

Performance-Barrier-Based Event-Triggered Control with Applications to Network Systems

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Abstract—This paper proposes a novel framework for resource-aware control design termed performance-barrier-based triggering. Given a feedback policy, along with a Lyapunov function certificate that guarantees its correctness, we examine the problem of designing its digital implementation through event-triggered control while ensuring a prescribed performance is met and triggers occur as sparingly as possible. Our methodology takes into account the *performance residual*, i.e., how well the system is doing in regards to the prescribed performance. Inspired by the notion of control barrier function, the trigger design allows the certificate to deviate from monotonically decreasing, with leeway specified as an increasing function of the performance residual, resulting in greater flexibility in prescribing update times. We study different types of performance specifications, with particular attention to quantifying the benefits of the proposed approach in the exponential case. We build on this to design intrinsically Zeno-free distributed triggers for network systems. A comparison of event-triggered approaches in a vehicle platooning problem shows how the proposed design meets the prescribed performance with a significantly lower number of controller updates.

I. INTRODUCTION

Trading computation and decision making for less actuator, sensing, or communication effort offers great promises for the autonomous operation of both individual and interconnected cyberphysical systems. The advent of increasingly capable devices operating in complex scenarios raises the importance of using the available resources efficiently in order to meet task specifications, prolong battery life, and provide algorithmic solutions that can scale up. Resource-aware control examines the tight coupling between physical and cyber processes to prescribe, in a principled way, when to use the available resources while still guaranteeing a desired quality of service. Motivated by these observations, this paper develops an event-triggered control framework that, given a prescribed performance specification, incorporates in the decision making criteria the performance residual to provide design flexibility for general nonlinear systems.

Literature Review: The event-triggered framework [2]–[4] seeks to determine criterions to employ opportunistically the available control resources (e.g., actuation, sensing, communication) in order to produce efficient implementations on digital systems. Such criterions, called triggers, are commonly obtained by examining the evolution under aperiodic sample-and-hold executions of the Lyapunov certificates valid for their continuous-time counterparts. This can be done in a derivative-based fashion, i.e., by monitoring the time derivative

of the certificate, see e.g., [2], [5]–[8], or in a function-based fashion, i.e., by directly monitoring the value of the certificate, see e.g., [9]–[11]. Both approaches are widely applicable. However, derivative-based approaches tend to be conservative because they are evaluated at the current system state without taking into account how much the certificate has decreased since the last update. This is tackled in dynamic event-triggering [12] by introducing an extra variable to store an estimate of this decrease and incorporate it into the trigger design. On the other hand, function-based designs suffer from lack of robustness to disturbances in the value of the certificate. The work [13] uses both frameworks to mitigate these drawbacks by estimating how much the certificate will decrease after each trigger, which constitutes another source of conservatism, together with its reliance on time triggering. Here, we take a different approach to combine the derivative- and function-based design methodologies inspired by the concept of control barrier functions, and particularly, Nagumo’s Theorem, see e.g., [14]–[17]. The basic insight is to incorporate into the trigger design the performance residual, i.e., how well the system is doing in regards to a prescribed performance specification. This specification plays the role of the “barrier” that the system should not exceed. This makes it possible to allow the certificate to deviate from monotonically decreasing at all times, with the amount of deviation allowed specified as a function of the size of the performance residual. Interestingly, the dynamic event-triggered approach mentioned above can be naturally interpreted within the framework proposed here.

Our technical approach also builds on the literature of event-triggered approaches applied to the distributed control of network systems, see e.g., [18]–[24] and references therein. One known issue in this context is that Zeno behavior may arise as a result of the partial availability of information to individual agents, despite it being ruled out for its centralized counterpart. In such scenarios, it is common to use time regularization [19]–[21], i.e., preventing by design any update before certain fixed time (usually the minimum inter-event time from the centralized design) has elapsed. This requires an offline computation and the resulting executions may behave like periodic time-triggered ones. An alternative way of avoiding Zeno behavior is to allow for the violation of the monotonic decrease of the certificate at all times, see e.g., [25], [26], at the cost of only achieving practical stability. Other works avoid Zeno behavior by either requiring stronger system assumptions on the type of certificates [27], [28] or their solutions are problem-specific [24], [29]. Here, we combine the performance-barrier-based framework with dynamic average consensus [30] to synthesize a Zeno-free distributed design that ensure asymptotic convergence for a general class of nonlinear systems.

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Statement of Contributions: This paper considers closed-loop continuous-time systems evolving under a robustly stabilizing feedback endowed with a certificate in the form of an ISS-Lyapunov function. We address the problem of developing a digital feedback implementation that simultaneously retains the stability properties, opportunistically updates the controller, and meets a prescribed performance. The contributions of the paper are threefold. The first contribution is the synthesis of a novel framework for event-triggered control termed performance-barrier-based design. We combine derivative- and function-based designs by incorporating into the trigger criterion both the time derivative and the value of the certificate. The flexibility of the proposed approach stems from allowing the certificate to deviate from having to monotonically decrease at all times. In our design, a larger performance residual, measured as the difference between the prescribed performance and the value of the certificate, results in a larger amount potential deviation allowed. By construction, at any given state, the performance-barrier-based design enjoys a longer inter-event time than the derivative-based approach, while still achieving the prescribed performance. Our second contribution is the characterization of the implementability and asymptotic stability properties of nonlinear systems under the proposed framework. We introduce the concept of class- \mathcal{K} performance specification function and establish, for general nonlinear systems, a uniform lower bound in the inter-event times of the proposed design, thereby ruling out the possibility of Zeno behavior. For the particular case of exponential performance specifications, which includes the case of linear control systems, we provide an explicit expression of an improved minimum inter-event time with respect to the derivative-based approach. Our third contribution builds on this characterization to develop distributed triggers for network systems using the performance-barrier-based approach that ensure asymptotic correctness. Our distributed design makes use of dynamic average consensus to estimate, with some tracking error, the terms in the trigger criterion that require global information to be evaluated. The guarantees on the design then rely on its ability to tolerate the tracking errors. This is where we leverage the flexibility provided by the performance-barrier-based approach to rule out Zeno behavior in the network executions without using any time regularization. We conclude the paper by illustrating the effectiveness of the proposed framework in a vehicle platooning problem.

II. PRELIMINARIES

This section presents basic preliminaries on graph theory and dynamic average consensus¹.

¹Throughout the paper, we use the following notation. We denote by \mathbb{N} , \mathbb{R} and \mathbb{R}_+ , the set of natural, real and nonnegative real numbers, respectively. We let $\mathbb{1}$ denote the vector with all its entries equal to one. For $n \in \mathbb{N}$, we use $[n]$ to denote $\{1, \dots, n\}$. Given $x \in \mathbb{R}^n$, $\|x\|$ denotes its Euclidean norm. We denote by $\mathbf{I} \in \mathbb{R}^{n \times n}$ the identity matrix. A function $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is locally Lipschitz if, for every compact set $S_0 \subset \mathbb{R}^n$, there exists $L > 0$ such that $\|f(x) - f(y)\| \leq L\|x - y\|$, for all $x, y \in S_0$. We use $\exp(\cdot)$ to denote the exponential function. We let \mathcal{L}_f denote the Lie derivative along the vector field $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$. A continuous function $h : \mathbb{R} \rightarrow \mathbb{R}$ is of class- \mathcal{K} if it is strictly increasing and $h(0) = 0$. In addition, the function is class- \mathcal{K}_∞ if it also satisfies $\lim_{r \rightarrow \infty} h(r) = \infty$.

Graph Theory: Our exposition follows [31]. We denote a graph by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with \mathcal{V} as the set of vertices and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ as the set of edges. We consider undirected graphs, where $(i, j) \in \mathcal{E}$ implies $(j, i) \in \mathcal{E}$. A path between two vertices $i, j \in \mathcal{V}$ is an ordered sequence of vertices starting with i and ending with j such that all pairs of consecutive vertices are elements of the set \mathcal{E} . A graph is connected if there exists a path between any two vertices. Vertices $i, j \in \mathcal{V}$ are neighbors if $(i, j) \in \mathcal{E}$. We let \mathcal{N}_i denote the set composed of vertex i and all its neighbors. We add the subscript $x_{\mathcal{N}_i}$ to represent the subvector of a vector x formed from the entries associated with \mathcal{N}_i . The adjacency matrix $\mathbf{A} \in \mathbb{R}^{|\mathcal{V}| \times |\mathcal{V}|}$ has entries $\mathbf{A}_{ij} = \mathbf{A}_{ji} = 1$ if i and j are neighbors, and $\mathbf{A}_{ij} = \mathbf{A}_{ji} = 0$ otherwise. The degree of a node i is $d(i) := \sum_{j \in \mathcal{N}_i} \mathbf{A}_{ij}$. The degree matrix \mathbf{D} is the diagonal matrix with $\mathbf{D}_{ii} = d(i)$. The Laplacian matrix $\mathbf{L} := \mathbf{D} - \mathbf{A}$ has nonnegative real eigenvalues and a simple eigenvalue of 0 with an eigenvector $\mathbb{1}$ iff the graph \mathcal{G} is connected.

Dynamic Average Consensus: Consider a group of N agents communicating over an undirected graph \mathcal{G} . Each agent $i \in \mathcal{V} = [N]$ has a continuously differentiable reference signal $W_i : [0, \infty) \rightarrow \mathbb{R}$. Dynamic average consensus aims at making the agents track asymptotically the average of the reference signals. For convenience, let $W = (W_1, \dots, W_N)$. Here we employ the dynamic average consensus algorithm [30],

$$\dot{y} = \dot{W} - \rho \mathbf{L}y, \quad (1)$$

where each component of $y \in \mathbb{R}^N$ is the agents' estimate of the average, $\rho > 0$ is a rate of convergence parameter, and \mathbf{L} is the Laplacian matrix of the graph. The following result shows that with the correct initialization and a suitable assumption on the evolution of W , each state y_i asymptotically tracks the average $\mathbb{1}^\top W(t)/N$ of the reference signal. The result is a refinement of [30, Thm.2] to reference signals whose time derivative is bounded exponentially and its proof is presented in the appendix.

Lemma II.1. (*Tracking Error Bound*): *Consider the dynamic average consensus dynamics (1) with a reference signal W whose time derivative is bounded exponentially, i.e., $\|\dot{W}(t)\| \leq c_{\dot{W}} \exp(-rt)$ with a constant $c_{\dot{W}} > 0$, for time $t \in [0, s)$. Define the tracking error as $\epsilon := y - \mathbb{1} \mathbb{1}^\top W/N$. If the initialization of y is such that $\mathbb{1}^\top y(0) = \mathbb{1}^\top W(0)/N$, then the tracking error is also bounded for time $t \in [0, s)$ as*

$$\|\epsilon(t)\| \leq \frac{c_{\dot{W}}}{\rho \lambda_2 - r} \exp(-rt) + \left(\|\epsilon(0)\| - \frac{c_{\dot{W}}}{\rho \lambda_2 - r} \right) \exp(-\rho \lambda_2 t) \quad (2)$$

where λ_2 is the second smallest eigenvalue of the Laplacian matrix \mathbf{L} . \square

III. PROBLEM FORMULATION

Consider a nonlinear control system of the form

$$\dot{x} = F(x, u), \quad x \in \mathbb{R}^n, \quad u \in \mathbb{R}^m,$$

with $F : \mathbb{R}^n \times \mathbb{R}^m \rightarrow \mathbb{R}^n$. The digital implementation of a desired feedback policy $\kappa : \mathbb{R}^n \rightarrow \mathbb{R}^m$ as $u = \kappa(x)$

can be accomplished through a sample-and-hold strategy. This consists of updating the control signal at a specific time t_k , for $k \in \{0\} \cup \mathbb{N}$, and keeping it constant up until t_{k+1} , when the evaluation of the feedback policy provides the next adjustment. As a result, the closed-loop system is

$$\dot{x} = F(x, \kappa(x + e)) = f(x, e), \quad (3)$$

where the error $e = x_k - x$ is the state deviation from the last update at iteration k (here, we use the shorthand notation $x_k = x(t_k)$). The challenge is then how to prescribe the sequence of update times $\{t_k\}$ in order to ensure that the digital implementation retains the convergence and performance properties of the original continuous-time system.

