Using data informativity for online stabilization of unknown switched linear systems

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***Abstract*—This work studies data-driven switched controller design for discrete-time switched linear systems. Instead of having access to the full system dynamics, an initialization phase is performed, during which noiseless measurements of the state and the input are collected for each mode. Under certain conditions on these measurements, we develop a stabilizing switched controller for the switched system. To be precise, the controller switches between identifying the active mode of the system and applying a predetermined stabilizing feedback. We prove that if the system switches according to certain specifications, this controller stabilizes the closed-loop system. Simulations on a network example illustrate our approach.**

I. Introduction

A switched system is a dynamical system that consists of several modes, or subsystems. A logical rule, called the switching signal, governs the switching between these modes [1]. Because such systems have been shown to model many applications, the study of switched systems has attracted a lot of research interest in the previous decades. One of the main topics of interest is the stabilization of switched systems [2]. Known controller designs include system matrix-based methods [3], [4], methods based on common Lyapunov functions [5], [6] or multiple Lyapunov functions [7]–[9]. However, all the aforementioned design approaches are model-based, that is, they require knowledge of the precise system dynamics in order to stabilize the switched system. In practice, this assumption is often quite restrictive, since the systems can be too complex to model or uncertainties make precise modeling impossible. The problem of controlling uncertain switched systems is studied in [10], [11], where some parameters of the system dynamics are assumed to be unknown but within a given range.

Uncertainty on models can be reduced by employing measurements and constructing data-driven controls. This has become an active area of research, leading to various papers such as [12], [13], of special relevance to this work. Data-driven controller design for switched systems is more challenging and has only been recently considered; see for instance [14], [15]. The type of systems studied here have no external inputs; and the only controlling element is the switching signal. On the other hand, the works [16], [17] focus on finding feedback control laws that stabilize a switched linear system under arbitrary and unknown switching signals. In this case, the desired feedback control needs to uniformly stabilize all the modes simultaneously. Therefore, such a controller may not exist in general and the corresponding algorithms will be necessarily restrictive. Another relevant work is [18], which proposes an online data-driven feedback control. In this way, a stabilizing feedback control is found based on measurements of the currently active mode of the system. Naturally, only switched systems for which the system switches infrequently can be stabilized.

In this paper, we study a stabilizing control design problem for switched linear systems. In particular, we consider a situation in which we do not have access to a model of the separate modes, nor the precise switching signal of the system. Clearly, in order to be able to design such a controller, it is necessary for each of the modes to be stabilizable separately. As such, to compensate for the fact that the dynamics of the modes are unknown, we assume that an *initialization phase* is performed. In this phase, measurements of the state and the input are collected on a finite time interval for each of these modes. In this sense, we have *partial information* on the modes of the system. We will employ the methods of the informativity framework (see e.g. [13]) to develop necessary and sufficient conditions on these measurements that guarantee that each mode is stabilizable with a given decay rate. In particular, we note that these measurements are not necessarily informative enough to uniquely identify a model for each mode.

After this initialization phase, we consider the problem of operating the switched system online. To achieve a stable behavior, our proposed controller alternates between two phases, which consist of a *mode detection phase* and a *stabilization phase*. If the controller detects a modal switch, the mode-detection phase applies excitatory inputs to measure system outputs and identify the active mode. Thus, a main problem we consider is the characterization of necessary and sufficient conditions on the *online* measurements that uniquely determine the current mode of the system. In particular, these conditions must be such that they are guaranteed to hold after a bounded number of steps. After determining the active mode, our proposed controller applies a stabilizing feedback corresponding to this mode in a *stabilization phase*. The controller will switch back to a mode detection phase as soon as the solution of the system does not converge fast enough (which is quantified in terms of Lyapunov functions). This leads to the final problem considered in this paper: Obtain conditions that guarantee that the stabilization phase controllers compensate for the potential destabilization that occurs in the mode-detection phase, so that overall closed-

This work was partially supported by AFOSR Award FA9550-19-1-0235.

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loop system is stable.

