Frequency Regulation with Heterogeneous Energy Resources: A Realization using Distributed Control

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Abstract-This paper presents one of the first real-life demonstrations of coordinated and distributed resource control for 2 secondary frequency response in a power distribution grid. A 3 series of tests involved up to 69 heterogeneous active distributed 4 energy resources consisting of air handling units, unidirectional 5 and bidirectional electric vehicle charging stations, a battery energy storage system, and 107 passive distributed energy resources 7 consisting of building loads and solar photovoltaic systems. The 8 distributed control setup consists of a set of Raspberry Pi endpoints exchanging messages via an ethernet switch. Actuation 10 commands for the distributed energy resources are obtained by 11 solving a power allocation problem at every regulation instant 12 using distributed ratio-consensus, primal-dual, and Newton-like 13 algorithms. The problem formulation minimizes the sum of 14 distributed energy resource costs while tracking the aggregate set-15 point provided by the system operator. We demonstrate accurate 16 and fast real-time distributed computation of the optimization 17 solution and effective tracking of the regulation signal over 18 40 min time horizons. An economic benefit analysis confirms 19 eligibility to participate in an ancillary services market and 20 demonstrates up to \$53k of potential annual revenue for the 21 selected population of distributed energy resources. 22

I. INTRODUCTION

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24 Many recent efforts seek to integrate renewable energy resources with the power grid to reduce the carbon footprint. The 25 high variability associated with wind and solar power can be 26 balanced using distributed energy resources (DERs) providing 27 ancillary services such as frequency regulation. Consequently, 28 there is a growing interest among market operators in DER 29 aggregations with flexible generation and load capabilities 30 to balance fluctuations in grid frequency and minimize area 31 control errors (ACE). The fast ramping rate and minimal 32 marginal standby cost put many DERs at an advantage against 33 conventional generators and make them suitable for participa-34 tion in the frequency regulation market. 35

The fast ramping rates reduce the required power capacity of DERs to only 10% of an equivalent generator to balance

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DERs have small capacities, typically on the order of kWs 39 compared to 10 s of MW for conventional frequency control 40 resources. Commanding the required thousands to millions 41 of DERs to replace existing frequency regulation resources 42 over a large balancing area entails aggregating DERs that 43 are distributed at end points all over the grid on customer 44 premises. The dynamic nature, large number, and distributed 45 location of DERs requires coordination. This is in contrast to 46 existing frequency regulation [2] implementation with conven-47 tional energy resources. For example, California Independent 48 System Operator (CAISO) requires all generators to submit 49 their bids once per regulation interval. Then, the setpoints 50 are assigned centrally to all resources every 2-4 s without 51 any consideration of operational costs [3]. While distributed 52 control has the potential to enable DER participation in the 53 frequency regulation market (e.g., [4]), there is a general lack 54 of large-scale testing to prove its effectiveness for widespread 55 adoption by system operators. The 2017 National Renew-56 able Energy Laboratory Workshop on Autonomous Energy 57 Grids [5] concluded that "A major limitation in developing new 58 technologies for autonomous energy systems is that there are 59 no large-scale test cases (...). These test cases serve a critical 60 role in the development, validation, and dissemination of new 61 algorithms". 62

a frequency drop within 30 s [1]. However, most individual

The results of this paper are the outcome of a project under the ARPA-e Network Optimized Distributed Energy Systems (NODES) program¹, which postulates DER aggregations as virtual power plants that enable variable renewable penetrations of at least 50%. The vision of the NODES program was to employ state-of-the-art tools from control systems, computer science, and distributed systems to optimally respond to dynamic changes in the grid by leveraging DERs while maintaining customer quality of service. The NODES program required testing with at least 100 DERs at power. Here, we demonstrate the challenges and opportunities of testing on a heterogeneous fleet of DERs for eventual operationalization of optimal distributed control at frequency regulation time scales.

Literature Review. To the best of our knowledge, real-world 76 testing of frequency regulation by DERs has been limited. 77 A Vehicle-to-Grid (V2G) electric vehicle (EV) [6] and two 78 Battery Energy Storage System (BESS) [7] provided frequency 79 regulation. 76 bitumen tanks were integrated with a simplified 80 power system model to provide frequency regulation via a 81 decentralized control algorithm in [8]. In buildings, a decen-82 tralized control algorithm controlled lighting loads in a test 83

¹https://arpa-e.energy.gov/arpa-e-programs/nodes

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room [9], centralized frequency control was applied to an air 84 handling unit (AHU) [10], [11], an inverter and four household 85 appliances [12], and four heaters in different rooms [13]. A 86 laboratory home with an EV and an AHU, and a number 87 of simulated homes were considered for demand response 88 in [14] through an aggregator at a 10 s level. Technologies for 89 widespread, but centrally controlled, cycling of air condition-90 ers directly by utilities cf. [15] and aggregators are common 91 place for peak shifting, but occur over time scales of minutes 92 to hours. Industrial solutions enabling heterogeneous DERs to 93 94 track power signals also exist, but they are either centralized, cf. [16] or require all-to-all communication [17]. 95