Event-triggered control looks past time-periodic implementations to identify a state-dependent trigger criterion to determine the update times. To come up with such a criterion for a general nonlinear system, a common starting point is to assume that there exists an Input-to-State Stability (ISS) Lyapunov function for (3), see e.g., [2], [5], [6]. Formally, we assume there exists a smooth function $V : \mathbb{R}^n \rightarrow \mathbb{R}$ and class- \mathcal{K}_∞ functions $\underline{\alpha}$, $\bar{\alpha}$, α , and γ satisfying

$$\underline{\alpha}(\|x\|) \leq V(x) \leq \bar{\alpha}(\|x\|), \quad (4a)$$

$$\mathcal{L}_f V(x, e) \leq -\alpha(\|x\|) + \gamma(\|e\|). \quad (4b)$$

The seminal work [2] provides the trigger design

$$t_{k+1} = \{t \geq t_k \mid -\sigma\alpha(\|x(t)\|) + \gamma(\|e(t)\|) = 0\}, \quad (5)$$

with design parameter $\sigma \in (0, 1)$. Under (5), the rate of change of the Lyapunov function along (3) satisfies

$$\frac{d}{dt} V(x(t)) \leq (\sigma - 1)\alpha(\|x(t)\|).$$

Therefore, by design, the certificate V decreases along the trajectories of the sample-and-hold implementation. Stability cannot be established from this fact alone, however, due to the possibility of Zeno behavior: the state-dependency of the trigger criterion makes it possible for the inter-event time between consecutive updates to become increasingly small. This, in turn, leaves open the possibility of an infinite number of updates within a finite period of time. A common strategy to rule out Zeno behavior is to establish the existence of a minimum inter-event time (MIET). For the trigger design (5), the existence of a MIET can be established under mild assumptions, cf. [2].

Triggering according to state-triggered criteria like (5) might lead to fewer controller updates than a time-triggered implementation at the cost of impacting performance (as measured, for instance, by the rate of decrease of the certificate V). Ideally, one would like the system to trigger as sparingly as possible while still guaranteeing a prescribed performance regarding convergence. In that regard, (5) tends to overprescribe updates, as the criterion looks exclusively at the derivative of the certificate without taking into account how much the certificate has decreased since the last update, cf. Figure 1(a). We refer to the difference between the prescribed performance and the value of the certificate as the *performance residual*. Presumably, allowing the certificate to momentarily violate the derivative condition, with leeway specified as an increasing

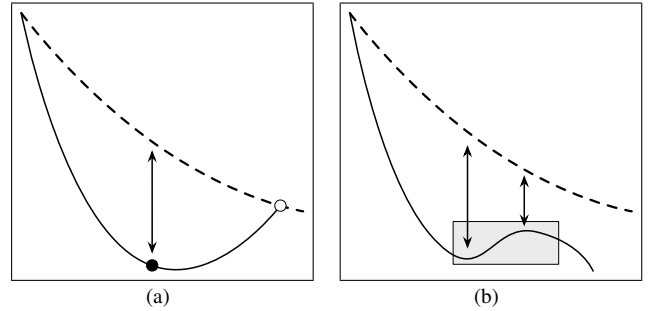


Fig. 1. Prescribed performance (dashed line) and evolution of the certificate (solid line) under state-dependent triggering. (a) the controller update (black circle) prescribed by (5) does not take into account the performance residual, which would otherwise be positive until the curve of the certificate meets the prescribed performance (empty circle). (b) a possible evolution of the certificate that momentarily violates (gray area) the derivative condition on the certificate specified by (5), does not require a controller update while always meeting the performance specification.

function of the performance residual, could result in executions with even fewer controller updates that still meet the performance requirements, cf. Figure 1(b). In the context of network systems, the overprescription of controller updates is also related to the fact that the design of distributed event-triggered schemes based on (5) might result, in general, in sample-and-hold implementations that do not have a MIET, see [3], [20], [25], [32].

The formalization of the ideas described above leads us to propose the *performance-barrier-based* design methodology for trigger design. In Section IV, we limit our discussion to linear systems to motivate and introduce the basic idea. We develop it further for general nonlinear systems in Section V. As we show in our exposition, the new approach naturally leads to longer inter-event times while meeting the specified performance. This provides the necessary groundwork for tackling the design of Zeno-free distributed event-triggered schemes for network systems in Section VI.

IV. PERFORMANCE-BARRIER-BASED EVENT-TRIGGERED CONTROL DESIGNS FOR LINEAR SYSTEMS

Here we introduce the performance-barrier-based ETC framework. In this section, we limit our discussion to linear systems for simplicity of exposition. Consider the sample-and-hold linear control system

$$\dot{x} = Ax + BKx_k = (A + BK)x + BKe, \quad (6)$$

with matrices $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times m}$ and $K \in \mathbb{R}^{m \times n}$ so that $A + BK$ is Hurwitz. In this case, it is easy to guarantee the existence of an ISS Lyapunov function satisfying (4). In fact, using the fact that $A + BK$ is Hurwitz, there exists positive definite matrices P and Q such that

$$V(x) = x^\top P x \quad (7a)$$

is an ISS Lyapunov function with

$$\begin{aligned} \mathcal{L}_f V(x, e) &= -x^\top Q x + 2x^\top P B K e \\ &\leq \left(\frac{\|P B K\|}{\theta} - \lambda_{\min}(Q) \right) \|x\|^2 + \theta \|P B K\| \|e\|^2 \\ &:= -c_\alpha \|x\|^2 + c_\gamma \|e\|^2, \end{aligned} \quad (7b)$$

where $\lambda_{\min}(Q)$ is the minimum eigenvalue of Q and Young's inequality [33] is applied with $\theta > 0$ selected appropriately so that c_α, c_γ are positive. In particular, for the original continuous-time system ($e \equiv 0$ in (6)), one obtains the performance guarantee

$$V(x(t)) \leq V(x_0) \exp(c_\alpha \|P\|^{-1}t), \quad (8)$$

where x_0 denotes the initial condition. We next turn to the trigger design.

A. Derivative- and Function-Based Trigger Designs

For the sample-and-hold linear system (6), the derivative-based trigger design (5) takes the form

$$t_{k+1} = \min \{t \geq t_k \mid -\sigma c_\alpha \|x\|^2 + c_\gamma \|e\|^2 = 0\},$$

with the certificate along any trajectory satisfying $\frac{d}{dt}V(x(t)) \leq (\sigma - 1)c_\alpha \|x(t)\|^2$. Using this inequality, the evolution of the certificate satisfies

$$V(x(t)) \leq V(x_0) \exp((\sigma - 1)c_\alpha \|P\|^{-1}t). \quad (9)$$

A higher value of $\sigma \in (0, 1)$ results in a longer inter-event time and a slower exponential rate on the evolution of the certificate. This presents a trade-off for design. In order to compare different designs fairly, it would seem reasonable to establish a common performance criterion. Given the exponential convergence characteristic of linear systems, prescribing a desired rate of convergence $r > 0$ is a natural candidate. Formally, we specify

$$V(x(t)) \leq V(x_0) \exp(-rt), \quad (10)$$

at all time and for any initial condition. Given the performance (8) of the continuous state-feedback system, we require $r < c_\alpha \|P\|^{-1}$. Since the derivative-based trigger is guaranteed to perform according to (9), one can see that $\sigma = 1 - \frac{r\|P\|}{c_\alpha}$ is the value that yields the longest inter-event time (for the derivative-based design) while still satisfying the performance specification. The following result summarizes the asymptotic convergence properties under the derivative-based trigger design.

Lemma IV.1. (*Derivative-Based Design – Linear Case*): Consider the sample-and-hold linear system (6) with an ISS Lyapunov function (7). Given a desired rate of convergence $r < c_\alpha \|P\|^{-1}$ and $\sigma \in (0, 1 - \frac{r\|P\|}{c_\alpha})$, let $g : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$ be any function such that

$$\mathcal{L}_f V(x, e) \leq g(x, e) \leq (\sigma - 1)c_\alpha \|x\|^2 + c_\gamma \|e\|^2.$$

Define the derivative-based trigger time as

$$t_{k+1}^d = \min \{t \geq t_k \mid g(x(t), e(t)) + rV(x(t)) \geq 0\}. \quad (11)$$

There exists a MIET $\tau_\sigma^d > 0$ such that if $V(x(t_k)) \leq V(x_0) \exp(-rt_k)$, then $t_{k+1}^d - t_k \geq \tau_\sigma^d$. As a consequence, if the trigger sequence $\{t_k\}_{k=0}^\infty$ is defined iteratively via the derivative-based trigger, then $V(x(t)) < V(x_0) \exp(-rt)$ for all $t > 0$, and the origin is globally exponentially stable. ■

Lemma IV.1 is essentially presented in [2]. We omit its proof as it is a special case of Proposition IV.3 below. The basic idea

behind the design (11) is to keep the time derivative of the Lyapunov function below an amount that, by application of the Comparison Lemma [34, Lemma 3.4], would make the system satisfy the desired performance, i.e., $\frac{d}{dt}V(x(t)) < -rV(x(t))$. As a result, the gap $V(x_0) \exp(-rt) - V(x(t))$ between the desired performance and the Lyapunov function, which we call *performance residual*, is always increasing until the next update, see Figure 1. While meeting the desired specifications means keeping the performance residual nonnegative, doing so by having it always increase is overly conservative. To produce a less conservative design, one can instead look at the value of the Lyapunov function itself (rather than its time derivative), as specified in the following result.

Lemma IV.2. (*Function-Based Design – Linear Case*): Consider the sample-and-hold linear system (6) with an ISS Lyapunov function (7). Given a desired rate of convergence $r < c_\alpha / \|P\|$, define the function-based trigger time as

$$t_{k+1}^f = \min \{t > t_k \mid 0 \geq V(x_0) \exp(-rt) - V(x(t))\}. \quad (12)$$

There exists a MIET $\tau_r^f > 0$ such that if $V(x(t_k)) \leq V(x_0) \exp(-rt_k)$, then $t_{k+1}^f - t_k \geq \tau_r^f$. As a consequence, if the trigger sequence $\{t_k\}_{k=0}^\infty$ is defined iteratively via the function-based trigger, then $V(x(t)) \leq V(x_0) \exp(-rt_k)$, and the origin is globally exponentially stable. ■

The function-based design relies on the idea of directly enforcing $V(x(t)) \leq V(x_0) \exp(-rt)$. A problem with this design, however, is that it waits until the last moment, i.e., when the performance residual becomes zero (empty circle in Fig. 1(a)), to prescribe a controller update. Consequently, the implementation is not robust to errors (e.g., delays in evaluation or actual implementation). The performance-barrier-based trigger design, proposed next, is motivated by the idea of overcoming the conservatism of the derivative-based design and the lack of robustness of the function-based one.

B. Performance-Barrier-Based Trigger Design

Our ensuing design builds on the observation that to ensure that the evolution of V satisfies the specified performance, V needs to decrease faster than (or at the same rate as) the specification only when their values are equal. Formally, this can be established using Nagumo theorem [16]: $V(x(t)) \leq V(x_0) \exp(-rt)$ if and only if

$$\frac{d}{dt}V(x(t)) \leq -rV(x(t)) \text{ when } V(x(t)) = V(x_0) \exp(-rt). \quad (13)$$

Note that this condition does not restrict how fast V changes when $V(x(t)) < V(x_0) \exp(-rt)$, no matter how small the performance residual is. One can readily see that the condition (13) suffers from the same lack of robustness as the function-based design. To address this, and inspired by how control barrier functions [15], [17] restrict the speed of their own evolution as the state approaches the boundary of the safe set, we instead prescribe

$$\frac{d}{dt}V(x(t)) + rV(x(t)) \leq c_\beta (V(x_0) \exp(-rt) - V(x(t))),$$

with a nonnegative constant $c_\beta \geq 0$. The key idea is restricting how fast V can increase proportionally to the performance residual. The following result summarizes the asymptotic convergence properties under this type of prescription.