The paper is structured as follows. First, necessary background notions and the problem are introduced and formulated in Section II. Then, the main elements of our data-driven switched controller design are explained in detail in Section III, which is followed by the stability analysis of the closed-loop system in Section IV. Our main results are then supported by a numerical example in Section V and finally Section VI concludes our paper. All proofs are omitted for reasons of space and will appear elsewhere.

II. Problem formulation

Consider[1] a switched linear system

$$x(t+1) = A_{\sigma(t)}x(t) + B_{\sigma(t)}u(t), \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, $\sigma : \mathbb{N} \mapsto \{1, 2, \cdots, p\} =: \mathcal{P}$ is the switching signal and $u \in \mathbb{R}^m$ is the control. The dimensions n, m and the number of modes p are known. However, the dynamics of the modes are unknown; that is, for each mode $i \in \mathcal{P}$, the matrices $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are unknown. Additionally, we assume that the switching signal σ is also unknown; that is, we do not know in advance when the switches happen and once a switch happens, which of the modes is active.

To offset this lack of knowledge, we have access to a finite set of measurements of the system. Specifically, in an *initialization* phase, we assume measurements of the individual modes are available. After this, once the system is running, we have access to *online* measurements of the currently active mode. We consider the situation in which we can not necessarily identify the model of the separate modes based on the initialization data, nor the currently active mode based on the online measurements. Given this setup, our goal is to solve the following problem.

Problem 1 (Online switched controller design)**.** Given initialization and online measurements, design a control law $u : \mathbb{N} \mapsto \mathbb{R}^m$ such that the resulting interconnection (1) is guaranteed to satisfy the following *input-to-state stability* (ISS)-like property: there exist constants $c > 0, \zeta \in (0, 1)$ and $r \geqslant 0$ (depending on the input) such that

$$|x(t)| \leqslant c\,\zeta^t |x(0)| + r, \tag{2}$$

for all initial states $x(0) \in \mathbb{R}^n$ and all time $t \in \mathbb{N}$.

To solve Problem 1, we employ the following multi-pronged approach. Since unique models for each of the modes cannot necessarily be determined, we employ the concept of *data informativity* to formulate conditions under which the initialization measurements are enough to guarantee the existence of a stabilizing feedback controller for each of the modes. Based on this, our switched controller operates in two phases. In the *mode detection phase*, the controller selects inputs that allow us to determine, within a bounded number of steps, the active mode (which, in general, requires less measurements than fully identifying the system). Once the mode is identified, the controller switches to the *stabilization phase*, where the controller found in the initialization step corresponding to the mode is applied. Section III formally describes the controller and Section IV characterizes its properties.

[1]Throughout the paper, we use the following notation. We denote by $\mathbb{N}$ and $\mathbb{R}$ the sets of non-negative integer and real numbers, respectively. We let $\mathbb{R}^{n\times m}$ denote the space of $n \times m$ real matrices. For any $M \in \mathbb{R}^{n\times m}$, $\|M\|$ denotes the standard 2-norm. For $P \in \mathbb{R}^{n\times n}$, $P \succeq 0$ (resp. $P \succ 0$) denotes that P is positive semi-definite (resp. definite).

III. Switched controller design

In this section, we describe the three components of the switched controller design: the initialization step, the mode detection phase, and the stabilization phase.

A. Initialization step

We start by considering the problem of finding stabilizing controllers from pre-collected measurements, for which we resort to the notion of data informativity. Given measurements of the state x and input u signals on the time interval $\{0, \dots, T\}$, we define the matrices:

$$X := \begin{bmatrix} x(0) & \cdots & x(T) \end{bmatrix}, \quad U_- := \begin{bmatrix} u(0) & \cdots & u(T-1) \end{bmatrix}.$$

In the following, we identify these matrices with the measurements. For convenience, we also define

$$X_+ := \begin{bmatrix} x(1) & \cdots & x(T) \end{bmatrix}, \quad X_- := \begin{bmatrix} x(0) & \cdots & x(T-1) \end{bmatrix}.$$

The set of consistent systems is given by

$$\Sigma(U_-, X) := \left\{ (A, B) : X_+ = AX_- + BU_- \right\} \subseteq \mathbb{R}^{n\times n} \times \mathbb{R}^{n\times m}.$$

We are interested in characterizing properties of the true system based on the measurements. However, the set $\Sigma(U_-, X)$ might contain more than one pair of system matrices. This means that, we can only conclude that, for instance a feedback gain K stabilizes the true system if this gain stabilizes any system whose system matrices are in $\Sigma(U_-, X)$. The following notion captures when the data is sufficiently informative to do this with a uniform decay rate.