Our literature review exposes the following limitations: (i) 96 centralized control or need for all-to-all communication [6], 97 [7], [10]–[17], which does not scale to millions of DERs; 98 (ii) small numbers of DERs [6], [7], [10]-[14]; (iii) lack of 99 diversity in DERs [6]–[11], [13], with associated differences in 100 tracking time scales and accuracy. No trial has been reported 101 that demonstrated generalizability to a real scenario with (i) 102 scalable distributed control and a (ii) large number of (iii) 103 heterogeneous DERs. 104

Statement of Contributions. To advance the field of real-105 world testing of DERs for frequency control, we conduct a 106 series of tests using a group of up to 69 active and 107 107 passive heterogeneous DERs on the University of California, 108 San Diego (UCSD) microgrid [18]. To the best of the authors' 109 knowledge, this is the first work to consider such a large, di-110 verse portfolio of real physical DERs for secondary frequency 111 response. As such, the major contributions of this work are: 112

- A detailed account of the testbed, including the DER actuation and sampling interfaces, the distributed optimization setup, and communication framework.
- A description of techniques to work around technical barriers, provision of lessons learned, and suggestions for future improvement.
- Evaluation of the performance of both the cyber and physical layers, including an evaluation of eligibility requirements for and the economic benefit of participating in the ancillary services market.

Paper Overview. Frequency regulation is simulated on the 123 UCSD microgrid using real controllable DERs (Section III-C) 124 to follow the Pennsylvania-New Jersey-Maryland Interconnec-125 tion (PJM) RegD signal [19] interpolated from 0.5Hz to 1Hz 126 (Sections III-B). The DER setpoint tracking is formulated 127 as a power allocation problem at every regulation instant 128 (Section III-A), and uses three types of provably convergent 129 distributed algorithms from [20]-[23] to solve the optimization 130 problem; see Appendix A of [24], removed from this version 131 for brevity. Setpoints are computed distributively on multiple 132 Raspberry Pi's communicating via ethernet switches (Section 133 III-D). The setpoints are implemented on up to 176 DERs at 134 power using dedicated command interfaces via TCP/IP com-135 munication (Section III-E), the DER power outputs monitored 136 137 (Section III-F), and their tracking performance evaluated (Section III-G). Results (Section V) for the various test scenarios 138 described in Section IV show that the test system tracks the 139 signal with reasonable error despite delays in response and 140 inaccurate tracking behavior of some groups of DERs, and 141

qualifies for participation in the PJM ancillary services market 142

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II. PROBLEM SETTING 144

This paper validates real-world DER controllability for par-145 ticipation in secondary frequency regulation through demon-146 stration tests implemented on a real distribution grid. The 147 tests showcase the ability of aggregated DERs to function as 148 a single market entity that responds to frequency regulation 149 requests from the independent system operator (ISO) by opti-150 mally coordinating DERs. The goal is to monitor and actuate 151 a set of real controllable DERs to collectively track a typical 152 automatic generation control (AGC) signal issued by the ISO. 153

Three different distributed coordination schemes optimize 154 the normalized contribution of each DER to the cumulative 155 active power signal. Unlike simulated models, the use of real 156 power hardware exposes implementation challenges associated 157 with measurement noise, sampling errors, data communica-158 tion problems, and DER response. To that end, precise load 159 tracking is pursued at timescales that differ by DER type 160 consistent with individual DER responsiveness and commu-161 nication latencies, yet meet frequency regulation requirements 162 in aggregation. 163

The 69 kV substation and 12 kV radial distribution system 164 owned by UCSD to operate the 5 km² campus was the 165 chosen demonstration testbed. It has diverse energy resources 166 with real-time monitoring and control capabilities, allowing 167 for active load tracking. This includes over 3 MW of solar 168 photovoltaic (PV) systems, 2.5 MW/5 MWh of BESS, building 169 heating ventilation and air conditioning (HVAC) systems in 170 14 million square feet of occupied space, and over 200 171 unidirectional V2G (V1G) [25] and V2G EV chargers. The 172 demonstration tests used a representative population of up to 173 176 such heterogeneous DERs to investigate tracking behavior 174 of specific DER types as well as their cooperative tracking 175 abilities. While the available DER capacity at UCSD far 176 exceeds the minimum requirements for an ancillary service 177 provider set by most ISOs (typically ~ 1 MW), logistical 178 considerations and controller capabilities dictated the choice 179 of a DER population size with less aggregate power capacity 180 (up to 184 kW) for this demonstration. Since this magnitude 181 of power is insufficient to measurably impact the actual grid 182 frequency, we chose to simulate frequency regulation by 183 following a frequency regulation signal. 184

III. TEST ELEMENTS

Here, we elaborate on the different elements of the vali-186 dation tests. These include the optimization formulation em-187 ployed to compute DER setpoints (Section III-A), the ref-188 erence AGC signal (Section III-B) and types of DERs used 189 to track it (Section III-C), the computing platform (Section 190 III-D), the actuation (Section III-E) and monitoring interfaces 191 (Section III-F), the performance metrics used to assess the cy-192 ber and physical layers, and eligibility for market participation 193 (Section III-G). 194

