^{t+}. In order to track P_{tot}^{t+}, the MGCC solves the following economic dispatch problem,

$$
\begin{aligned}
\textbf{(P2)}\quad & \min_{P} \sum_{i\in\mathcal{D}} p_i^d P_i, \\
\text{s.t.}\ & \sum_{i\in\mathcal{N}\setminus\mathcal{D}} P_i^t + \sum_{i\in\mathcal{D}} P_i = P_{\text{tot}}^{t+}, \qquad (5)\\
& \underline{P}_i \le P_i \le \overline{P}_i,\ P_i^t - r_i^d T \le P_i \le P_i^t + r_i^d T,\ \forall i\in\mathcal{D},
\end{aligned}
$$

where T is the sampling time of the regulation signal P_{tot}^{t+}, P_i^t is the active power of node $i\in\mathcal{N}\setminus\mathcal{D}$ at time t, and $P\in\mathbb{R}^{|\mathcal{D}|}$ is the collection of active powers, P_i, $i\in\mathcal{D}$. The optimum of **(P2)**, $P^\star$, serves as a solution for tracking P_{tot}^{t+} at time $t_+ > t$. Note that for nodes $i\in\mathcal{N}\setminus\mathcal{D}$, we use P_i^t as an estimation of the nodal power at time t_+. The inequality constraints in **(P2)** ensure that the power limits and ramp constraint of every node $i\in\mathcal{D}$ are satisfied. Note that optimization **(P2)** is a simple linear programming problem and the time to solve it is negligible for reasonably sized microgrid networks.

After solving **(P2)**, the MGCC broadcasts the optimum to all the flexible resources $\mathcal{D}$. Every node $i\in\mathcal{D}$ then tracks its assigned power. Ideally, the tie-line power should match P_{tot}^{t+}. However, we note that **(P2)** may lead to an oversimplified problem because:

(a) in lossy microgrids, constraint (5) may not hold.
(b) the power dispatch $P^\star$ may not be feasible due to the power flow and voltage constraints of the microgrid.
(c) the power of the buses in $\mathcal{N}\setminus\mathcal{D}$, P_i^t, may not be obtained in real time for **(P2)**. The MGCC can only estimate P_i^t, $i\in\mathcal{N}\setminus\mathcal{D}$, for constraint (5).

For the reasons above, solving **(P2)** alone is not satisfactory, especially when considering the high tracking performance that is required in the frequency-regulation market. We therefore propose that every node in the microgrid coordinates to solve a distributed optimization problem to post-adjust $P^\star$, as described next.

B. Distributed Optimization with Consideration of OPF

In this section, we formulate the distributed OPF problem that accounts for both the power flow constraints and $P^\star$. As it turns out, while such OPF problem formulation addresses all the issues (a)-(c), it is challenging to solve it in real time, which is a major impediment for participation in frequency regulation. Therefore, we apply a sequence of simplifications so that the resulting distributed optimization problem still encodes the power flow constraints, while making the solution implementable in real time.

We formulate the OPF problem with the phasor voltage $V_i\in\mathbb{C}$, $i\in\mathcal{N}$, as the decision variable. Let $\hat{V}_i\in\mathbb{C}^{N_i}$, $N_i = |\mathcal{N}_i|$, as a vector collecting the phasor voltages of all $k\in\mathcal{N}_i$. The following constraints are imposed

$$\underline{V}_i^2 \le |V_i|^2 \le \overline{V}_i^2, \quad \forall i\in\mathcal{N}, \tag{6a}$$

$$\underline{P}_i \le P_i(\hat{V}_i) \le \overline{P}_i, \quad \forall i\in\mathcal{N}, \tag{6b}$$

$$\underline{Q}_i \le Q_i(\hat{V}_i) \le \overline{Q}_i, \quad \forall i\in\mathcal{N}, \tag{6c}$$