Definition III.1 (Informativity for uniform stabilization)**.** The data (U_-, X) is *informative for uniform stabilization by state feedback with decay rate* $\lambda \in (0, 1)$ if there exist $K \in \mathbb{R}^{m\times n}$ and $P \in \mathbb{R}^{n\times n}, P \succ 0$ such that

$$(A + BK)^\top P(A + BK) \preceq \lambda P \quad \forall (A, B) \in \Sigma(U_-, X). \tag{3}$$

When (U_-, X) satisfies this definition and the matrices K and P are known, one can apply the control $u = Kx$ to the system, choose the function $x \mapsto V(x) = x^\top Px$ and conclude

$$V(x(t+1)) \leqslant \lambda x(t)^\top Px(t) = \lambda V(x(t)), \tag{4}$$

for all $t \in \mathbb{N}$. This means that, even though the exact dynamics of the system are unknown, the feedback gain K is stabilizing, with Lyapunov certificate V with decay rate λ. Nevertheless, Definition III.1 does not provide a constructive way of finding K and P satisfying (3). The next result formulates the problem of finding such K and P as a *linear matrix inequality* (LMI) problem and it also shows that the feasibility of this inequality is a necessary and sufficient condition for the data (U_-, X) to be informative for uniform stabilization with decay rate λ by state feedback.

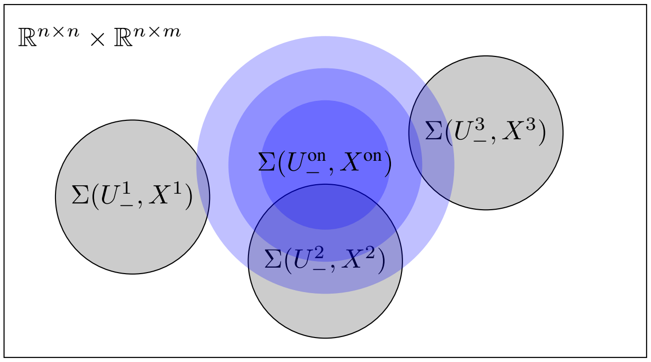

Figure 1: Graphical interpretation of the mode detection scheme. Initially, $\Sigma(U_-^{\text{on}}, X^{\text{on}})$ intersects the sets corresponding to three different modes. As more data is collected, $\Sigma(U_-^{\text{on}}, X^{\text{on}})$ decreases in size (cf. darker blue disks). When enough data is available, $\Sigma(U_-^{\text{on}}, X^{\text{on}})$ eventually becomes compatible only with mode 2.

Theorem III.2 (Conditions for informativity for uniform stabilization)**.** *The data (U_-, X) is informative for uniform stabilization by state feedback with decay rate λ iff there exist $Q \in \mathbb{R}^{n\times n}, Q \succ 0$ and $L \in \mathbb{R}^{m\times n}$ such that*

$$\begin{bmatrix} \lambda Q & 0 & 0 & 0 \\ 0 & 0 & 0 & Q \\ 0 & 0 & 0 & L \\ 0 & Q & L^\top & Q \end{bmatrix} + \begin{bmatrix} X_+ \\ -X_- \\ -U_- \\ 0 \end{bmatrix} \begin{bmatrix} X_+ \\ -X_- \\ -U_- \\ 0 \end{bmatrix}^\top \succeq 0. \quad (5)$$

Moreover, the matrices $K := LQ^{-1}, P := Q^{-1}$ satisfy (3).