195 A. Optimization Formulation

The optimization model for AGC signal tracking using DERs can be mathematically stated as a separable resource allocation problem subject to box constraints as follows:

$$\min_{p \in \mathbb{R}^n} f(p) = \sum_{i=1}^n f_i(p_i),$$
s.t.
$$\sum_{i=1}^n p_i = P_{\text{ref}},$$

$$p_i \in [p_., \overline{p}_i], \quad \forall i \in \mathcal{N} = \{1, \dots, n\}.$$
(1)

The agents $i \in \mathcal{N}$ each have local ownership of a decision 199 variable $p_i \in \mathbb{R}$, representing an active power generation or 200 consumption quantity (setpoint), a local convex cost function 201 f_i , and local box constraints $[p, \overline{p}]$, representing active power 202 capacity limits. Pref is a given active power reference value 203 determined by the ISO and transmitted to a subset of the agents 204 as problem data, see e.g. [26]. Pref is a signal that changes 205 over time, so a new instance of (1) is solved in real-time 1 s 206 intervals corresponding to these changes. Note that with just 207 1 s difference between the instances, the box constraints might 208 also change due to the limited ramp rates of DERs. In this 209 work we consider them constant and assume (1) is feasible. 210

For the validation tests, we used two types of cost functions: 211 constant and quadratic. Constant functions were used for the 212 Ratio-Consensus (RC) solver [20], which turns the optimiza-213 tion into a feasibility problem. Quadratic functions were used 214 for the primal-dual based (PD) [21], [22] and Distributed 215 Approximate Newton Algorithm (DANA) [23] methods. In 216 short, RC prescribes dynamics which seek to achieve consen-217 sus on a ratio of operating capacity with respect to p_i, \overline{p}_i so 218 that the agents achieve $\sum_i p_i = P_{\text{ref}}$. PD and DANA each 219 are Lagrangian-based dynamics; in particular, PD is gradient-220 based ("first-order") and DANA is Newton-based ("second-221 order"). See Appendix A of the extended version [24] for more 222 technical detail on these algorithms. The quadratic functions 223 were artificially chosen to produce satisfactorily diverse and 224 representative solutions to (1) for each DER population. Costs 225 associated with a physical or economic metric (e.g. deviation 226 from a building setpoint for AHUs, user-specified charging 227 demands for V1G and V2Gs, and resistive losses in a BESS) 228 are of great interest, but are far from trivial to model and thus 229 not the focus of this study. We split the total time period of 230 the signal, P_{ref} into three equal segments, and implemented 231 RC, PD, and DANA in that order. Box constraints $[p_i, \overline{p}_i]$ are 232 given in Table I and were centered at zero for simplicity; for 233 example, an AHU *i* with 2 kW capacity has $[p_i, \overline{p}_i] = [-1, 1]$, 234 while a V2G j with ± 5 kW capacity has $[\underline{p}_i, \overline{p}_j] = [-5, 5]$. 235

236 B. Regulation Signal

The 40 min RegD signal published by PJM [19] served as the reference AGC signal for the validation tests, and was used to obtain the value for P_{ref} in (1). The normalized RegD signal, contained in [-1, 1] (see Figure 1), was interpolated from 0.5 Hz to 1 Hz. The signal was then treated by subtracting the normalized contributions of building loads and PV systems, cf. Section III-C. Finally, the normalized signal was scaled by

a factor proportional to the total DER capacity $\sum_{i} (\overline{p}_{i} - \underline{p}_{i})$ 244 before sending to the optimization solvers. More precisely, 245

$$P_{\text{ref}} = \beta \frac{\sum_{i} (\overline{p}_{i} - \underline{p}_{i})}{\|P_{\text{RegD}} + P_{\text{PV}} - P_{\text{b}}\|_{\infty}} \left(P_{\text{RegD}} + P_{\text{PV}} - P_{\text{b}}\right), \quad (2)$$

where P_{RegD} refers to the normalized RegD signal data, P_{PV} 246 and P_b respectively refer to the normalized PV generation and 247 building load data obtained from the UCSD ION server as 248 described in Section III-F, and $0 < \beta < 1$ is an arbitrary 249 scaling constant. Note that this results in a different target 250 signal Pref for the different test scenarios considered in Sec-251 tion IV due to the different power ratings of the DERs (cf. 252 Section III-C) used across the tests. For most test scenarios, 253 $\beta = 0.75$ to prevent extreme set points that would require 254 all DERs to operate at either \overline{p}_i or p_i simultaneously, which 255 may be infeasible in some time steps due to slower signal 256 update times, see Table II. Each P in (2) is a vector with 257 2401 elements corresponding to each 1 s time step's instance 258 of (1) over the 40 min time horizon. 259



Fig. 1: Normalized PJM RegD signal.