Proposition IV.3. (*Performance-Barrier-Based Design – Linear Case*): Consider the sample-and-hold linear system (6) with an ISS Lyapunov function (7). Given a desired rate of convergence $r < c_\alpha/\|P\|$ and $\sigma \in (0, 1 - \frac{r\|P\|}{c_\alpha})$, let g be as in Lemma IV.1. Define the performance-barrier-based trigger time as

$$t_{k+1}^p = \min \left\{ t \geq t_k \mid g(x(t), e(t)) + rV(x(t)) \geq c_\beta(V(x_0) \exp(-rt) - V(x(t))) \right\}. \quad (14)$$

Let $G(\tau) = \exp(A\tau) + \int_0^\tau \exp(A(\tau-s))dsBK$ and

$$M(\tau) = c_\beta P \exp(-r\tau) - c_\gamma \|\mathbf{I} - G(\tau)\|^2 - G(\tau)^\top ((c_\beta + r)P + (\sigma - 1)c_\alpha \mathbf{I})G(\tau). \quad (15)$$

The constant

$$\tau_\sigma^p := \min\{\tau > 0 \mid \det(M(\tau)) = 0\}, \quad (16)$$

is a MIET such that if $V(x(t_k)) \leq V(x_0) \exp(-rt_k)$, then $t_{k+1}^p - t_k \geq \tau_\sigma^p$. As a consequence, if the trigger sequence $\{t_k\}_{k=0}^\infty$ is defined iteratively via the performance-barrier-based trigger, then $V(x(t)) \leq V(x_0) \exp(-rt)$ for all time, and the origin is globally exponentially stable.

Proof. First, we note that we can derive from the trigger design, $V(x(t)) \leq V(x_0) \exp(-rt)$ for every interval $[t_k, t_{k+1}^p)$, but we have omitted the proof here because it will appear in the proof of Proposition V.3 later for the more general case. Nevertheless, we will prove here the result on the MIET, which will rule out the the sequence $\{t_k\}_{k=0}^\infty$ converging to a finite value (Zeno behavior). We start by deducing for each update

$$\begin{aligned} V(x(t_k)) &\leq V(x_0) \exp(-rt_k) \\ V(x(t_k)) \exp(-r\Delta t_k) &\leq V(x_0) \exp(-rt) \end{aligned}$$

for the time $t \in [t_k, t_{k+1}]$ where $\Delta t_k = t - t_k$. Using this bound to lower bound the right-hand side of the trigger condition in (14), as well as using the definition of g to upper bound the left-hand side, we derive the condition

$$\begin{aligned} x^\top (rP + (\sigma - 1)c_\alpha \mathbf{I})x + c_\gamma \|e\|^2 \\ = c_\beta (V(x_k) \exp(-r\Delta t_k) - x^\top P x) \end{aligned} \quad (17)$$

which must be met earlier. Note we have replaced inequality with equality due to continuity of all the terms along the trajectory. Under system (6), we can find the expression for the state during each iteration as $x(t) = G(\Delta t_k)x_k$. Substituting the state and moving everything of the left-hand side to the right, (17) becomes

$$0 = x_k^\top M(\Delta t_k)x_k.$$

We know that $M(0) \succ 0$ because the right-hand side of (17) is zero, and the left-hand side is negative at time t_k due to the definition of r . The MIET is given by when $M(\tau)$ transits from positive definite to semi-positive definite which is when there exists an x_k such that the condition is

satisfied. Therefore, the MIET is given by (16). As a result, $V(x(t)) < V(x_0) \exp(-rt)$ for all time. Lastly, the origin can be deemed exponentially stable as we can derive

$$\|x\| \leq \|x_0\| \frac{\|P\|^{1/2}}{\lambda_{\min}(P)^{1/2}} \exp(-rt/2),$$

concluding the proof. \square

Proposition IV.3 generalizes both Lemmas IV.1 and IV.2. Note that the trigger design (11) is recovered by selecting $c_\beta = 0$ in (14), and the trigger design (12) corresponds to the limit of (14) as $c_\beta \rightarrow \infty$. Directly from the construction of the trigger designs, one can deduce $t_{k+1}^d \leq t_{k+1}^p \leq t_{k+1}^f$ (inequalities are strict if g is continuous). Therefore, we can adjust the parameter c_β to control the inter-event times, which is also evident in the expression for the MIET. Note that the performance-barrier-based design enjoys longer inter-event times than the derivative-based one while still being able to achieve the prescribed performance. Although the performance-barrier-based strategy does not have a MIET as large as the function-based one, it does not suffer from the same lack of robustness to errors. The design also includes the flexibility of using the surrogate function g if it is more convenient or easier to evaluate. Finally, Proposition IV.3 also provides a method to calculating the MIET using the design (14) for linear control systems. The expression only depends on time (not on the state), which means that it can be calculated offline.

V. PERFORMANCE-BARRIER-BASED EVENT-TRIGGERED CONTROL DESIGNS FOR NONLINEAR SYSTEMS

In this section we expand our presentation of the performance-barrier-based event-triggered control design to general nonlinear systems (3). Our starting point is the availability of an ISS Lyapunov function (4) in tandem with the feedback policy κ . Unlike the case of linear systems, the evolution of the Lyapunov function along the trajectories of the closed-loop system might not be exponentially decaying, and this raises the question of how to suitably define a performance specification. We do this by considering a continuously differentiable, time-dependent function $S(\cdot; x_0) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$, parametrized by the initial condition x_0 , encoding the desired behavior as

$$V(x(t)) \leq S(t; x_0). \quad (18)$$

We use Nagumo theorem [16] to write an equivalent condition (assuming that $V(x_0) \leq S(0; x_0)$) to the requirement (18) as

$$\frac{d}{dt}V(x(t)) \leq \frac{d}{dt}S(t; x_0) \text{ when } V(x(t)) = S(t; x_0). \quad (19)$$

With this in mind, we seek to identify different types of performance specification functions S that allow us to establish the existence of a MIET. In the following, we discuss several classes of specification functions.

A. Class- \mathcal{K} Derivative Performance Specification

This class of specification function is an extension of the exponential decrease of the linear case. In particular, note that the desired convergence rate r is limited in the linear case by

the performance (8) of the original continuous-time system. Similarly, in the nonlinear case, we look at the performance under the continuous-time controller implementation ($e \equiv 0$ in (3)). Hence, let $h : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be such that

$$\mathcal{L}_f V(x, 0) \leq -\alpha(\|x\|) < -h(V(x)),$$

for all x . In other words, h expects a slower convergence than the natural convergence of the system with a continuous controller.

Definition V.1. (*Class- \mathcal{K} Derivative Specification*): For $\sigma^* \in (0, 1)$, let $h : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be locally Lipschitz and class- \mathcal{K} with $h(V(x)) \leq (1 - \sigma^*)\alpha(\|x\|)$ for all x . A function $S(\cdot; x_0) : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is a class- \mathcal{K} derivative performance specification if it is the unique solution to the differential equation

$$\dot{S} = -h(S), \quad S(0; x_0) \geq V(x_0),$$

for any initial condition x_0 . •

According to this definition, S is strictly decreasing in time and $\lim_{t \rightarrow \infty} S(t; x_0) = 0$ for all x_0 , and is increasing in $\|x_0\|$, cf. [34, Lemma 4.4] (with a slight abuse of notation, writing the specification in the form $S(\|x_0\|, t)$ makes it a class \mathcal{KL} function). Note that the exponential rate specification is a particular case of Definition V.1 (by setting $h(s) = -rs$). The following result expands the treatment in [2] regarding derivative-based triggers to account for this notion of performance specification and follows a similar line of reasoning.

Proposition V.2. (*Derivative-Based Design – Class- \mathcal{K} Derivative*): Consider the sample-and-hold nonlinear system (3) with an ISS Lyapunov function (4). Given a class- \mathcal{K} derivative performance specification S and $\sigma \in (0, \sigma^*)$, let $g : \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$ be any function such that

$$\mathcal{L}_f V(x, e) \leq g(x, e) \leq (\sigma - 1)\alpha(\|x\|) + \gamma(\|e\|).$$

Define the derivative-based trigger time as

$$t_{k+1}^d = \min \{t \geq t_k \mid g(x(t), e(t)) + h(V(x(t))) \geq 0\}. \quad (20)$$

Under the assumption that $F, \kappa, \gamma, \alpha^{-1}$ are locally Lipschitz, there exists a MIET $\tau_\sigma^d > 0$ such that if $V(x(t_k)) \leq S(t_k; x_0)$, then $t_{k+1}^d - t_k \geq \tau_\sigma^d$. As a consequence, if the sequence $\{t_k\}_{k=0}^\infty$ is defined iteratively via the derivative-based trigger, then $V(x(t)) \leq S(t; x_0)$ for all time, and the origin is globally asymptotically stable.

Proof. The trigger design directly enforces $g(x(t), e(t)) < -h(V(x(t)))$ for $t \in [t_k, t_{k+1}^d)$. Therefore,

$$\frac{d}{dt} V(x(t)) = \mathcal{L}_f V(x(t), e(t)) \leq -h(V(x(t))).$$

Consequently, if $V(x(t_k)) \leq S(t_k; x_0)$, one can guarantee $V(x(t)) \leq S(t; x_0)$ for all $t \in [t_k, t_{k+1}^d]$ via the Comparison Lemma [34, Lemma 3.4]. Next, we prove the existence of a MIET. Because the sublevel set $\{x \in \mathbb{R}^n \mid V(x) \leq S(0; x_0)\}$ is forward invariant and compact, $\|e\| = \|x - x_k\|$ must be bounded by some constant $E > 0$ and hence the error remains in the compact set $\{e \mid \|e\| \leq E\}$. On these compact sets, let

L_γ and $L_{\alpha^{-1}}$ denote the Lipschitz constants for the functions γ and α^{-1} , respectively. Then,

$$\begin{aligned} t_{k+1}^d &= \min \{t \geq t_k \mid g(x(t), e(t)) + h(V(x(t))) \geq 0\} \\ &\geq \min \{t \geq t_k \mid g(x(t), e(t)) + (1 - \sigma^*)\alpha(\|x(t)\|) \geq 0\} \\ &\geq \min \{t \geq t_k \mid (\sigma - \sigma^*)\alpha(\|x(t)\|) + \gamma(\|e(t)\|) = 0\} \\ &\geq \min \{t \geq t_k \mid \frac{L_\gamma}{\sigma^* - \sigma} \|e\| = \alpha(\|x(t)\|)\} \\ &\geq \min \{t \geq t_k \mid \frac{L_{\alpha^{-1}} L_\gamma}{\sigma^* - \sigma} \|e\| = \|x(t)\|\} \\ &= \min \{t \geq t_k \mid \frac{\|e\|}{\|x\|} = D^{-1}\}, \end{aligned}$$

where $D = \frac{L_{\alpha^{-1}} L_\gamma}{\sigma^* - \sigma}$. Using Lemma A.1, the time at which the condition in the last equation is met is lower bounded by $t_k + \frac{1}{L_f D + L_f}$, where L_f is the Lipschitz constant for f with respect to (x, e) (which exists because F and κ are locally Lipschitz). This establishes the existence of a positive MIET bound, ruling out the possibility of Zeno behavior in the sequence $\{t_k\}_{k=0}^\infty$. Finally, asymptotic stability follows from the fact that S is strictly decreasing and $\lim_{t \rightarrow \infty} S(t; x_0) = 0$, concluding the proof. □

Next, we build on the ideas presented in Sections III and IV to introduce the performance-barrier-based trigger design (21) for the nonlinear case. The proposed design is based on enforcing the condition (19) to ensure the performance specification is met. In doing so, we take advantage of the *performance residual* $S(t; x_0) - V(x(t))$ to avoid overconstraining the evolution of the Lyapunov certificate V when $V(x(t)) < S(t; x_0)$.