Theorem III.2 can be concluded using the methods of from [19], [20]. With these notions in place, we make the following assumption.

Assumption 1 (Initialization phase)**.** Let $\lambda \in (0, 1)$. For each mode $i \in \mathcal{P}$, the data (U_-^i, X^i) is informative for uniform stabilization by state feedback with decay rate λ.

B. Mode detection phase

After the initialization step, the system is in operation and additional *online* measurements, denoted by $(U_-^{\text{on}}, X^{\text{on}})$, are collected from the system when it dwells in one mode. We then face the problem of determining which of the p different systems has generated the online measurements $(X^{\text{on}}, U_-^{\text{on}})$. In this section, we propose an algorithm to find the data-generating mode.

The notion of data compatibility plays a key role in achieving this goal.

Definition III.3 (Compatibility of data)**.** For $i \in \mathcal{P}$, the data (U_-^i, X^i) and $(U_-^{\text{on}}, X^{\text{on}})$ are *compatible* if there exists a system consistent with both, i.e., $\Sigma(U_-^i, X^i) \cap \Sigma(U_-^{\text{on}}, X^{\text{on}}) \neq \varnothing$.

The following result characterizes this property.

Lemma III.4 (Conditions for data compatibility)**.** *For $i \in \mathcal{P}$, the data (U_-^i, X^i) and $(U_-^{\text{on}}, X^{\text{on}})$ are compatible iff*

$$\ker \begin{bmatrix} X_-^i & X_-^{\text{on}} \\ U_-^i & U_-^{\text{on}} \end{bmatrix} \subseteq \ker \begin{bmatrix} X_+^i & X_+^{\text{on}} \end{bmatrix}. \quad (6)$$

In order for any mode detection mechanism to be successful, we assume that the initialization data are pairwise incompatible.

Assumption 2 (Initialization phase –cont'd)**.** For each mode $i \in \mathcal{P}$ the matrix pair (A_i, B_i) is controllable. Furthermore, the data $\{(U_-^i, X^i)\}_{i\in\mathcal{P}}$ are such that (U_-^i, X^i) and (U_-^j, X^j) are incompatible for each pair $i \neq j \in \mathcal{P}$.

Figure 1 provides the intuition for the mode detection scheme. Since the online measurements are generated by one of the modes $i \in \mathcal{P}$, there must exists at least one $i \in \mathcal{P}$ such that $(U_-^{\text{on}}, X^{\text{on}})$ and (U_-^i, X^i) are compatible. Since the initial data are pairwise incompatible, when $\Sigma(U_-^{\text{on}}, X^{\text{on}})$ is sufficiently "small", the mode i giving (U_-^i, X^i) compatible with $(U_-^{\text{on}}, X^{\text{on}})$ must be unique and hence the active mode is detected. This leads us to the following definition.

Definition III.5 (Informativity for mode detection)**.** The initialization data $\{(U_-^i, X^i)\}_{i\in\mathcal{P}}$ and online data $(U_-^{\text{on}}, X^{\text{on}})$ are *informative for mode detection* if $(U_-^{\text{on}}, X^{\text{on}})$ and (U_-^i, X^i) are compatible for exactly one $i \in \mathcal{P}$.

As a result of these observations, we are interested in *generating* online data such that, after a bounded number of steps, $\Sigma(U_-^{\text{on}}, X^{\text{on}})$ becomes small enough so that the data are informative for mode detection. More precisely, given the initialization data, the problem is to find a time horizon T^{on} and inputs $u^{\text{on}}(0), \ldots, u^{\text{on}}(T^{\text{on}} - 1)$ such that the corresponding online data $(U_-^{\text{on}}, X^{\text{on}})$ are informative for mode identification. To obtain such inputs, we adopt the experiment design method of [21], which constructs inputs $u^{\text{on}}(0), \ldots, u^{\text{on}}(n + m - 1)$ such that the corresponding $\Sigma(U_-^{\text{on}}, X^{\text{on}})$ is a singleton set. Clearly, such online measurements would be informative for mode identification. We apply these specific inputs to obtain an upper bound to the number of steps required for the mode detection phase. Note, however, that in general mode detection is achieved with less measurements than system identification.