C. DERs

The reference AGC signal was to be collectively tracked 261 using DERs consisting of HVAC AHUs, BESS, V1G and 262 V2G EVs, PV systems, and whole-building loads. Since PV 263 systems and (non-AHU) building loads were not controllable, 264 they participated in the test as passive DERs. Consequently, 265 the active DERs were commanded to track a modified target 266 signal derived by subtracting the net active power output of 267 passive DERs from the reference AGC signal and applying 268 appropriate scaling (cf. Section III-B). Table I lists the typical 269 net power capacity $\overline{p}_i - \underline{p}_i$ of the different active DER types. 270

TABLE I: Typical power rating of active DER types

DER Type	AHU	V1G EV	V2G EV	BESS
Typical power rating per DER type	2 kW	3.3 kW (Tests 0 & 1), 4.9 kW (Test 2)	\pm 5 kW	\pm 3 kW

The contribution of each active DER to the target signal 271 was defined with respect to a baseline power, around which 272 $[p_i, \overline{p}_i]$ was centered, to enable tracking of both positive and 273 negative ramps in the target signal. For DERs like V2G EVs 274 and BESS, which were capable of power adjustments in both 275 directions, the baseline was 0 kW. The baseline for V1G EVs 276 was defined to be halfway between their allowed minimum 277 and maximum charging rates, where the former was restricted 278 by the SAE J1772 charging standard to 1.6 kW. Similarly, the 279 baseline for AHUs was defined to be half of their power draw 280 when on. Further, since AHUs were limited to binary on-off 28 operational states, the continuous and arbitrarily precise AHU 282 setpoints obtained by solving (1) were rounded to the closest 283

discrete setpoint obtained from a combination of on-off statesbefore actuation.

AHU control was restricted, by UCSD Facilities Management, to specifying only DER setpoints and duration of actuation; since building automation controllers could not be modified, model-based designs were impossible. This was to avoid malfunctioning or disruptions to real physical infrastructure in the networked building management system that also controls lighting, security, and fire protection systems.

293 D. Computing Setup

The DER active power setpoints were computed for the 294 entire 40-min test horizon prior to any device actuation using 295 a set of 9 Linux-based nodes. The nodes C1-C9 communicate 296 with each other over an undirected ring topology, cf. Fig. 2. 297 As one of the sparsest network topologies, where message 298 passing occurs only between a small number of neighbors, the 299 ring topology presents a challenging scenario for distributed 300 control. Since there were more active DERs than computing 301 nodes, the 9 nodes were mapped subjectively to the 69 active 302 DERs such that nodes C1-C2 computed the actuation setpoints 303 for the AHUs, C3 for V1G EVs, C4-C8 for V2G EVs and C9 304 for the BESS. 305

The computing steps are summarized in Algorithm 1. Each 306 computing node generated actuation commands as CSV files 307 containing the power setpoints for their respective group of 308 DERs at a uniform update rate of 1 Hz. Preliminary testing 309 revealed different response times across DER types, with 310 AHUs and V1G EVs exhibiting slower response than other 311 active DER types. DERs with response times greater than 312 1 s were subject to a stair-step control signal with a signal 313 update time consistent with DER responsiveness and constant 314 setpoints during intermediate time steps. Table II lists the 315 signal update times for the different DER types. 316

Algorithm 1 Computing process

Require: Map $f: C_i \to \text{DER-type}$ 1: Initialize time of last solution update $t_{sol-update_i} = 0$, initial setpoints for DERs mapped to computing node C_i as $P_{f(C_i)}, \forall i \in \{1, ..., 9\}$ 2: for $k = 0, \dots, 2400$ do for i = 1, ..., 9 do 3: if $k - t_{\text{sol-update}_i} == t_{\text{signal-update}_i}$ then Solve (1) to update $P_{f(C_i)}(k)$ 4: 5: $t_{\text{sol-update}_i} = k$ 6: end if 7: $P_{f(C_i)}(k) \leftarrow P_{f(C_i)}(t_{\text{sol-update}_i})$ 8: if mod(k, 60) == 0 then 9: Send $P_{f(C_i)}(k)$ to DER type, $f(C_i)$ 10: end if 11: end for 12: 13: end for

through a custom Visual Basic program that interfaced with the Johnson Control Metasys building automation software. The power rate of the BESS was set via API-based communication with a dedicated computer that controlled the battery inverter. The V1G and V2G EVs charging rates were adjusted through proprietary smart EV charging platforms of the charging station operators. EVs using ChargePoint[®] V1G stations were manually controlled via the load shedding feature of ChargePoints station management software. The actuation of EVs using PowerFlex[®] V1G chargers and Nuvve[®] V2G chargers was automated and commands were issued via API-

F. Power Measurements

based communication.

The active power of all DERs was metered at a 1 Hz 334 frequency. The power outputs of individual PV systems and 335 building loads were obtained prior to the test from their 336 respective ION meters by logging data from the UCSD ION 337 Supervisory Control and Data Acquisition (SCADA) system 338 and aggregated to obtain the total power output of all PVs 339 and building loads. A moving average filter with a 20 s 340 time horizon was used to remove noise from the aggregate 341 measured data for these passive DERs. V2G EVs and BESS 342 power data were acquired using the same interfaces that were 343 used for their actuation, which logged data from dedicated 344 power meters. 345