Proposition V.3. (*Performance-Barrier-Based Design – Class- \mathcal{K} Derivative*): Consider the sample-and-hold nonlinear system (3) with an ISS Lyapunov function (4). Given a class- \mathcal{K} derivative performance specification S and $\sigma \in (0, \sigma^*)$, let g be as in Proposition V.2 and let β be any \mathcal{K}_∞ function on $[0, \infty)$. Define the performance-barrier-based trigger time as

$$t_{k+1}^p = \min \{t \geq t_k \mid g(x(t), e(t)) + h(V(x(t))) \geq \beta(S(t; x_0) - V(x(t)))\}. \quad (21)$$

Under the assumption that $F, \kappa, \gamma, \alpha^{-1}$ are locally Lipschitz, there exists a MIET $\tau_\sigma^p > 0$ such that if $V(x(t_k)) \leq S(t_k; x_0)$, then $t_{k+1}^p - t_k \geq \tau_\sigma^p$. As a consequence, if the sequence $\{t_k\}_{k=0}^\infty$ is defined iteratively via the performance-barrier-based trigger, then $V(x(t)) \leq S(t_k; x_0)$ for all time, and the origin is globally asymptotically stable.

Proof. The trigger design directly enforces

$$g(x(t), e(t)) + h(V(x(t))) < \beta(S(t; x_0) - V(x(t))), \quad (22)$$

for $t \in [t_k, t_{k+1}^p)$. Thus, when $S(t; x_0) = V(x(t))$, we find $g(x(t), e(t)) + h(S(t; x_0)) < 0$, and hence $\mathcal{L}_f V(x(t), e(t)) < -h(S(t; x_0))$ from the properties of g . Since S is a derivative performance specification, it follows that

$$\frac{d}{dt} V(x(t)) < \frac{d}{dt} S(t; x_0),$$

implying (19). Consequently, $V(x(t)) \leq S(t; x_0)$ for all $t \in [t_k, t_{k+1}^p)$. We can also use this fact to deduce

$$t_{k+1}^p \geq \min \{t \geq t_k \mid g(x(t), e(t)) + h(V(x(t))) = 0\},$$

implying that the performance-barrier-based trigger time must occur after the derivative-based trigger one (20). Thus, τ_σ^d from Proposition V.2 is a valid MIET for the performance-barrier-based trigger as well, ruling out the possibility of Zeno behavior in $\{t_k\}_{k=0}^\infty$. Finally, asymptotic stability follows from the properties of S . \square

Note that the function β in Proposition V.3 restricts the speed of evolution of the Lyapunov certificate V when $V(x(t)) < S(t; x_0)$ as a function of the performance residual.

Remark V.4. (*Comparison with Derivative-Based Approach: Longer Inter-Event Times*): As pointed out by Propositions V.2 and V.3, both the derivative- and performance-barrier-based approaches meet the performance specification defined by S . However, since the performance residual on the right-hand side of (21) always remains greater than zero by design, the performance-barrier-based approach, for a given system state, has a longer inter-event time than the derivative-based one, and is therefore less conservative. In general, it is challenging to provide an explicit bound between the respective MIETs due to the generality of the system dynamics and the performance requirement. We show later in Section V-B that in the case of exponential performance specification this difference in MIETs can be quantified analytically. \bullet

Remark V.5. (*Comparison with Function-Based Approach: Robustness to Input Disturbances*): A purely function-based design would correspond to (21) with the left-hand side substituted by zero. Note that the error term does not show up explicitly in such design, in contrast to the performance-barrier-based approach. Much like how one can use the ISS notion to deal with disturbances, the performance-barrier-based design allows for the analysis and mitigation of input disturbances. This is the intention of the presence of the parameter σ in the definition of g , that reserves a part of the negativity of the Lyapunov function decay. \bullet

Remark V.6. (*Connection with Dynamic Trigger Design*): We note that dynamic triggering can be interpreted as a particular case of the performance-barrier-based trigger design, where the performance function is specified in an online fashion. We elaborate on this point here. Formally, and with the same notation employed in Proposition V.3, the dynamic trigger [12] would take the form

$$t_{k+1}^{\text{dyn}} = \min \{t \geq t_k \mid \theta g(x(t), e(t)) \geq \eta(t)\}, \quad (23a)$$

for $\theta > 0$, where the variable η follows the dynamics

$$\dot{\eta} = -\iota(\eta) - g(x, e) \quad (23b)$$

with a locally Lipschitz class- \mathcal{K}_∞ function ι . The basic idea is to store the decrease of V in the variable η through (23b) and use it to increase the inter-event times in (23a). The term $\iota(\eta)$ represents a decay in the stored amount, ensuring that the system as a whole loses total “energy” over time.

Interestingly, the dynamic design (23) can be interpreted from the perspective of performance-barrier-based ETC. Selecting the performance specification function $S(t; x_0) = \eta(t) + V(x(t; x_0))$, one can see that the design (23) ensures

$$\frac{d}{dt}V(x(t; x_0)) - \frac{d}{dt}S(t; x_0) < \beta(S(t; x_0) - V(x(t; x_0)))$$

with $\beta(\eta) = \iota(\eta) + \eta/\theta$ (note the parallelism with the performance-barrier-based design (21)), implying (19) is satisfied. Note that this performance specification S is not known a priori and is instead determined in an online fashion, tailored to the concrete initial condition of the system trajectory. In particular, this means that the explicit performance guarantee of the design is difficult to obtain unless additional assumptions are made on the dynamics. A final observation is that errors in the evaluation of the decrease of V might jeopardize the convergence properties of dynamic triggering, whereas the evaluation of the performance residual in a feedback fashion characteristic of the performance-barrier-based ETC approach makes it naturally robust to errors. \bullet

B. Exponential Performance Specification

Here we discuss the exponential performance specification. This is a subfamily of the class- \mathcal{K} derivative performance specifications in Section V-A for which an explicit analysis of the performance residual leads us to an improved MIET with respect to the derivative-based approach.

In this case, in lieu of the conditions (4) for the ISS Lyapunov function $V : \mathbb{R}^n \rightarrow \mathbb{R}$, assume the following stronger set of conditions hold: there exist positive constants c_1, c_2, c_3 , and c_4 such that

$$c_1 \|x\|^2 \leq V(x) \leq c_2 \|x\|^2, \quad (24a)$$

$$\frac{dV}{dx} f(x, 0) \leq -c_3 \|x\|^2, \quad (24b)$$

$$\left\| \frac{dV}{dx} \right\| \leq c_4 \|x\|, \quad (24c)$$

for all $x \in \mathbb{R}^n$. Under the additional assumption that F and κ are globally Lipschitz, and using Young’s inequality [33], the following inequality holds for all (x, e) ,

$$\begin{aligned} \mathcal{L}_f V(x, e) &= \frac{dV}{dx} f(x, e) \leq -c_3 \|x\|^2 + c_4 L_f \|x\| \|e\| \\ &\leq -c_\alpha \|x\|^2 + c_\gamma \|e\|^2, \end{aligned} \quad (25)$$

for some positive constants L_f, c_α , and c_γ . Notice that the functions $\underline{\alpha}, \bar{\alpha}, \alpha$, and γ for this ISS Lyapunov function are defined as quadratic functions with constants $c_1, c_2, c_\alpha, c_\gamma$, respectively. Note that in the absence of error, the value of V converges exponentially. Hence, we consider the exponential performance specification $S(t; x_0) = V(x_0) \exp(-rt)$ with $r < c_\alpha/c_2$, which is of class- \mathcal{K} since it is the unique solution to $\dot{S} = -rS$, cf. Definition V.1.

The next result provides an expression for the MIET for the performance-barrier-based trigger design (21) and shows it is strictly larger than the MIET τ_σ^d of the derivative-based trigger design.

Proposition V.7. (*Performance-Barrier-Based Design – Exponential Performance*): Consider the sample-and-hold nonlinear system (3) with a Lyapunov function (24). Given an exponential performance specification S and $\sigma \in (0, 1 - \frac{rc_2}{c_\alpha})$, let g be as in Proposition V.2, and $\beta(z) = c_\beta z$ with a positive c_β . Define

$$\tau_\sigma^{\text{exp}} := \min \left\{ \tau \geq 0 \mid (\xi(\tau) + r) \exp \left(\int_0^\tau \xi(s) ds \right) \right.$$

$$= c_\beta \left(\exp(-r\tau) - \exp\left(\int_0^\tau \xi(s)ds\right) \right) \} \quad (26)$$

where

$$\tau_\sigma^d = \xi^{-1}(-r) := \frac{\sqrt{((1-\sigma)c_\alpha - r)/c_\gamma}}{L_f + L_f \sqrt{((1-\sigma)c_\alpha - r)/c_\gamma}},$$

$$\xi(\tau) = \begin{cases} ((\sigma-1)c_\alpha + c_\gamma\phi(\tau)^2)/c_2 & 0 \leq \tau < \tau^* \\ ((\sigma-1)c_\alpha + c_\gamma\phi(\tau)^2)/c_1 & \tau^* \leq \tau \end{cases},$$

$$\phi(\tau) = \frac{L_f\tau}{1-L_f\tau}, \quad \tau^* = \xi^{-1}(0) := \frac{\sqrt{(1-\sigma)c_\alpha/c_\gamma}}{L_f + L_f \sqrt{(1-\sigma)c_\alpha/c_\gamma}}.$$

Under the assumption that F and κ are globally Lipschitz, τ_σ^{exp} is a MIET such that if $V(x(t_k)) \leq V(x_0) \exp(-rt_k)$, then $t_{k+1}^p - t_k \geq \tau_\sigma^{\text{exp}} > \tau_\sigma^d$. As a consequence, if the trigger sequence $\{t_k\}_{k=0}^\infty$ is defined iteratively with the exponential performance-barrier-based trigger (21), then $V(x(t)) \leq V(x_0) \exp(-rt)$ for all time, and the origin is globally exponentially stable.

Proof. The statements on performance and stability follow with the same arguments used in the proof of Proposition V.3. Here, we only establish the MIET expression given for τ_σ^{exp} . First, we use (24a) and Lemma A.1 to find

$$g(x(t), e(t)) \leq ((\sigma-1)c_\alpha + c_\gamma\phi(t-t_k)^2)\|x\|^2 \leq \xi(t-t_k)V(x),$$

where we have used that $\phi(\tau^*)^2 = (1-\sigma)c_\alpha/c_\gamma$. This gives the bound

$$t_{k+1}^p \geq \min \left\{ t \geq t_k \mid (\xi(t-t_k) + r)V(x(t)) \geq c_\beta(V(x_0) \exp(-rt) - V(x(t))) \right\}.$$

In addition, we can bound the Lyapunov function along the trajectory using the differential form of Gronwall's inequality [34, Lemma A.1] as

$$V(x(t)) \leq V(x_k) \exp\left(\int_{t_k}^t \xi(s-t_k)ds\right). \quad (27)$$

This helps us isolate the state component, which in turn allows us to bound the trigger time with only the time variable as follows

$$t_{k+1}^p \geq \min \left\{ t \geq t_k \mid (\xi(t-t_k) + r) \exp\left(\int_{t_k}^t \xi(s)ds\right) \geq c_\beta \left(\exp(-r(t-t_k)) - \exp\left(\int_{t_k}^t \xi(s-t_k)ds\right) \right) \right\}.$$

With the change of variables $\tau = t - t_k$, and using continuity, the condition defining the set is as in (26). Next, because ξ is strictly increasing and $\xi(\tau_\sigma^d) = -r$, the left-hand side of the condition is nonpositive for $\tau \leq \tau_\sigma^d$. At the same time, the right-hand side of the condition must always be positive. Hence, the condition must be met at $\tau_\sigma^{\text{exp}} > \tau_\sigma^d$, concluding the proof. \square

Note that the expression (26) in Proposition V.7 for the MIET of the performance-barrier-based design with exponential specification does not depend on the state, and can therefore be calculated a priori, before the actual implementation

of the controller. We take advantage of the ability to quantify the benefits of the performance-barrier-based approach for exponential specifications when discussing its application to network systems in our forthcoming discussion.