$$|V_i - V_k|^2 \le \overline{V}_{ik}, \quad \forall\{i,k\}\in\mathcal{E}, \tag{6d}$$

where $P_i(\hat{V}_i)$ and $Q_i(\hat{V}_i)$ are given by the well-established power flow equations (PFEs)

$$P_i = \text{Tr}\{Y_i\hat{V}_i\hat{V}_i^*\}, \qquad Q_i = \text{Tr}\{\overline{Y}_i\hat{V}_i\hat{V}_i^*\}, \tag{7}$$

and $Y_i\in\mathcal{H}^{N_i}$ and $\overline{Y}_i\in\mathcal{H}^{N_i}$ are derived from the admittance matrix of the network. Notice that every constraint in Eq. (6)-(7) only depends on the local voltages of a node i and its neighbors. If there are communication links overlaying the physical microgrid network, then we can derive a distributed OPF problem that directly incorporates all the constraints (6)-(7). Define $W_i := \hat{V}_i\hat{V}_i^*\in\mathcal{H}^{N_i}$. We formulate the distributed OPF problem as follows

$$
\begin{aligned}
\textbf{(P3)}\quad & \min_{W_i\in\mathcal{W}_i,\forall i\in\mathcal{N}} \sum_{i\in\mathcal{D}} |P_i(W_i) - P_i^\star|, \\
& \text{s.t. } G_{ik}(W_i,W_k) = 0, \quad \forall\{i,k\}\in\mathcal{E}, \\
& \quad \text{rank}(W_i) = 1, \quad \forall i\in\mathcal{N},
\end{aligned}
$$

where the set $\mathcal{W}_i$ is defined by the constraints (6)-(7), and $G_{ik}(W_i,W_k)$ is linear in W_i and W_k. The constraint $G_{ik}(W_i,W_k) = 0$ ensures that all copies of the voltage at every node are the same.[1] We aim to simplify **(P3)** to an optimization problem that can be solved distributively with only few iterations. For instance, given the typical regulation-signal sampling time of about 2 seconds, we aim for 50 iterations per node, with each iteration taking about 40 milliseconds (which is a reasonable mark for commercial off-the-shelf hardware, see e.g., [15]).

We start our simplification of **(P3)** by applying the distributed SDP convex relaxation method of [6], which drops the rank constraint. This leads to

$$\textbf{(P4)}\ \min_{\substack{W_i\in\overline{\mathcal{W}}_i,\forall i\in\mathcal{N}\\ a_i\ge 0,\forall i\in\mathcal{D}}} \sum_{i\in\mathcal{D}} a_i,\ \text{ s.t. } G_{ik}(W_i,W_k) = 0,\ \forall\{i,k\}\in\mathcal{E}\,.$$

Note that in **(P4)**, we also introduce variables a_i, $i\in\mathcal{D}$, to replace the cost $|P_i(W_i) - P_i^\star|$, while adding the constraint $-a_i \le P_i(W_i) - P_i^\star \le a_i$ to the constraints (6)-(7) defining the new set $\overline{\mathcal{W}}_i$. We next consider the dual problem of **(P4)**. The derivation of the dual problem can be found in [6], so we will not repeat the lengthy derivation here. A compact form of the dual problem of **(P4)** is shown in the following

$$
\begin{aligned}
\textbf{(D4)}\quad & \min_{\substack{\gamma_{ik},\ \forall\{i,k\}\in\mathcal{E}\\ \eta_i\ge 0,\ \forall i\in\mathcal{N}}} \sum_{i\in\mathcal{N}} d_i^\top \eta_i, \\
& \text{s.t. } A_i(\eta_i,\gamma_i) = \sum_{l=1}^{i_n} A_{il}\eta_{il} + \sum_{k=1}^{N_k} B_{ik}\gamma_{ik} \succeq 0,\ \forall i\in\mathcal{N}, \\
& \quad \gamma_{ik} = \gamma_{ki}, \quad \forall\{i,k\}\in\mathcal{E},
\end{aligned}
$$

where i_n is the number of inequality constraints in $\overline{\mathcal{W}}_i$, $d_i\in\mathbb{R}^{i_n}$ is a constant vector derived from the constants in Eq. (6), $\eta_i\in\mathbb{R}^{i_n}$ is the dual variable associated with