Since neither AHUs nor the ChargePoint V1G EVs had 346 dedicated meters, they were monitored via their respective 347 building ION meters by subtracting a baseline building load 348 from the building meter power output. Assuming constant 349 baseline building load, any change in the meter outputs can be 350 attributed to the actuation of AHUs and V1G EVs. This as-351 sumption is justifiable considering the tests were conducted at 352 0400 PT to 0600 PT on a weekend, when building occupancy 353 was likely zero and building load remained largely unchanged. 354 Noise in the ION meter outputs observed as frequent 15 -355 30 kW spikes in the measured data for AHUs (Fig. 3) and 356 ChargePoint V1G EVs was treated by removing outliers and 357 passing the resulting signal through a 4 s horizon moving 358 average filter. Here, outliers refer to points that change in 359 excess of 50% of the mean of the 40 min signal in a 1 s 360 interval. 361

G. Performance Metrics

The performance of the distributed implementation (cyberlayer) was measured by the normalized mean-squared-error (MSE) between the distributed and true (i.e. exact) centralized optimization solutions. The true solutions were computed for each instance of (1) using a centralized CVX solver in MATLAB [27]. The MSE was normalized by dividing by the mean of the squares of the true solutions. 369

The tracking performance of the DERs was evaluated ³⁷⁰ through (i) the root-mean-squared-error (RMSE) in tracking ³⁷¹

$$RMSE = \sqrt{\frac{\sum_{t=1}^{T} (P_t^{prov} - P_t^{tar})^2}{\sum_{t=1}^{T} (P_t^{tar})^2}},$$
(3)

317 E. Actuation Interfaces and Communication Framework

The actuation commands were issued using fixed IP computers through dedicated interfaces that varied by DER type as depicted in Fig. 2. The setpoints for AHUs were issued

where P_t^{prov} is the total power that was provided (measured), and P_t^{tar} is the target (commanded) regulation power at time

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Fig. 2: Communication architecture for computation and actuation of control policies.

step $t \in \{1, \ldots, T = 2401\}$; and (ii) the tracking delay, 374 computed as the time shift of the measured signal which yields 375 the lowest RMSE between the commanded and measured 376 signals. The sum of the delays due to local computation and 377 communication between the computing nodes is capped by 378 the algorithm computation time, and would be less than 1 s. 379 Therefore, these delays are not explicitly considered in the 380 tracking delay calculation, and the computed tracking delay 381 only includes the device response times and measurement 382 delays. 383

The PJM Performance Score S following [28, Section 4.5.6] was computed as a test for eligibility to participate in the ancillary services market, and is given by the mean of a Correlation Score S_c , Delay Score S_d , and Precision Score S_p :

$$\begin{split} S_c &= \frac{1}{T-1} \sum_{t=1}^T \frac{(P_t^{\text{prov}} - \mu^{\text{prov}})(P_t^{\text{tar}} - \mu^{\text{tar}})}{\sigma^{\text{prov}}\sigma^{\text{tar}}}, \\ S_d &= \left| \frac{\delta - 5\min}{5\min} \right|, \quad S_p = 1 - \frac{1}{T} \sum_{t=1}^T \left| \frac{P_t^{\text{prov}} - P_t^{\text{tar}}}{\mu^{\text{tar}}} \right|, \\ S &= 1/3(S_c + S_d + S_p), \end{split}$$

where P_t^{prov} and P_t^{tar} are as in (3), μ^{prov} , μ^{tar} and σ^{prov} , σ^{tar} denote their respective means and standard deviations, and δ is the corresponding maximum delay in DER response for when S_c was maximized. A performance score of at least 0.75 is required for participating in the PJM ancillary services market.

IV. TEST SCENARIOS

In this section, we describe the test scenarios carried out on the UCSD microgrid elaborating on the challenges we faced and the differences across the tests, summarized by type of DER in Table II.

394 A. Commonalities

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A series of three tests were conducted on December 12, 2018 (Test 0), April 14, 2019 (Test 1) and December 17, 2019

(Test 2). All three tests involved a 40 min preparatory run 397 followed by a 40 min final test. Table II lists the type of 398 DERs across the tests. All tests were carried out during non-399 operational hours (between 0400 PT and 0540 PT) to avoid 400 potential disruptions to building occupants with the exception 401 of V1G EVs in Test 2, which were tested at the start of the 402 work day (0900 - 1010 PT) to maximize fleet EV availability 403 (cf. Section IV-D). Day-time PV output data from February 24, 404 2019 was used as a proxy for an actual daytime PV signal. 405

TABLE II: Characteristics of each test by DER type.

DER Type	AHU	V1G EV	V2G EV	BESS
# DERs - Test 0	7	4	5	1
# DERS - Test 1	34	29	5	1
# DERs - Test 2	34	17	6	1
Signal updates	1 m	5 m (Tests 0 & 1), 1 m (Test 2)	1 s	20 s
DER Actuation	Synchronous (Tests 0 & 1), Two-stage: Stage 1 (Test 2)	Synchronous (Tests 0 & 1), Two-stage: Stage 2 (Test 2)		
Operation Mode	Automatic	Manual (Tests 0 & 1), Automatic (Test 2)		atic
Time of test	0400 - 0500 PT	0400 - 0500 PT (Tests 0 & 1), 0900 - 1010 PT (Test 2)		
Computing setup	Semi-centralized using ROS (Tests 0 & 1), Fully distributed using Raspberry Pi (Test 2)			

B. Test 0

Test 0 was a preliminary calibration that was used to examine the response times and tracking behavior of every DER type and detect issues related to communication and actuation. 410

1) DERs: Test 0 used only a representative sample of 17 411 DERs. The V1G and V2G population was composed of UCSD 412 ⁴¹³ fleet EVs plugged in at ChargePoint and Nuvve charging ⁴¹⁴ stations, respectively.