VI. PERFORMANCE-BARRIER-BASED TRIGGERING FOR NETWORK SYSTEMS

In this section we discuss the application of the performance-barrier-based triggering approach to the design of distributed triggers for network systems. Specifically, we consider exponential performance specifications and take advantage of the additional flexibility provided by the performance residual to ensure the existence of a MIET.

Consider a network of N agents whose interconnection is represented by a connected undirected graph $\mathcal{G} = ([N], \mathcal{E})$. By this, we mean that each agent can only communicate with its neighbors, and hence has access to limited information about the system. We make the assumption that the Lyapunov function satisfying (24) can be expressed as an aggregate

$$V(x) = \sum_{i=1}^N V_i(x_{\mathcal{N}_i}),$$

with each function V_i depending on the local information available to agent i . We assume each V_i to be continuously differentiable with Lipschitz gradient. Our goal is to design distributed triggers that can be evaluated by individual agents with the information available to them.

A. Challenges for ETC in Network Systems

Here we describe the challenges in transcribing the derivative-based trigger approach to network systems. The direct transcription of (20) to the network setting would result in a centralized trigger that requires global information to be evaluated. Making use of the aggregate decomposition of V , one can instead define

$$t_{k+1} = \min \left\{ t \geq t_k \mid \exists i \in [N] \ni (\sigma-1)c_\alpha \|x_i(t)\|^2 + c_\gamma \|e_i(t)\|^2 + rV_i(x_{\mathcal{N}_i}(t)) \geq 0 \right\}. \quad (28)$$

Note the slight abuse of notation here, where x_i and e_i now refer to the states associated with agent i , rather than the i -th component of vectors x and e , resp. This trigger corresponds to partitioning (20) across the network into multiple triggers, one per agent, that can be individually evaluated with local information. Note that the design means that when an agent triggers, a controller update request is sent network-wide. This relies on the observation that such messages, which do not require any state information, can be easily propagated through the network. The design is more conservative than the centralized one and, as a consequence, results in shorter inter-event times for an arbitrary network state. In fact, this type of distributed trigger schemes can suffer from Zeno behavior, see e.g., [3], [20], [25], [32]. A common practice to address this is to explicitly incorporate a MIET at the design stage, a process known as time regularization, see e.g., [19]–[21]. For instance, with a slight modification to suit our context, [19] proposes the following

$$t_{k+1} = \min \left\{ t \geq t_k + \tau_\sigma^d \mid \exists i \in [N] \ni \right. \quad (29)$$

$$(\sigma - 1)c_\alpha \|x_i(t)\|^2 + c_\gamma \|e_i(t)\|^2 + rV_i(x_{\mathcal{N}_i}(t)) \geq b_i(t_k)\},$$

where $b \in \mathbb{R}^n$ is a budget variable satisfying $\mathbb{1}^\top b = 0$, which we discuss below. Time regularization discards the possibility of Zeno behavior by forcing the inter-event time to be above the MIET known from the centralized design. The design builds on the fact that, from the analysis in Section V, we know controller updates are not necessary for τ_σ^d seconds after the last update in order to meet the performance specification. Consequently, agents can ignore the trigger conditions for this amount of time and only start enforcing them thereafter.

However, note that time regularization does not change the fact that the error $\|e_i\|$ might have already surpassed the level at which the trigger would occur as soon as the trigger condition starts getting monitored, see e.g., [32]. The variable b seeks to address this by re-balancing the budget that each agent has in its trigger condition, allowing for the possibility of allocating at the triggering times some budget from a node where the condition has not been violated to another node where it has (in order to have the latter not trigger immediately next time once τ_σ^d seconds have elapsed). Among the potential disadvantages of the design (29) from a network perspective, we point out the following:

- (i) the computation of the MIET τ_σ^d can be challenging and requires the execution of a dedicated distributed algorithm prior to the controller implementation. Moreover, the value obtained may turn out to be too conservative, making the trigger occur more frequently than necessary;
- (ii) the proposed scheme requires a central entity, albeit only at each triggering time, to calculate and assign budgets to all the agents;
- (iii) without further assumptions on the nonlinear system, the evolution of the trigger condition cannot be predicted, and consequently there is no guarantee that the selected budgets b will successfully extend the inter-event time.

Our proposed method addresses these problems by designing a trigger that intrinsically exhibits a MIET and relying on distributed computation and communication among the agents to calculate their budgets.

B. Intrinsically Zeno-Free Distributed ETC Design

We use two different elements to propose a distributed trigger scheme: dynamic average consensus algorithm and the performance-barrier-based trigger design. We approach the Zeno problem by attacking directly its root cause in distributed settings: partial information of the system states is insufficient to inform agents of system's overall performance. For this reason, our distributed trigger design makes use of dynamic average consensus algorithm to estimate, with some tracking errors, the global terms in the centralized version of the trigger. Doing so transforms the problem into ascertaining how well the trigger design can tolerate errors. This is where we leverage the additional flexibility provided by the performance-barrier-based approach over the derivative-based one regarding handling of the tracking errors. Particularly, as we will show later in the analysis of our design, the performance residual term

offered by performance-barrier-based ETC plays a key role in ruling out Zeno behavior.

We begin by defining some notation functions for compactness of presentation. Let

$$\begin{aligned} \mathcal{W}^x(x) &= (\sigma - 1)c_\alpha \|x\|^2 + (r + c_\beta)V(x), \\ \mathcal{W}^{xe}(x, e) &= \mathcal{W}^x(x) + c_\gamma \|e\|^2. \end{aligned}$$

These functions can be decomposed as sums of the following functions, respectively,

$$\begin{aligned} \mathcal{W}_i^x(x_{\mathcal{N}_i}) &= (\sigma - 1)c_\alpha \|x_i\|^2 + (r + c_\beta)V_i(x_{\mathcal{N}_i}), \\ \mathcal{W}_i^{xe}(x_{\mathcal{N}_i}, e_i) &= \mathcal{W}_i^x(x_{\mathcal{N}_i}) + c_\gamma \|e_i\|^2. \end{aligned}$$

For convenience, we let W^x and W^{xe} be vector-valued functions with components $W_i^x = \mathcal{W}_i^x$ and $W_i^{xe} = \mathcal{W}_i^{xe}$, respectively. We omit the dependency on $x_{\mathcal{N}_i}$ and e_i when it is clear from the context. Notice that $\mathbb{1}^\top W^x = \mathcal{W}^x$ and $\mathbb{1}^\top W^{xe} = \mathcal{W}^{xe}$. The centralized performance-barrier-based trigger design (21) can be rewritten compactly as

$$t_{k+1}^{\text{exp}} = \min \{t \geq t_k \mid \mathcal{W}^{xe}(t) = c_\beta V(x_0) \exp(-rt)\}. \quad (30)$$

This trigger has a MIET, cf. Proposition V.7, but the direct computation of \mathcal{W}^{xe} requires global information. However, given the aggregate decomposition $\mathbb{1}^\top W^{xe} = \mathcal{W}^{xe}$ and the fact that agent i knows \mathcal{W}_i^{xe} , a dynamic average consensus algorithm enables the agents to estimate the average \mathcal{W}^{xe}/N . This leads to the following trigger design,

$$t_{k+1} = \min \{t \geq t_k \mid \exists i \in [N] \ni a_i(t) = c_\beta V(x_0) \exp(-rt)/N\}, \quad (31a)$$

$$\dot{a} = \dot{W}^{xe} - \rho_a \mathbf{L}a, \quad (31b)$$

where $\rho_a > 0$ and $\mathbf{L} \in \mathbb{R}^{N \times N}$ is the graph's Laplacian. With this formulation, we denote the tracking error by $\epsilon_a := a - \mathbb{1} \mathbb{1}^\top W^{xe}/N$. In order for the dynamic average consensus to track the right variable, it is crucial to initialize a so that $\mathbb{1}^\top \epsilon_a = 0$. As such, we assume that $a(0)$ is so that $\epsilon_a(0) = 0$ at the initial time $t = 0$. Since the tracking error's mean $\mathbb{1}^\top \epsilon_a$ is conserved along the dynamics (31b), this ensures $\mathbb{1}^\top \epsilon_a = 0$ until the next triggering time. However, the value of \mathcal{W}^{xe} jumps to \mathcal{W}^x at each trigger time t_k due to e being reset to zero, and therefore the average estimate a must be reinitialized at each trigger time t_k to keep the tracking error's mean zero. To do this, we use another dynamic average consensus to keep track of \mathcal{W}^x as

$$\dot{z} = \dot{W}^x - \rho_z \mathbf{L}z \quad (31c)$$

where $\rho_z > 0$, with the initial condition $z(0) = \mathbb{1} W^x(x_0)/N$. Similarly, we denote the tracking error by $\epsilon_z := z - \mathbb{1} \mathbb{1}^\top W^x/N$. Note that the variable z does not depend on e , so it does not need to be reinitialized at each t_k . With the new tracking variable, we reinitialize a to z at each trigger time with a jump map,

$$a^+ = z, \quad t \in \{t_k\}_{k=0}^\infty. \quad (31d)$$

Remark VI.1. (Distributed Implementation): The design (31) does not require a central entity to estimate the evolution of the trigger condition, relying instead on dynamic average consensus. To implement (31), the i -th agent, with local

exchange information on a_i and z_i , can evaluate the dynamic average consensus dynamics (31b) and (31c) if the time derivative of the reference signals \dot{W}_i^{xe} and \dot{W}_i^x are available to it. Each agent i has the information of the states x_i and e_i and the dynamics \dot{x}_i and \dot{e}_i . However, due to dependency on $x_{\mathcal{N}_i}$, the calculation of \dot{W}_i^{xe} and \dot{W}_i^x requires knowledge of $x_{\mathcal{N}_i}$ and $\dot{x}_{\mathcal{N}_i}$. The computation of the latter requires two-hop communication in the graph (alternatively, only one-hop communication is required if the decomposition of the Lyapunov function takes the form $V(x) = \sum_{i=1}^N V_i(x_i)$). •

Remark VI.2. (*Extensions to Discrete-Time Consensus and Directed Graphs*): Instead of the continuous-time algorithms in (31b) and (31c), the design (31) could employ discrete-time implementations of the dynamic average consensus algorithm, see e.g., [30]. Since the effective timescales of (31b) and (31c) scale linearly with ρ_a and ρ_z , respectively, cf. Lemma II.1, the stepsizes of such discrete-time implementations would scale linearly with $1/\rho_a$ and $1/\rho_z$, respectively. A technical analysis analogous to the one presented in Section VI-C below could be developed, albeit we do not pursue it here for simplicity of exposition. A similar observation can be made about the interconnection structure of the network, which could easily be extended from undirected to weight-balanced, strongly connected directed graphs, cf. [31]. •

C. Convergence Analysis

In this section we show that the proposed distributed trigger design (31), with suitable choices of the parameters c_β , ρ_a , and ρ_z , makes the origin asymptotically stable. Our analysis includes establishing performance satisfaction and a MIET. Regarding the former, from the definition of the trigger, we have that

$$\mathcal{W}^{xe}(t)/N + \epsilon_{a,i}(t) = a_i(t) < c_\beta V(x_0) \exp(-rt)/N \quad (32)$$

along the trajectory for all $i \in [N]$. Using the fact that $\mathbb{1}^\top \epsilon_a = 0$ at all time and summing (32), we deduce that $\mathcal{W}^{xe}(t) < c_\beta V(x_0) \exp(-rt)$, i.e., the same condition enforced by the centralized trigger (30). This shows the satisfaction of performance. Establishing MIET is more complicated. The inequality (32) suggests that $\epsilon_{a,i}$ being nonzero can make the distributed trigger (31) occur prematurely in comparison to the centralized trigger (30). However, our analysis below shows that, by tuning different parameters appropriately, we can ensure that at least for the time interval $[t_k, t_k + \tau_\sigma^d]$, the presence of $\epsilon_{a,i}$ does not have this effect, and (31) is not triggered. Before establishing this fact, we show next that the reference signals W^{xe} and W^x have an exponentially bounded time derivative. Its proof is given in the appendix.