[1] The distributed formulation **(P3)** is already a relaxation of standard OPF problems, see [14] for details.

the constraint $W_i \in \overline{\mathcal{W}}_i$, $\gamma_{ik} \in \mathbb{R}^4$ is the dual variable associated with $G_{ik}(W_i, W_k) = 0$, and A_{il} and B_{ik} are N_i-dimensional Hermitian matrices. In the formulation above, each terminal node of $\{i, k\}$ has a copy of the dual variable for $G_{ik}(W_i, W_k) = 0$, written as γ_{ik} and γ_{ki}.

Remark IV.1. ***(Complexity comparison between (P4) and (D4)).*** The number of decision variables of **(P4)** and **(D4)** is close if the average degree of the microgrid graph is small (≈ 2), as is the case for instance of tree networks. In addition, the size of the packet exchanges through every communication link is similar between the two. However, in **(D4)**, every constraint except $A_i \succeq 0$ ($\forall i \in \mathcal{N}$) only relates to one decision variable, while the constraints in **(P4)** couple the variables together. We therefore expect **(D4)** to have a lower complexity than **(P4)**. The simulations support this observation with significant less computational iterations for **(D4)** than **(P4)**. □

Solving **(D4)** may not directly lead to the solution of the primal problem **(P4)**. Fortunately, under mild assumptions, solving **(D4)** is sufficient for frequency regulation. We elaborate on this point next. Let $(\eta^{\text{opt}}, \gamma^{\text{opt}})$ be the solution of **(D4)**. With a slight abuse of notation, let $A_i^{\text{opt}} = A_i(\eta_i^{\text{opt}}, \gamma_i^{\text{opt}})$, where $\gamma_i^{\text{opt}} \in \mathbb{R}^{4(N_i-1)}$ collects the optimal γ_{ik}, $k \in \mathcal{N}_i$. We make the following assumptions to relate **(D4)** to **(P3)**.

Assumption IV.2. ***(Rank Deficiency of A_i^{opt}).*** *Rank*$(A_i^{\text{opt}}) = N_i - 1$ *for all* $i \in \mathcal{N}$.

Assumption IV.3. ***(Strong duality).*** *The optima of **(P4)** and **(D4)** are the same.*

Recent work [6] shows that Assumption IV.2 holds for many existing power networks. Strong duality for the SDP convexified OPF formulation is a widely adopted assumption in the literature. Our next result relates the optima of **(D4)** and **(P3)**.

Proposition IV.4. ***(Optimum of (P3)).*** *If Assumptions IV.2 and IV.3 hold, then we have the optimum of **(P3)** given as*

$$W_i^{\text{opt}} = \alpha_i (v_i e_i)(v_i e_i)^*, \tag{8}$$

where $v_i \in \text{null}(A_i^{\text{opt}})$, *for some* $e_i \in \mathbb{C}$, $|e_i| = 1$, *and* $\alpha_i \in \mathbb{R}_+$, *for all* $i \in \mathcal{N}$.

To determine the optimal solution of **(P3)**, we need to find α_i and e_i, $\forall i \in \mathcal{N}$. Selecting e_i is in fact equivalent to finding a reference angle for the entire network. In other words, once e_i is chosen for a node i, there is a unique selection of e_k, $\forall k \in \mathcal{N} \setminus \{i\}$, which satisfies the functional relation given by Proposition IV.4 with respect to the optimal solution W_l^{opt}, $l \in \mathcal{N}$. To see this, note that as e_i is chosen at node i, every neighboring node $k \in \mathcal{N}_i$ can use $G_{ik}(W_i^{\text{opt}}, W_k^{\text{opt}}) = 0$ to compare the ratio of the real and imaginary parts of W_i^{opt} and W_k^{opt} to find e_k. Such iteration propagates from node i to reach the entire network. The number of such iterations may approach $|\mathcal{N}|$. Recall that our target maximal number of iterations is 50 and those iterations are mostly reserved for solving **(D4)**. Finding e_i for all $i \in \mathcal{N}$ requires too many iterations on top of the distributed algorithm for **(D4)**. Fortunately, finding e_i is unnecessary for several reasons. Proposition IV.5 discusses why this is the case.