2) Computing Setup: 9 laptops running a Robotic Operat ing System (ROS) communicated via local Wi-Fi hotspot to
 implement the distributed coordination algorithms and com pute the DER setpoints.

419 3) Actuation: All DERs were actuated synchronously.

420 C. Test 1

Test 1 was identical to Test 0 except in the number of DERs utilized.

1) DERs: Test 1 used a larger population of 69 active DERs and 107 passive DERs.

2) Computing Setup: The same semi-centralized ROS-425 based computing setup as in Test 0 was used in Test 1. Given 426 that the available power capacity of fast-responding DERs such 427 as V2G and BESS was smaller than slow-responding DERs, 428 the steep ramping demands of the target signal were met by 429 upscaling the power of the fast responding DERs in solving 430 for the contribution of individual DERs. Another option would 431 have been to reduce the number of slow responding DERs, 432 but the funding agency stipulated prioritizing the number and 433 types of heterogeneous DERs over accuracy in signal tracking. 434 A real DER aggregator would instead require a more balanced 435 capacity of slow and fast DERs to ensure feasibility of tracking 436 these ramp features. 437

3) Actuation: All DERs were actuated synchronously. 438 Since the ChargePoint V1G EVs in Test 1 were operated via 439 manual input of DER setpoints (an interface to their API had 440 not been developed yet), to avoid overloading the (human) 441 operators, they were grouped into three groups and actuated 442 in a staggered fashion such that each of the three groups 443 maintained a signal update time of 5 min but were commanded 444 1 min apart from each other. 445

446 D. Test 2

Test 2 also used the entire population of DERs but substituted the cumbersome V1G population with more capable
V1G chargers and used a new distributed computing setup and
method of actuation based on lessons learned from Test 1.

1) DERs: The V1G EVs used in Test 1 performed poorly 451 owing to an unreliable actuation-interface that experienced 452 seemingly random stalling and lacked automated control ca-453 pabilities. Therefore, 17 PowerFlex V1G charging stations 454 at one location replaced the distributed 29 V1G charging 455 stations used in Test 1. Since the PowerFlex interface did not 456 permit actuating individual stations, the 17 charging stations 457 participated in the test as a single aggregate DER. The 0930 458 1010 PT timing of the V1G EV part of the test coincided 459 with the start of the workday and a V1G EV population 460 that had only recently plugged in and therefore had ample 461 remaining charging capacity. The EVs were contributed by 462 UCSD employees and visitors randomly plugging in at the 463 PowerFlex charging stations just before the start of the trial. 464 An aggregate signal of 15 kW to 19 kW was distributed 465 equally amongst the 17 EVs. 466

In addition to the new V1G EVs, the V2G population in Test 2 was replaced with a different set of Nuvve chargers to resolve a tracking/noise issue during discharge-to-grid observed in Test 1 and expanded to include an additional charger. 470

2) Computing Setup: Test 2 featured a fully distributed 471 architecture that consisted of a network of Raspberry Pis 472 that asynchronously communicated with each other via an 473 ethernet switch. In addition, a modified synchronization tech-474 nique was implemented in the software which improved the 475 fidelity and robustness of message-passing. This upgraded 476 message-passing framework and synchronization technique for 477 both software and hardware resulted in significantly faster 478 communication between nodes. 479

3) Actuation: The order of AHU actuation was modified 480 in Test 2 to allow for device settling time and prevent in-481 terference. In particular, in Tests 0 and 1, individual AHUs 482 were ordered and actuated using a protocol that was not 483 cognizant of settling times or building groupings, while the 484 protocol was revised in Test 2 to systematically command the 485 entire population of AHUs in a manner which maximized time 486 between consecutive actuations for an individual unit. 487

Test 2 also featured a two-stage approach of actuation 488 that was a result of the DER tracking behavior in Test 1. 489 Some DERs, such as BESS, V1G EVs and V2G EVs, tracked 490 quickly and accurately, whereas others, such as AHUs, tracked 491 poorly. The overall tracking performance in Test 2 was im-492 proved by using "well-behaved" DERs to compensate for 493 AHU tracking errors by incorporating the error signal from 494 actuating AHUs in Stage 1 to the cumulative target signal 495 for BESS, V1G EVs and V2G EVs in Stage 2. Although 496 synchronous actuation of all participating DERs is preferred 497 in practice, the two-stage approach highlights the significance 498 of systematic characterization of DERs in minimizing ACE. 499

V. TEST RESULTS

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A. Distributed Optimization/Cyber-Layer Results

In Table III, we present MSE results of our 1 s real-time Raspberry-pi distributed optimization solutions (the "cyberlayer" of the system). 502