Lemma VI.3. (*Exponential Bounds for Reference Signals*): Consider the distributed trigger design (31) for the sample-and-hold nonlinear system (3) with Lipschitz F and κ . Assume that each V_i is continuously differentiable with Lipschitz gradient. Given a desired rate of convergence $r < c_\alpha/c_2$ and $\sigma \in (0, 1 - \frac{rc_2}{c_\alpha})$, there exists $\Omega^{xe} > 0$ such that, for all $k \in \{0\} \cup \mathbb{N}$,

$$\|\dot{W}^{xe}(t)\| \leq \Omega^{xe} V(x_k) \exp(-r\Delta t_k)$$

for $t \in [t_k, t_k + \tau_\sigma^d]$. Furthermore if $c_\beta > (1 - \sigma)(c_\alpha/c_1) - r$, there exists $\Omega^x > 0$ such that

$$\|\dot{W}^x(t)\| \leq \Omega^x V(x_0) \exp(-rt)$$

for all time along the trajectory. □

Lemma VI.3 ensures that the requirements to apply Lemma II.1 hold, allowing us to bound ϵ_a and ϵ_z . We are now ready to state the main result of this section.

Theorem VI.4. (*Distributed ETC with Exponential Performance*): Consider the sample-and-hold nonlinear system (3) with a Lyapunov function (24). Given a desired rate of convergence $r < c_\alpha/c_2$ and $\sigma \in (0, 1 - \frac{rc_2}{c_\alpha})$, let t_{k+1} be determined iteratively according to the performance-barrier-based distributed trigger (31) with $c_\beta > (1 - \sigma)(c_\alpha/c_1) - r$. Under the assumption that F and κ are Lipschitz and that each V_i is continuously differentiable with Lipschitz gradient, let the constant τ_σ^d be defined as in Proposition V.7. Then, there exist ρ_a and ρ_z large enough such that

$$t_{k+1} - t_k \geq \tau_\sigma^d,$$

for all $k \in \{0\} \cup \mathbb{N}$. Consequently, the performance requirement $V(x(t)) \leq V(x_0) \exp(-rt)$ is enforced for all time and the origin is rendered globally exponentially stable.

Proof. Our proof strategy is to show that, for each $k \in \{0\} \cup \mathbb{N}$, $\max_{i \in [N]} a_i - c_\beta V(x_0) \exp(-rt)/N < 0$ during the time period $[t_k, t_k + \tau_\sigma^d]$, which implies that no trigger occurs in said period. Note the bound

$$\max_{i \in [N]} a_i = \max_{i \in [N]} \epsilon_{a,i} + \mathcal{W}^{xe}/N \leq \|\epsilon_a\| + \mathcal{W}^{xe}/N.$$

Therefore, it is enough to prove instead that

$$\|\epsilon_a\| + \frac{1}{N} (\mathcal{W}^{xe} - c_\beta V(x_0) \exp(-rt)) < 0. \quad (33)$$

We start bounding the second summand. Using the bounds $\|e\| \leq \phi(t - t_k)\|x\|$ from Lemma A.1 and $\|x\|^2 \geq V(x)/c_2$ from (24a),

$$\begin{aligned} \mathcal{W}^{xe} &\leq ((\sigma - 1)c_\alpha + c_\gamma \phi(\Delta t_k)^2) \|x\|^2 + (r + c_\beta)V(x) \\ &\leq \left(\frac{(\sigma - 1)c_\alpha + c_\gamma \phi(\Delta t_k)^2}{c_2} + r + c_\beta \right) V(x) \\ &= (\xi(\Delta t_k) + r + c_\beta)V(x) \end{aligned}$$

for $t \in [t_k, t_k + \tau_\sigma^d]$. Notice from the second inequality that with $c_\beta > (1 - \sigma)(c_\alpha/c_1) - r$, the coefficient of $V(x)$ is positive, so we can use the upper bound of V from (27) to get

$$\begin{aligned} \mathcal{W}^{xe} - c_\beta V(x_0) \exp(-rt) &= \mathcal{W}^{xe} - c_\beta V(x_k) \exp(-r\Delta t_k) \\ &\quad - c_\beta (V(x_0) \exp(-rt) - V(x_k) \exp(-r\Delta t_k)) \\ &\leq (\xi(\Delta t_k) + r)V(x_k) \exp\left(\int_0^{\Delta t_k} \xi(s) ds\right) \\ &\quad - c_\beta V(x_k) \left(\exp(-r\Delta t_k) - \exp\left(\int_0^{\Delta t_k} \xi(s) ds\right) \right) \\ &\quad - c_\beta (V(x_0) \exp(-rt) - V(x_k) \exp(-r\Delta t_k)). \end{aligned}$$

Consider the first two terms in this expression. Both terms are strictly negative in the time interval $[t_k, t_k + \tau_\sigma^d]$, so the

maximum value of their sum must be negative. Therefore, there exists $\Omega^* > 0$ (which can be found explicitly by examining its derivative and endpoints on the time interval $\Delta t_k \in [0, \tau_\sigma^d]$) independent of x_k such that

$$\begin{aligned} & \mathcal{W}^{xe} - c_\beta V(x_0) \exp(-rt) \\ & \leq -\Omega^* V(x_k) - c_\beta (V(x_0) \exp(-rt) - V(x_k) \exp(-r\Delta t_k)) \\ & = -\Omega^* V(x_k) - c_\beta (V(x_0) \exp(-rt_k) - V(x_k)) \exp(-r\Delta t_k) \\ & \leq -\Omega^* V(x_k) - c_\beta (V(x_0) \exp(-rt_k) - V(x_k)) \exp(-r\tau_\sigma^d). \end{aligned} \quad (34)$$

Note here that both terms in the bound are non-positive.

Regarding the first summand $\|\epsilon_a\|$ in (33), we resort to Lemma II.1 to bound it. We write (2), with a change of variable to shift time by t_k , for ϵ_a ,

$$\begin{aligned} \|\epsilon_a(t)\| & \leq \frac{\Omega^{xe} V(x_k)}{\rho_a \lambda_2 - r} \exp(-r\Delta t_k) \\ & \quad + \left(\|\epsilon_a(t_k)\| - \frac{\Omega^{xe} V(x_k)}{\rho_a \lambda_2 - r} \right) \exp(-\rho \lambda_2 \Delta t_k). \end{aligned}$$

Over the time interval $\Delta t_k \geq 0$, the bound either achieves the maximum value at $\Delta t_k = 0$ or where its time derivative is zero on the positive interval $\Delta t_k > 0$. In other words, $\|\epsilon_a(t)\| \leq \max\{\|\epsilon_a(t_k)\|, \frac{\Omega^{xe} V(x_k)}{\rho_a \lambda_2 - r}\}$. We consider these two scenarios separately.

First, consider the case where the $\|\epsilon_a(t_k)\| \leq \frac{\Omega^{xe} V(x_k)}{\rho_a \lambda_2 - r}$. By selecting $\rho_a > (1/\lambda_2)(N\Omega^{xe}/\Omega^* + r)$, we can ensure that $\frac{\Omega^{xe} V(x_k)}{\rho_a \lambda_2 - r} < \Omega^* V(x_k)/N$. This shows that the first term in the upper bound (34) is enough to dominate $\|\epsilon_a(t)\|$, guaranteeing that (33) holds.

Next, consider the case where $\|\epsilon_a(t_k)\| > \frac{\Omega^{xe} V(x_k)}{\rho_a \lambda_2 - r}$. Because $W^{xe}(t_k) = W^x(t_k)$ holds at the update time t_k , we deduce from the jump map (31d) that $\epsilon_a(t_k) = \epsilon_z(t_k)$. Thus, the size of $\|\epsilon_a(t_k)\|$ directly depends on how well the dynamic average consensus (31c) performs, so we tune ρ_z appropriately so that (33) holds. Particularly, we look at the possibility that

$$\|\epsilon_z(t_k)\| \geq c_\beta (V(x_0) \exp(-rt_k) - V(x_k)) \exp(-r\tau_\sigma^d)/N$$

(otherwise, the second term of the upper bound (34) already dominates $\|\epsilon_a(t)\|$). From (2), and given the initialization of z with $\epsilon_z(0) = 0$, we have

$$\|\epsilon_z(t_k)\| \leq \frac{\Omega^x V(x_0)}{\rho_z \lambda_2 - r} (\exp(-rt_k) - \exp(-\rho_z \lambda_2 t_k)).$$

Since $\exp(-\rho_z \lambda_2 t_k) \geq 0$, we obtain the relationship

$$\begin{aligned} \frac{\Omega^x V(x_0)}{\rho_z \lambda_2 - r} \exp(-rt_k) & \geq \|\epsilon_z(t_k)\| \\ & \geq c_\beta (V(x_0) \exp(-rt_k) - V(x_k)) \exp(-r\tau_\sigma^d)/N. \end{aligned} \quad (35)$$

After some algebraic manipulations, this implies

$$V(x_0) \exp(-rt_k) \leq \frac{c_\beta \exp(-r\tau_\sigma^d)}{c_\beta \exp(-r\tau_\sigma^d) - \frac{N\Omega^x}{\rho_z \lambda_2 - r}} V(x_k),$$

if the denominator of the right-hand side is positive. For this to be the case, we have to make sure that our choice of ρ_z

satisfies $\rho_z > (1/\lambda_2)(\frac{N\Omega^x}{c_\beta \exp(-r\tau_\sigma^d)} + r)$. Substituting the bound above into the upper bound in (35), we get

$$\|\epsilon_z(t_k)\| \leq \frac{\Omega^x c_\beta \exp(-r\tau_\sigma^d)}{c_\beta \exp(-r\tau_\sigma^d)(\rho_z \lambda_2 - r) - N\Omega^x} V(x_k).$$

Now, any selection of ρ_z such that

$$\rho_z > \frac{1}{\lambda_2} \left(\frac{N\Omega^x}{\Omega^*} + \frac{N\Omega^x}{c_\beta \exp(-r\tau_\sigma^d)} + r \right),$$

ensures that

$$\frac{\Omega^x c_\beta \exp(-r\tau_\sigma^d)}{c_\beta \exp(-r\tau_\sigma^d)(\rho_z \lambda_2 - r) - N\Omega^x} < \frac{\Omega^*}{N},$$

and therefore $\|\epsilon_z(t_k)\| < \Omega^* V(x_k)/N$, implying that the first term of the upper bound in (34) dominates $\|\epsilon_a(t)\|$. Therefore, (33) holds for $t \in [t_k, t_k + \tau_\sigma^d]$, and τ_σ^d is a MIET for the distributed trigger design (31). With the existence of the MIET, performance satisfaction and global exponential stability follow. \square

Theorem VI.4 shows that, with the appropriate tuning of the design parameters, (31) is an intrinsically Zeno-free event-triggered design for network systems with exponential performance (without the need to prescribe the MIET in the design as in (29)). This property relies critically on the performance-barrier-based design approach, particularly on the robustness to errors provided by the performance residual.