Proposition IV.5. ***(Power invariance to reference angle).*** *Both P_i^{opt} and Q_i^{opt} are invariant under e_i, for all $i \in \mathcal{N}$. Namely,*

$$P_i^{\text{opt}} = \alpha_i \textbf{Tr}\{Y_i (v_i e_{i1})(v_i e_{i1})^*\} = \alpha_i \textbf{Tr}\{Y_i (v_i e_{i2})(v_i e_{i2})^*\}, \tag{9}$$

$$Q_i^{\text{opt}} = \alpha_i \textbf{Tr}\{\overline{Y}_i (v_i e_{i1})(v_i e_{i1})^*\} = \alpha_i \textbf{Tr}\{\overline{Y}_i (v_i e_{i2})(v_i e_{i2})^*\},$$

for all $e_{i1}, e_{i2} \in \mathbb{C}$ *such that* $|e_{i1}| = |e_{i2}| = 1$.

One can prove Proposition IV.5 by direct computation of (9) for any e since $|e| = 1$. Due to Proposition IV.5, every node i can capture P_i^{opt} and Q_i^{opt} without knowledge of e_i. Recall that every node i only needs to know the optimal active power P_i^{opt} for the frequency regulation, so finding e_i is unnecessary and we can avoid the effort of finding e_i. The nice consequence of this is that, if we extract P_i^{opt} and Q_i^{opt}, from **(D4)** by finding α_i and choosing $e_i = 1$, $\forall i \in \mathcal{N}$, there exists a voltage profile $V^{\text{opt}} \in \mathbb{C}^N$ that computes P_i^{opt}, Q_i^{opt}, and satisfies the network constraints (6)-(7). If the nodal power injection of every i is controlled as P^{opt}, Q^{opt}, then the network could to reach V^{opt} autonomously.

Thus, the last step is to find the scaling factor $\alpha_i > 0$, for all $i \in \mathcal{N}$, which allows for the computation of P_i^{opt} and Q_i^{opt}. Recall that $\eta_i \in \mathbb{R}^{i_n}$ for all i are the Lagrange multipliers associated with the constraints (6), where every constraint can be written in the form of $\textbf{Tr}\{M_{il} W_i\} \leq h_{il}$ for some $h_{il} \in \mathbb{R}$ and $M_{il} \in \mathcal{H}^{N_i}$. For any node i with $\eta_{il}^{\text{opt}} > 0$, for some $l = 1, \ldots, i_n$, we have $\textbf{Tr}\{M_{il} W_i^{\text{opt}}\} = h_{il}$ by the complementary slackness condition on **(D4)**. This equality can be used to find α_i as $\alpha_i = h_{il} / \textbf{Tr}\{M_{il} v_i v_i^*\}$. In this way, if α_i is known to a node i, all $k \in \mathcal{N}_i$ can find α_k by applying $G_{ik}(W_i, W_k) = 0$ and using α_i. The solution can then propagate through the entire network. The main difference is that when **(D4)** is solved, the number of nodes with known α_i is much bigger than the case of e_i, which is only one. As shown in Table I, empirical simulations on IEEE testbeds indicate that many nodes have at least one dual variable which is strictly positive. Hence, it requires less communication hops for every node to find its α_i compared to e_i case. We can characterize the number of communication hops needed by defining a subgraph that induced by $\mathcal{N}_0 \subset \mathcal{N}$, where $\mathcal{N}_0$ is the set of nodes with $\eta_i = 0$. The graph induced by $\mathcal{N}_0$ has a small diameter D. The nodes can find their α_i with at most $D + 1$ iterations, where D is typically much less than $|\mathcal{N}|$ as shown in Table I. We summarize the proposed two-layer power tracking in Algorithm 1.