TABLE III: Normalized mean-squared-error of distributed solutions obtained from real-time 1 s intervals compared to centralized solver solution for Test 2 (Section III-G)

DER Type	RC	PD	DANA	all	
AHU	0	1.4×10^{-7}	2.8×10^{-9}	$4.6 imes 10^{-8}$	
V1G EVs	0	$7.0 imes 10^{-8}$	1.7×10^{-9}	$2.3 imes 10^{-8}$	
V2G EVs	0	$6.6 imes 10^{-5}$	$5.0 imes 10^{-7}$	$2.1 imes 10^{-5}$	
BESS	0	2.0×10^{-6}	9.1×10^{-8}	6.5×10^{-7}	
Total	0	1.8×10^{-5}	1.1×10^{-7}	4.9×10^{-6}	

RC converged to the exact solution in all instances. This is 505 unsurprising, as the RC problem formulation does not account 506 for individual DER costs and thus, is a much simpler problem 507 with a closed-form solution. For PD and DANA, we obtained 508 excellent convergence, with errors on the order of 0.001%509 in the worst cases. In general, DANA tended to converge 510 faster than PD in the sense that the obtained solutions were 511 more accurate under the same fixed 1 s computation time. 512 For our application with 1 s real-time windows, accuracy and 513 convergence differences did not affect the physical layer re-514 sults in any tangible way, but applications with more stringent 515 accuracy or speed requirements may benefit from using a 516

faster algorithm like DANA. The differences between DER 517 populations can be largely attributed to the faster time scale 518 of the V2G EVs (and to a lesser extent the BESS), see Table II. 519 Since the V2G EVs were responsible for the high-frequency 520 component of P_{ref} , the solver was required to converge to 521 new solutions at every time step, which induced more error 522 compared to the slow V1G EVs and AHUs with relatively 523 static solutions. 524

525 B. Physical-Layer Test Results

We now present the results of the tracking performance 526 pertaining to the physical-layer of the experiment. We provide 527 only some selective plots for Test 0 and Test 1 in Fig. 3, and a 528 complete set of plots for each Test 2 DER population in Fig. 4. 529 Error and tracking delay data defined in Section III-G is given 530 in Table IV for Test 1 and Test 2. Data for Test 0 is omitted 531 due to its preliminary nature. The optimal shift described in 532 Section III-G is applied to each time series and hence some 533 areas in plots may appear like the provided signal anticipated 534 the target. 535



Fig. 3: Selected plots from Tests 0 and 1. **Top:** AHU response in Test 0. Note the poor tracking and spikes in the measured response. **Middle:** V2G response in Test 1. Note the inaccuracy in tracking during discharge-to-grid phases. **Bottom:** Total response in Test 1. Note the large-magnitude, low-frequency features demonstrating some broad tracking behavior, but overall poor performance.

Signal tracking accuracy in Test 0 was generally poor 536 despite the small number of DERs employed, largely due to 537 inexperience in actuating the AHUs and V1Gs. In particular, 538 Fig. 3 reveals some oscillations in the AHU response. It 539 is overall difficult to determine if even large-feature, low-540 frequency components of the signal were tracked. Further, 541 data gathering for V1Gs and AHUs was done via noisy and 542 unreliable building ION meters, which motivated the need for 543 outlier treatment (Section III-F) in Tests 1 and 2, and resulted 544 in the smoother and better tracking signal in the top plot of 545 Fig. 4. 546

Test 1 yielded a 111% RMSE for AHUs. We speculate that the small 4 s delay in Test 1 is not representative of the actual AHU delay due to random correlations dominating the time shift for this large error. This is confirmed by a much better AHU response in Test 2 with RMSE 12%, where a 105 s



Fig. 4: Test 2 results. From **top to bottom**, AHU, V2G EVs, V1G EVs, BESS, and total responses. Note the substantially improved AHU, V2G, and total tracking performance compared to Figure 3.

delay is more likely to be representative of the true AHU 552 actuation delay. Given the poor visibility into AHU and V1G 553 controllers explained in Section IV, it is challenging to identify 554 the source of the poor tracking behavior. We speculate that 555 DER metering at the building level rather than the DER level 556 was a major source of error for AHU and V1G in Test 1. 557 This was largely resolved in Test 2 by utilizing a different 558 population of V1Gs with dedicated meters and by modifying 559 the actuation scheme for AHUs to be less susceptible to 560 metering errors as described in Section IV-D. Additionally, 561 the actuation-interface stalling for V1G EVs, described in 562 Section IV-C, was dominant in Test 1, resulting in the poor 563 tracking for V1Gs. Actuating-interface issues were resolved in 564 Test 2 by utilizing an automated control scheme for the V1Gs, 565 which led to significantly lower error. 566

The BESS emerged as the star performer achieving very 567 accurate tracking across all tests with no delay. The V2G EVs 568 also performed relatively well aside from a signal overshoot 569 issue observed during the discharge cycle in Test 1 seen in 570 Fig. 3. The issue was resolved in Test 2 by using V2G EV 571 charging stations from a different manufacturer (Princeton 572 Power), as described in Section IV-D. The V2G charging 573 stations deployed for these tests were pre-commercial or early 574 commercial models that had a few operating issues, such as 575 the overshoot issue during Test 1. 576

The inability of the AHUs to respond to steep, short ramps (Fig. 4) could be due to slow start-up sequences programmed into the building automation controllers to increase device longevity or due to transients associated with driving their AC induction electric motors. Tackling this would require dynamic

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models and parameter identification of signal response and delay. With the new V1G EV population in Test 2, tracking delay reduced from 40 s to 10 s and the tracking accuracy improved significantly. The 1 kW bias seen in Fig. 4 is likely due to rounding errors arising from the inability of PowerFlex charging stations to accept non-integer setpoints.