Remark VI.5. (*Conservativeness in Design Parameters*): The required bounds for the design parameters ρ_a and ρ_z developed in the proof of Theorem VI.4 are conservative and, in fact, we have observed in practice that values that violate these bounds also result in successful executions. Such bounds must be computed offline, a requirement that is also shared by the time-regularization method regarding the computation of the MIET. However, the key difference, beyond the fact that the method proposed here overcomes the challenges (i)-(iii) described in Section VI-A, is that conservativeness in the MIET computation leads to higher actuation resource usage, whereas conservativeness in the bounds of Theorem VI.4 imposes requirements on the communication and computational resources of the agents, without affecting the timing of the triggers. \bullet

VII. SIMULATIONS ON VEHICLE PLATOONING

To illustrate the effectiveness of the performance-barrier-based trigger design approach, we consider a vehicle platooning problem with $N = 5$ vehicles driving in a line formation along a rectilinear curve. Following [35], we seek to take advantage of the inter-agent communication resources to minimize the usage in actuation resources. The goal is to synchronize the speed v_i of each vehicle $i \in \{2, \dots, 5\}$ to the leader's desired speed v_{des} , and the vehicle's following distance d_i to a safe distance $d_{\text{des},i} = d_0 + T_v v_i$. Here, d_0 is the standstill following distance and T_v represents the factor for the additional distance to keep with respect to the vehicle's speed. Vehicle 1 is the leader and measures distance with respect to a virtual reference vehicle. We define $\delta_i := d_i - d_{\text{des},i}$ and $\nu := v_i - v_{\text{des}}$ to be the mismatch between the actual and

the desired variables. Each vehicle uses a dynamic feedback controller to compute its control input u_i , which directly affects the vehicle's acceleration q_i . The closed-loop dynamics of the leading car, with state $x_i = [\delta_i \ \nu_i \ q_i \ u_i]^\top$, can be written as

$$\begin{aligned} \dot{x}_1 &= \begin{bmatrix} 0 & -1 & -T_v & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{T_d} & \frac{1}{T_d} \\ \frac{k_p}{T_v} & -\frac{k_d}{T_v} & -k_d & -\frac{1}{T_v} \end{bmatrix} x_1 + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{T_d} \\ 0 \end{bmatrix} e_1 \\ &= \bar{A}_{\text{diag}} x_1 + \bar{E} e_1, \end{aligned}$$

where $e_1 = x_{1,4}(t_k) - x_{1,4}$ encodes the fact that the actual control input u is sampled at time t_k and held constant until t_{k+1} . We use the system parameters $k_p = 0.2$, $k_d = 0.7$, $T_v = 0.6$, and $T_d = 0.1$. Vehicles $\{2, \dots, 5\}$ have dynamics that depend on the cars in front of them, as follows

$$\dot{x}_i = \bar{A}_{\text{diag}} x_i + \bar{A}_{\text{off}} x_{i-1} + \bar{E} e_i,$$

where $e_i = x_{i,4}(t_k) - x_{i,4}$ is the sample-and-hold error and

$$\bar{A}_{\text{off}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & \frac{k_d}{T_v} & 0 & \frac{1}{T_v} \end{bmatrix}.$$

We next explain how we obtain an ISS Lyapunov function. First, we find $\mathcal{P} > 0$ such that $\bar{A}_{\text{diag}}^\top \mathcal{P} + \mathcal{P} \bar{A}_{\text{diag}} = -\mathbb{I}$ (this corresponds to ignoring the interconnection of each following vehicle with the one in front). Next, we define

$$V(x) = \sum_{i=1}^N \pi^{N-i} x_i^\top \mathcal{P} x_i,$$

where π is a weight factor to be chosen. Note that this definition naturally places more weight to the vehicles towards the front of the platoon. The Lie derivative of V is given by

$$\begin{aligned} L_f V(x, e) &= \sum_{i=2}^N \pi^{N-i} (-\|x_i\|^2 + 2x_i^\top \mathcal{P} \bar{A}_{\text{off}} x_{i-1}) \\ &\quad - \pi^{N-1} \|x_1\|^2 + \sum_{i=1}^N \pi^{N-i} 2x_i^\top \mathcal{P} \bar{E} e_i. \end{aligned}$$

Using Young's inequality [33], we can bound the cross terms as $2x_i^\top \mathcal{P} \bar{A}_{\text{off}} x_{i-1} \leq 5\|\mathcal{P} \bar{A}_{\text{off}}\|^2 \|x_{i-1}\|^2 + (1/5)\|x_i\|^2$. Selecting then $\pi = 31.25\|\mathcal{P} \bar{A}_{\text{off}}\|^2$, we find, after some algebraic manipulations, that

$$L_f V(x, e) \leq -0.145V(x) + \sum_{i=1}^N \pi^{N-i} 2x_i^\top \mathcal{P} \bar{E} e_i.$$

This implies a rate of convergence of $r^* = 0.145$ in the absence of sample-and-hold error e . In our simulations, we specify the desired exponential convergence rate $r = 0.08 < 0.75r^*$ for the triggered implementations.

With all the elements in place, we are ready to provide a comparison of different event-triggered control approaches.

We implement the centralized performance-barrier-based trigger design, specifically the linear one in (14), and compare it to the derivative-based design (11). For this, we use

$$\begin{aligned} g(x, e) &= 0.75 \sum_{i=2}^N \pi^{N-i} (-\|x_i\|^2 + 2x_i^\top \mathcal{P} \bar{A}_{\text{off}} x_{i-1}) \\ &\quad - 0.75 \pi^{N-1} \|x_1\|^2 + \sum_{i=1}^N \pi^{N-i} 2x_i^\top \mathcal{P} \bar{E} e_i, \end{aligned} \quad (36)$$

and $c_\beta = 1$. Each simulation lasts 400 seconds. Figure 2 shows the evolution of the Lyapunov functions in logarithmic scale for different trigger designs and Table I shows the empirical MIET (which might be larger than the actual MIET) and average number of controller updates across 50 different trajectories with random initial conditions. As expected, both designs satisfy the required performance. However, it is evident from Figure 2 that the derivative-based design outperforms the requirement, meaning that the number of updates could be significantly reduced. This is precisely what the performance-barrier-based design accomplishes by tuning the timing of the updates to the degree of satisfaction of the prescribed performance, reducing their number by almost 20-fold on average.

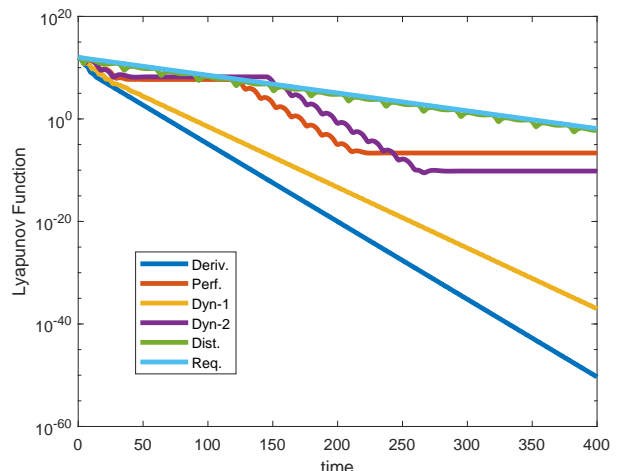


Fig. 2. Evolution of the Lyapunov function for different trigger designs.

TABLE I
EMPIRICAL MIET AND AVERAGE NUMBER OF UPDATES FROM 50
DIFFERENT RANDOM INITIAL CONDITIONS

Design	MIET (s)	Avg. no. of updates
Derivative-Based (11)	0.009	198.28
Performance-Barrier-Based (14)	0.009	9.94
Dynamic (23)	0.013	108.48
Dynamic (23) – small decay	0.013	7.18
Distributed (31)	0.003	96.38

We also compare the proposed designs with the dynamic trigger approach, cf. Remark V.6. To do so, we choose a linear decay function $\iota(\eta) = c_\iota \eta$, and consider different values of c_ι . According to (23), the degree of decay of the Lyapunov function V grows with the value of c_ι , but it is not possible to determine in advance whether a given value

of c_l will guarantee that the evolution meets the desired performance specification. We first use $c_l = 1$, and observe, cf. Figure 2, that the evolution of V is, similarly to that of the derivative-based design, too conservative. Consequently, we employ $c_l = 0.05$, which leads to a significant decrease in the number of updates, cf. Table I, at the cost of not meeting the performance specification any more, cf. Figure 2. One could go through the exercise of fine-tuning the value of c_l to make sure the trajectories meet the desired performance, but this would have to be verified a posteriori in an empirical way, rather than a priori by design, as the performance-barrier-based approach does.

Lastly, we also report the simulation results of the distributed trigger design (31) with $\rho_a = 10$ and $\rho_z = 20$. In fact, notice that both the Lyapunov function V and g in (36) can be expressed as the sum of functions, one per agent, whose value can be computed by each agent with local information,

$$\begin{aligned} V_i(x_i) &= \pi^{N-i} x_i^\top \mathcal{P} x_i, & W_1^x(x_1) &= -0.75\pi^{N-1} \|x_1\|^2, \\ W_i^x(x_{N_i}) &= 0.75\pi^{N-i} (-\|x_i\|^2 + 2x_i^\top \mathcal{P} \bar{A}_{\text{off}} x_{i-1}), & \forall i \geq 2 \\ W_i^{xe}(x_{N_i}, e_i) &= W_i^x(x_{N_i}) + \pi^{N-i} 2x_i^\top \mathcal{P} \bar{E} e_i, & \forall i \geq 1. \end{aligned}$$

The distributed implementation meets the prescribed performance, cf. Figure 2 and is free of Zeno behavior, as guaranteed by Theorem VI.4. This implementation triggers less often than the centralized derivative-based approach and, as expected, more often than the centralized performance-barrier-based design, cf. Table I.

VIII. CONCLUSIONS

We have developed a novel framework for event-triggered control design that meets a prescribed performance regarding convergence. The proposed approach allows for greater flexibility in prescribing update times by allowing the certificate to gradually deviate from strictly decreasing in proportion to the performance residual. We have shown analytically how, for exponential performance specifications, the resulting trigger design exhibits an improved MIET with respect to the derivative-based approach. We have taken advantage of the flexibility of the proposed approach to design intrinsically Zeno-free triggers for network systems that rely on distributed computation and communication and are applicable for a general class of systems. Future work will seek to generalize the guarantees on an improved MIET with respect to the derivative-based approach and the distributed trigger design for network systems beyond exponential performance specifications. We also plan to explore the extension of the performance-barrier-based trigger design framework to deal with Zeno-free output feedback stabilization, handle actuation delays, and cope with scenarios where triggers cannot be evaluated continuously.

APPENDIX

Proof of Lemma II.1. We begin the proof by writing the dynamics of the tracking error,

$$\begin{aligned} \dot{\epsilon} &= \dot{y} - \mathbb{1} \mathbb{1}^\top \dot{W} / N \\ &= \dot{W} - \rho \mathbf{L} (\epsilon - \mathbb{1} \mathbb{1}^\top W / N) - \mathbb{1} \mathbb{1}^\top \dot{W} / n \end{aligned}$$

$$= -\rho \mathbf{L} \epsilon + (\mathbf{I} - \mathbb{1} \mathbb{1}^\top / N) \dot{W}$$

where we have used the fact that $\mathbf{L} \mathbb{1} = 0$. Note also that $\mathbb{1}^\top \dot{\epsilon} = 0$, so $\mathbb{1}^\top \epsilon = 0$ by construction. Hence, at all time, there is no component of ϵ along the eigenvector $\mathbb{1}$ associated with the eigenvalue 0 of the Laplacian matrix \mathbf{L} . Consequently, we can bound

$$\begin{aligned} \frac{d}{dt} \|\epsilon\|^2 &= -\rho \epsilon^\top (\mathbf{L} + \mathbf{L}^\top) \epsilon + 2\epsilon^\top (\mathbf{I} - \mathbb{1} \mathbb{1}^\top / N) \dot{W} \\ &\leq -2\rho \lambda_2 \|\epsilon\|^2 + 2\|\epsilon\| \|\mathbf{I} - \mathbb{1} \mathbb{1}^\top / N\| \|\dot{W}\| \\ &\leq -2\rho \lambda_2 \|\epsilon\|^2 + 2c_{\dot{W}} \exp(-rt) \|\epsilon\| \end{aligned}$$

for time $t \in [t_k, t_{k+1})$. It can be verified through substitution that the solution

$$v = \frac{c_{\dot{W}}}{\rho \lambda_2 - r} \exp(-rt) + \left(v(0) - \frac{c_{\dot{W}}}{\rho \lambda_2 - r} \right) \exp(-\rho \lambda_2 t).$$

satisfies the Bernoulli differential equation [36]

$$v \frac{dv}{dt} = -\rho \lambda_2 v^2 + c_{\dot{W}} \exp(-rt) v.$$

Note that when $v \neq 0$, this reduces to

$$\frac{dv}{dt} = -\rho \lambda_2 v + c_{\dot{W}} \exp(-rt),$$

which is linear, with the right-hand side locally Lipschitz in v . Then, with $v(0) = \|\epsilon(0)\| \neq 0$, we can deduce $\|\epsilon\|^2 \leq v^2$ by applying the Comparison Lemma [34, Lemma 3.4]. Whenever $\|\epsilon\| = 0$, it is possible (depending on \dot{W}) for ϵ to remain zero for some time interval. On such interval, the Comparison Lemma does not apply; however, the case is trivial, and the bound $\|\epsilon\|^2 \leq v^2$ still holds. Finally, by noting that $v \geq 0$ because $v(0) \geq 0$, we obtain $\|\epsilon\| \leq |v| = v$ as stated. \square