V. Simulations

Through the simulations on several moderate size power networks, we demonstrate that the existing distributed algorithms can solve **(D4)** in less then the target 50 iterations per node. We choose IEEE 14, 30, 57 testbed and 47 bus microgrid example established in [16]. The generation nodes in those networks are viewed as nodes with DERs that can

Algorithm 1 Two-layer power tracking (TPT)

Initialize: $\alpha_i = 0$ and $e_i = 1$ for all $i \in \mathcal{N}$
RTO sends the regulation signal, P_{tot}^{t+}, to MGCC
MGCC solves (P2) and broadcast $P^\star$ to all $i \in \mathcal{N}$
Every $i \in \mathcal{N}$ cooperates to solve **(D4)**.
while $\exists i \in \mathcal{N}$ s.t. $\alpha_i = 0$
  For i s.t. $\alpha_i = 0$,
    If $\exists \eta_{il}^{\text{opt}} > 0$ or received $\alpha_k \in \mathcal{N}_i$
      Computes α_i and sends α_i to all $k \in \mathcal{N}_i$.
    end
end
Every $i \in \mathcal{N}$ finds W_i^{opt}, P_i^{opt}, Q_i^{opt} by Eq. (8)-(9).

TABLE I
CHARACTERIZATION OF $\mathcal{N}_0$ FOR EMPIRICAL TEST BEDS

	IEEE 14	IEEE 30	IEEE 57
Numb. of nodes with $\eta_i = 0$	3	4	5
Diam. of graph ind. by $\mathcal{N}_0$	1	0	1

adjust the power for frequency regulation. We implement the scheduled-asynchronous distributed optimization algorithm proposed in [17] to solve **(D4)**. The distributed algorithm gives one rank deficiency of A_i^{opt}, $\forall i \in \mathcal{N}$, for all the test cases above. As shown in Table V, the distributed algorithm can solve **(D4)** in the target iteration numbers (< 50). We further illustrate the convergence for IEEE 57 bus testbed in Figure. 2. The transformation to the dual problem is effective as the number of iterations needed is far less than the one in **(P4)** for the same distributed algorithm.

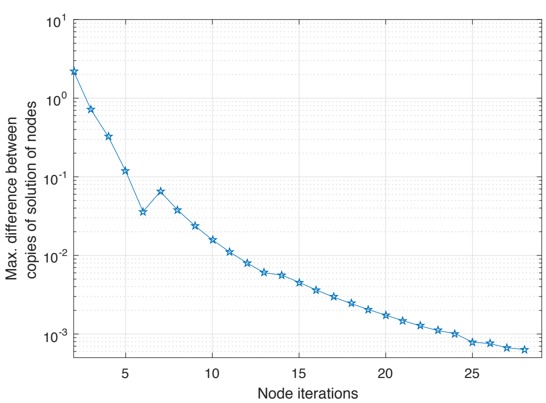

Fig. 2. Convergence of the scheduled-asynchronous distributed optimization algorithm for solving **(D4)** in the IEEE 57 testbed.

VI. CONCLUSIONS

We have considered the scenario of grid-connected microgrids supporting frequency regulation of the bulk grid. The microgrid acts as a single entity and groups heterogeneous

TABLE II
NUMBER OF ITERATIONS PER NODE FOR SOLVING **(D4)**

	IEEE 14	IEEE 30	47	IEEE 57
Iter. Numb. for **(D4)**	13	24	17	28
Iter. Numb. for **(P4)**	50	57	245	660

and small power consuming resources. We propose a two-layer framework that enables the microgrid to efficiently allocate the share of the regulation signal to the resources. The power allocation accounts the power flow constraints of the microgrid. In addition, the power allocation is near optimal in the sense that the total cost of power adjustment of the resources is minimized. Future work will further explore the possibility of grid-connected microgrid participation in the regulation and energy dispatch markets.