Algorithm 1 summarizes the mode detection procedure. The strategy updates the online data $(U_-^{\text{on}}, X^{\text{on}})$ and a list $\mathcal{P}_{\text{match}}$ containing all the modes that are compatible with the online data for each time instance. To bound the destabilizing effect of the mode detection phase, the algorithm modifies the experiment design method in [21] (Steps 1 to Steps 6) to add a parameter $u_{\max} > 0$ bounding the magnitude of the detection input. The existence of $\eta \in \mathbb{R}^n$ satisfying the conditions in Step 2 is guaranteed when each mode is controllable. As a consequence, it is straightforward to check that the rank of $\begin{pmatrix} X_-^{\text{on}} \\ U_-^{\text{on}} \end{pmatrix}$ increases with each step of the algorithm. As such, after $n + m$ steps this data-matrix has full column rank, and hence admits a unique compatible system. As such:

Corollary III.6. *Under Assumptions 1 and 2, the online data $(U_-^{\text{on}}, X^{\text{on}})$ generated by Algorithm 1 are informative for mode detection after at most $n + m$ time instances.*

C. Stabilization phase

Given Assumption 1, for each mode $i \in \mathcal{P}$, we can use the LMI (10) to find a feedback gain K_i and a positive

Algorithm 1 Mode detection algorithm per time instant

Input: $\mathcal{P}, \mathcal{P}_{\text{match}}, \{U_-^i, X^i\}_{i\in\mathcal{P}}, U_-^{\text{on}}, X^{\text{on}}, u_{\max}$
Output: $\mathcal{P}_{\text{match}}, U_-^{\text{on}}, X^{\text{on}}$

if $U_-^{\text{on}} \neq []$ and $x(t) \in \operatorname{im} X^{\text{on}}-$ **then** ▷ Choose the next input
  Pick $\begin{bmatrix}\xi\\ \eta\end{bmatrix} \in \ker \begin{pmatrix}X_-^{\text{on}}\\ U_-^{\text{on}}\end{pmatrix}^\top$ with $\eta \neq 0$
  Let $u(t) \in \mathbb{R}^m$ be such that $|u(t)| \leqslant u_{\max}$ and $\xi^\top x(t) + \eta^\top u(t) \neq 0$
else
  $u(t) \leftarrow 0$
end if
Get the next state $x(t+1)$
$U_-^{\text{on}} \leftarrow \begin{bmatrix}U_-^{\text{on}} & u(t)\end{bmatrix}$
$X^{\text{on}} \leftarrow \begin{bmatrix}X^{\text{on}} & x(t+1)\end{bmatrix}$ ▷ Append the data
for $i \in \mathcal{P}$ **do**
  if the inclusion (6) is violated **then** ▷ Data are incompatible
    $\mathcal{P}_{\text{match}} = \mathcal{P}_{\text{match}} \backslash \{i\}$ ▷ Eliminate mode i
  end if
end for

definite matrix P_i, such that $u = K_i x$ is a stabilizing feedback law corresponding to the Lyapunov function given by $V_i(x) := x^\top P_i x$. Once the current mode has been identified, this provides the control input for the stabilization phase.

Note, however, that the switching signal is unknown. This means that, once the system has switched its mode, the current controller may no longer be stabilizing the (new) closed-loop system and needs to be updated. As such, we introduce a method of *switch triggering* to have the controller go back to the mode detection phase. For this, we employ the decay rate guaranteed by the applied feedback gain K_i. To be precise, recall from (4) that when the controller applies the feedback corresponding with the active mode,

$$V_i(x(t+1)) \leqslant \lambda V_i(x(t)). \tag{7}$$

This inequality provides a criterium to trigger switches: the controller switches to the mode detection phase whenever the one-step inequality (7) is violated.