The superior performance of the BESS and V2Gs motivated 588 the two-stage actuation scheme described in Section IV-D, 589 which contributed to reducing the total RMSE from 50% in 590 Test 1 to 10% in Test 2 (compare the bottom plots of Figs 3) 591 and 4). The two-stage approach allows a sufficiently large 592 proportion of accurately tracking DERs to compensate for 593 the errors of the first stage, where tracking is worse. In this 594 way, poorly-tracking DERs, such as AHUs, can still contribute 595 by loosely tracking some large-feature, low-frequency com-596 ponents of the target signal. The low-frequency contribution 597 reduces the required total capacity of the strongly-performing 598 DERs in the second stage leading to more fine-tuned signal 599 tracking in aggregation. Some recommended rules of thumb 600 for two-stage approach are: (i) Total capacity of first-stage 601 DERs is less than or equal to total capacity of second-stage 602 DERs. (ii) DERs in the first stage are capable of tracking 603 with < 50% RMSE. (iii) DER cost functions are such that the 604 deviation from the baseline is lower cost for first-stage DERs 605 than for second-stage. (iii) allocates a significant portion of 606 the target signal initially to first-stage DERs, freeing up DER 607 capacity in the second-stage for error compensation. 608

TABLE IV: Left: Relative root mean-squared-error of tracking error by DER type. Right: Delay (optimal time-shift) of DER responses in seconds.

DER Type	Test 1	Test 2	DER Type	Test 1	Test 2
AHU	1.11	0.12	AHU	4	105
V1G EVs	0.68	0.077	V1G EVs	40	10
V2G EVs	0.30	0.060	V2G EVs	5	3
BESS	0.054	0.018	BESS	0	0
Total	0.50	0.097	Total	N/A	N/A

609 C. Economic Benefit Analysis

Here, we evaluate the economic benefit of the proposed test 610 system, which is vital for wider scale adoption of DERs as 611 a frequency regulation resource in real electricity markets. 612 To this end, we take an approach similar to [10] to first 613 demonstrate that the testbed is eligible to participate in the 614 PJM ancillary services market. Following the PJM Manual 615 12 [28] (Section III-G), we compute a Correlation Score S_c 616 = 0.98, Delay Score S_d = 0.65, and Precision Score S_p = 617 0.91 from data for Test 2, and obtain a Performance Score 618 $S = 0.85 \ge 0.75$, which confirms the eligibility to participate 619 in the PJM ancillary service market. 620

Next, we compute the estimated annual revenue assuming that the resources are available throughout the day. Using PJM's ancillary service market data² with our total (active) DER capacity of 184 kW and performance score of 0.85, the capability and performance credits for this population of resources (cf. [29, Section 4]) would respectively be \$135 and \$11, for July 9, 2020. This gives an estimated amount of \$53,290 as the total annual revenue. Note that the 184 kW DER capacity employed in this work represents less than 5% of the total DER capacity and less than 0.5% of the total capacity of the UCSD microgrid, cf. [18]. As such, the revenue would significantly increase if more microgrid resources are utilized for regulation, even with reduced availability. 633

VI. CONCLUSIONS

We have presented one of the first real-world demonstrations 635 of secondary frequency response in a distribution grid using 636 up to 176 heterogeneous DERs. The DERs include AHUs, 637 V1G and V2G EVs, a BESS, and passive building loads and 638 PV generators. The computation setup utilizes state-of-the-art 639 distributed algorithms to find the solution of a power allocation 640 problem. We show that the real-time distributed solutions are 641 close to the true centralized solution in an MSE sense. Tests 642 with real, controllable DERs at power closely track the given 643 active-power reference signal in aggregation. Further, our 644 economic benefit analysis shows a potential annual revenue of 645 \$53K for the chosen DER population. These tests highlight the 646 importance of dedicated and noise-free measurement sensors 647 and a well-understood and reliable DER control interface for 648 precise signal tracking. Extensions of this work are ongoing 649 under DERConnect³, a new project at UCSD that aims to 650 develop a testbed consisting of 2500 DERs that allows for 651 online implementation of various distributed algorithms. As 652 is already recognized by the power systems community and 653 federal funding agencies such as ARPA-e and National Science 654 Foundation, large-scale power-in-the-loop testing is needed 655 for transitioning distributed technologies to real distribution 656 systems. We hope that this work spurs further testing and ul-657 timately widespread adoption of coordinated resource control 658 algorithms by relevant players in industry. 659

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