ACKNOWLEDGMENTS

This research was supported in part by the ARPA-e NODES program, Cooperative Agreement DE-AR0000695.

REFERENCES

[1] "Order No. 755: Frequency regulation compensation in the organized wholesale power markets," 2011, available at http://www.ferc.gov/whats-new/comm-meet/2011/102011/E-28.pdf.
[2] J. Donadee and M. D. Ilic, "Stochastic optimization of grid to vehicle frequency regulation capacity bids," *IEEE Transactions on Smart Grid*, vol. 5, no. 2, pp. 1061–1069, 2014.
[3] J. L. Mathieu, S. Koch, and D. S. Callaway, "State estimation and control of electric loads to manage real-time energy imbalance," *IEEE Transactions on Power Systems*, vol. 28, no. 1, pp. 430–440, 2013.
[4] L. Zhao and W. Zhang, "A unified stochastic hybrid system approach to aggregated load modeling for demand response," in *IEEE Conf. on Decision and Control*, 2015, pp. 6668–6673.
[5] E. Crisostomi, M. Liu, M. Raugi, and R. Shorten, "Plug-and-play distributed algorithms for optimized power generation in a microgrid," *IEEE Transactions on Smart Grid*, vol. 5, no. 4, pp. 2145–2154, 2014.
[6] C.-Y. Chang and W. Zhang, "On near and exact optimal power flow solutions for microgrid applications," in *IEEE Conf. on Decision and Control*, Las Vegas, NV, 2016.
[7] E. Dall'Anese, H. Zhu, and G. B. Giannakis, "Distributed optimal power flow for smart microgrids," *IEEE Transactions on Smart Grid*, vol. 4, no. 3, pp. 1464–1475, 2013.
[8] F. Bullo, J. Cortés, and S. Martínez, *Distributed Control of Robotic Networks*, ser. Applied Mathematics Series. Princeton University Press, 2009, electronically available at http://coordinationbook.info.
[9] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2009.
[10] M. Kintner-Meyer, "Regulatory policy and markets for energy storage in North America," *Proceedings of the IEEE*, vol. 102, no. 7, pp. 1065–1072, 2014.
[11] "CAISO Business Practice Manual for Market Operation," 2017, version 51. Available at https://bpmcm.caiso.com/Pages/BPMDetails.aspx?BPM=Market%20Operations.
[12] A. K. David and F. Wen, "Strategic bidding in competitive electricity markets: a literature survey," in *IEEE Power Engineering Society Summer Meeting*, vol. 4, 2000, pp. 2168–2173.
[13] V. Krishna, *Auction theory*. Academic press, 2009.
[14] S. Bose, S. H. Low, T. Teeraratkul, and B. Hassibi, "Equivalent relaxations of optimal power flow," *IEEE Transactions on Automatic Control*, vol. 60, no. 3, pp. 729–742, 2015.
[15] S. T. Cady, A. D. Domínguez-García, and C. N. Hadjicostis, "A distributed generation control architecture for islanded AC microgrids," *IEEE Transactions on Control Systems Technology*, vol. 23, no. 5, pp. 1717–1735, 2015.
[16] M. Farivar, C. R. Clarke, S. H. Low, and K. M. Chandy, "Inverter var control for distribution systems with renewables," in *IEEE International Conference on Smart Grid Communications (SmartGridComm)*, 2011, pp. 457–462.
[17] C.-Y. Chang, J. Cortés, and S. Martínez, "A scheduled-asynchronous distributed optimization algorithm for the optimal power flow problem," in *American Control Conference*, Seattle, WA, May 2017, pp. 3968–3973.