Proof of Lemma VI.3. Note that, since F and κ are Lipschitz, then f is Lipschitz too. Consider the column vector composed of $\{V_i\}_{i=1}^N$ and let $J_V(x)$ be its Jacobian. Then, because each V_i have Lipschitz gradients, there exist constants L_{dV} and L_f on the compact sublevel set $\{x \mid V(x) \leq V(x_0)\}$ such that

$$\begin{aligned} \|\dot{W}^{xe}\| &= \|((\sigma - 1)c_\alpha x^\top - c_\gamma e^\top + (r + c_\beta) J_V(x)) f(x, e)\| \\ &\leq ((1 - \sigma)c_\alpha \|x\| + c_\gamma \|e\| + (r + c_\beta) L_{dV} \|x\|) \\ &\quad \times L_f (\|x\| + \|e\|). \end{aligned} \quad (37)$$

We next bound the quadratic terms $\|x\|^2$, $\|e\|^2$ and $\|x\| \|e\|$ in terms of $V(x_k) \exp(-r\Delta t_k)$ for the duration of the interval $[t_k, t_k + \tau_\sigma^d)$. First, knowing that $V(x) \leq V(x_k) \exp(-r\Delta t_k)$ over the interval, we can immediately bound $\|x\|^2 \leq V(x_k) \exp(-r\Delta t_k) / c_1$. Next, for $\|e\|^2$, recall that τ_σ^d is the minimum inter-event time for the derivative-based design, and we can therefore bound

$$\begin{aligned} \|e\|^2 &\leq (1/c_\gamma) ((1 - \sigma)c_\alpha \|x\|^2 - rV(x)) \\ &\leq (1/c_\gamma) ((1 - \sigma)c_\alpha / c_1 - r) V(x) \\ &\leq (1/c_\gamma) ((1 - \sigma)c_\alpha / c_1 - r) V(x_k) \exp(-r\Delta t_k), \end{aligned}$$

for $t \in [t_k, t_k + \tau_\sigma^d)$. Finally, it follows that

$$\|x\| \|e\| \leq \sqrt{\frac{(1 - \sigma)c_\alpha / c_1 - r}{c_1 c_\gamma}} V(x_k) \exp(-r\Delta t_k),$$

for $t \in [t_k, t_k + \tau_\sigma^d]$. Substituting the bounds back into (37) leads to the identification of $\Omega^{xe} > 0$, proving the claim for $\|\dot{W}^{xe}\|$.

For the bound of $\|\dot{W}^x\|$, we consider the entire time interval $t \in [t_k, t_{k+1})$. Using the performance satisfaction, we bound

$$\|x\|^2 \leq V(x)/c_1 \leq V(x_0) \exp(-rt)/c_1.$$

From the trigger condition and $c_\beta > (1 - \sigma)(c_\alpha/c_1) - r$,

$$\begin{aligned} c_\gamma \|e\|^2 &\leq c_\beta V(x_0) \exp(-rt) - (r + c_\beta + (\sigma - 1) \frac{c_\alpha}{c_1}) V(x) \\ &\leq c_\beta V(x_0) \exp(-rt) \end{aligned}$$

The result now follows using the same line of reasoning as in the proof of the bound for $\|\dot{W}^{xe}\|$ to conclude the existence of $\Omega^x > 0$ as stated. \square

For the sake of completeness, we state the following result on the sample-and-hold error bound.

Lemma A.1. (*Sample-and-Hold Error Bound [2, Thm III.1]*): *Consider the sample-and-hold nonlinear system (3). If the functions f is Lipschitz with a constant L_f , then for $t \in [t_k, t_k + 1/L_f]$, the state deviation is bounded as*

$$\|e\| \leq \phi(t - t_k) \|x\|$$

where $\phi(\tau) = \frac{L_f \tau}{1 - L_f \tau}$. \blacksquare

REFERENCES

- [1] P. Ong and J. Cortés, “Event-triggered control design with performance barrier,” in *IEEE Conf. on Decision and Control*, (Miami Beach, FL), pp. 951–956, Dec. 2018.
- [2] P. Tabuada, “Event-triggered real-time scheduling of stabilizing control tasks,” *IEEE Transactions on Automatic Control*, vol. 52, no. 9, pp. 1680–1685, 2007.
- [3] W. P. M. H. Heemels, K. H. Johansson, and P. Tabuada, “An introduction to event-triggered and self-triggered control,” in *IEEE Conf. on Decision and Control*, (Maui, HI), pp. 3270–3285, 2012.
- [4] L. Hetel, C. Fiter, H. Omran, A. Seuret, E. Fridman, J. P. Richard, and S. I. Niculescu, “Recent developments on the stability of systems with aperiodic sampling: An overview,” *Automatica*, vol. 76, pp. 309–335, 2017.
- [5] M. Abdelrahim, R. Postoyan, J. Daafouz, and D. Nešić, “Stabilization of nonlinear systems using event-triggered output feedback controllers,” *IEEE Transactions on Automatic Control*, vol. 61, no. 9, pp. 2682–2687, 2016.
- [6] R. Postoyan, P. Tabuada, D. Nešić, and A. Anta, “A framework for the event-triggered stabilization of nonlinear systems,” *IEEE Transactions on Automatic Control*, vol. 60, no. 4, pp. 982–996, 2015.
- [7] W. P. M. H. Heemels, M. C. F. Donkers, and A. R. Teel, “Periodic event-triggered control based on state feedback,” in *IEEE Conf. on Decision and Control*, (Orlando, FL), pp. 2571–2576, 2011.
- [8] B. A. Khashoeei, D. J. Antunes, and W. P. M. H. Heemels, “Output-based event-triggered control with performance guarantees,” *IEEE Transactions on Automatic Control*, vol. 62, no. 7, pp. 3646–3652, 2017.
- [9] M. Velasco, P. Martí, and E. Bini, “On Lyapunov sampling for event-driven controllers,” in *IEEE Conf. on Decision and Control*, (Shanghai, China), pp. 6238–6243, 2009.
- [10] S. Durand, N. Marchand, and J. F. Guerrero-Castellanos, “Simple Lyapunov sampling for event-driven control,” *IFAC Proceedings Volumes*, vol. 44, no. 1, pp. 8724–8730, 2011.
- [11] M. Mazo Jr., A. Anta, and P. Tabuada, “On self-triggered control for linear systems: Guarantees and complexity,” in *European Control Conference*, (Budapest, Hungary), pp. 3767–3772, Aug. 2009.
- [12] A. Girard, “Dynamic triggering mechanisms for event-triggered control,” *IEEE Transactions on Automatic Control*, vol. 60, pp. 1992–1997, 2015.
- [13] A. Seuret and C. Prieur, “Event-triggered sampling algorithms based on a Lyapunov function,” in *IEEE Conf. on Decision and Control*, (Orlando, FL), pp. 6128–6133, 2011.
- [14] P. Wieland and F. Allgöwer, “Constructive safety using control barrier functions,” *IFAC Proceedings Volumes*, vol. 40, no. 12, pp. 462–467, 2007.
- [15] A. D. Ames, X. Xu, J. W. Grizzle, and P. Tabuada, “Control barrier function based quadratic programs for safety critical systems,” *IEEE Transactions on Automatic Control*, vol. 62, no. 8, pp. 3861–3876, 2017.
- [16] F. Blanchini and S. Miani, *Set-Theoretic Methods in Control*. Birkhäuser Boston, 2007.
- [17] A. D. Ames, S. Coogan, M. Egerstedt, G. Notomista, K. Sreenath, and P. Tabuada, “Control barrier functions: theory and applications,” in *European Control Conference*, (Naples, Italy), pp. 3420–3431, June 2019.
- [18] P. Wan and M. D. Lemmon, “Event-triggered distributed optimization in sensor networks,” in *Symposium on Information Processing of Sensor Networks*, (San Francisco, CA), pp. 49–60, 2009.
- [19] M. Mazo Jr. and P. Tabuada, “Decentralized event-triggered control over wireless sensor/actuator networks,” *IEEE Transactions on Automatic Control*, vol. 56, no. 10, pp. 2456–2461, 2011.
- [20] P. Tallapragada and N. Chopra, “Decentralized event-triggering for control of nonlinear systems,” *IEEE Transactions on Automatic Control*, vol. 59, no. 12, pp. 3312–3324, 2014.
- [21] D. P. Borgers, V. S. Dolk, and W. P. M. H. Heemels, “Dynamic event-triggered control with time regularization for linear systems,” in *IEEE Conf. on Decision and Control*, (Las Vegas, NV), pp. 1352–1357, 2016.
- [22] D. V. Dimarogonas, E. Frazzoli, and K. H. Johansson, “Distributed event-triggered control for multi-agent systems,” *IEEE Transactions on Automatic Control*, vol. 57, no. 5, pp. 1291–1297, 2012.
- [23] C. Nowzari, E. Garcia, and J. Cortés, “Event-triggered control and communication of networked systems for multi-agent consensus,” *Automatica*, vol. 105, pp. 1–27, 2019.
- [24] J. Berneburg and C. Nowzari, “Robust dynamic event-triggered coordination with a designable minimum interevent time,” *IEEE Transactions on Automatic Control*, vol. 66, no. 8, pp. 3417–3428, 2021.
- [25] M. C. F. Donkers and W. P. M. H. Heemels, “Output-based event-triggered control with guaranteed L_∞ -gain and improved and decentralised event-triggering,” *IEEE Transactions on Automatic Control*, vol. 57, no. 6, pp. 1362–1376, 2012.
- [26] M. Guinaldo, D. Lehmann, J. S. Moreno, S. Dormido, and K. H. Johansson, “Distributed event-triggered control with network delays and packet losses,” in *IEEE Conf. on Decision and Control*, (Hawaii, USA), pp. 1–6, Dec. 2012.
- [27] X. Wang and M. D. Lemmon, “Event-triggering in distributed networked control systems,” *IEEE Transactions on Automatic Control*, vol. 56, no. 3, pp. 586–601, 2011.
- [28] E. Garcia and P. J. Antsaklis, “Decentralized model-based event-triggered control of networked systems,” in *American Control Conference*, (Montreal, Canada), pp. 6485–6490, 2012.
- [29] T. H. Cheng, Z. Kan, J. M. Shea, and W. E. Dixon, “Decentralized event-triggered control for leader-follower consensus,” in *IEEE Conf. on Decision and Control*, (Los Angeles, CA), pp. 1244–1249, 2014.
- [30] S. S. Kia, B. V. Scoy, J. Cortés, R. A. Freeman, K. M. Lynch, and S. Martinez, “Tutorial on dynamic average consensus: The problem, its applications, and the algorithms,” *IEEE Control Systems*, vol. 39, no. 3, pp. 40–72, 2019.
- [31] F. Bullo, J. Cortés, and S. Martinez, *Distributed Control of Robotic Networks*. Applied Mathematics Series, Princeton University Press, 2009.
- [32] D. P. Borgers and W. P. M. H. Heemels, “Event-separation properties of event-triggered control systems,” *IEEE Transactions on Automatic Control*, vol. 59, no. 10, pp. 2644–2656, 2014.
- [33] G. H. Hardy, J. E. Littlewood, and G. Polya, *Inequalities*. Cambridge, UK: Cambridge University Press, 1952.
- [34] H. K. Khalil, *Nonlinear Systems*. Prentice Hall, 3 ed., 2002.
- [35] V. S. Dolk, J. Ploeg, and W. P. M. H. Heemels, “Event-triggered control for string-stable vehicle platooning,” *IEEE Transactions on Intelligent Transportation Systems*, vol. 18, no. 12, pp. 3486–3500, 2017.
- [36] E. L. Ince, *Ordinary Differential Equations*. Dover Publications, 1956.



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