Remark III.7 (On switching to the mode detection phase). Note that the system (1) may switch its mode while the inequality (7) is preserved. In this case, our controller will not immediately change to the mode detection phase, causing the switching of the system and the switching of the controller to be unsynchronized. As we show later when analyzing the properties under the proposed controller, this does not affect stability, as the Lyapunov function $t \mapsto V_i(x(t))$ is still decreasing at the desired rate λ. •

Algorithm 2 summarizes the proposed controller. After the initialization step, the algorithm starts in the mode detection phase, indicated by the flag variable S_{phase}. During the mode detection phase, Algorithm 1 is executed for each iteration until the list of compatible modes contains only a single element. After that, the controller's determination of the active mode σ_d is assigned with that remaining element and S_{phase} is toggled to 1, indicating the switching into stabilization phase. During the stabilization phase. the control $u(t) = K_{\sigma_d} x(t)$ is applied. Moreover, when (7) is violated, the controller switches back to mode detection phase by toggling S_{phase} to 0. In the meantime, $X^{\text{on}}, U_-^{\text{on}}$ are reset to record the new online data.

Algorithm 2 Data-driven switched feedback controller

Input: $\mathcal{P}, \{U_-^i, X^i, K_i, P_i\}_{i\in\mathcal{P}}, \lambda$

$\mathcal{P}_{\text{match}} \leftarrow \mathcal{P}$
$S_{\text{phase}} \leftarrow 0$ ▷ Initialize to mode detection phase
$X^{\text{on}} \leftarrow x(0)$
$U_-^{\text{on}} \leftarrow []$ ▷ Initialize online data
while the system (1) is running **do**
  if $S_{\text{phase}} = 0$ **then** ▷ Mode detection phase
    Run Algorithm 1
    Update the variables $\mathcal{P}_{\text{match}}, X^{\text{on}}, U_-^{\text{on}}$
    if $|\mathcal{P}_{\text{match}}| = 1$ **then**
      $\sigma_d \in \mathcal{P}$ ▷ Set the mode of the controller
      $\mathcal{P}_{\text{match}} \leftarrow \mathcal{P}$
      $S_{\text{phase}} \leftarrow 1$ ▷ Toggle phase
    end if
  else ▷ Stabilization phase
    Apply control $u(t) = K_{\sigma_d} x(t)$
    Get the next state $x(t+1)$
    if $x(t+1)^\top P_{\sigma_d} x(t+1) > \lambda x(t)^\top P_{\sigma_d} x(t)$ **then** ▷ Trigger for phase change
      $X^{\text{on}} \leftarrow x(t)$
      $U_-^{\text{on}} \leftarrow []$ ▷ Reset online data
      $S_{\text{phase}} \leftarrow 0$
    end if
  end if
  $t \leftarrow t+1$ ▷ Update the time
end while

We recall that "exciting enough" inputs are used during the mode detection phase, which may have negative effect on the stability of the closed-loop system. Therefore, the alternation between mode detection and stabilization in our control mechanism makes it nontrivial to ensure the stability guarantee (2). Establishing it is the subject of the next section.

IV. Stability analysis of the closed-loop system

In this section, we analyze the stability properties of the closed-loop system and identify conditions so that it enjoys the ISS-like property (2). Note that the switching nature of the closed-loop system is due not only to the unknown switching signal σ, but also to the control switches induced by Algorithm 2. We make the following assumptions regarding the system switching frequency.

Assumption 3 (Assumptions on the switching signal)**.** Let $\mathbb{T}^m := \{t_1^m, t_2^m, \cdots\}$ (resp. $\mathbb{T}^s := \{t_1^s, t_2^s, \cdots\}$) be the ordered sets consisting of the first time instants of each mode identification phase (resp. stabilization phase). Then, the following holds:

(i) The system does not switch while the controller is in mode detection phase;

(ii) Let $N(t_a, t_b)$ be the total number of mode identification phases over the time interval $[t_a, t_b)$, that is, $N(t_a, t_b) := |[t_a, t_b) \cap \mathbb{T}^m|$, where $|\cdot|$ denotes cardinality. There exists τ and $N_0 \geqslant 1$ such that

$$N(t_a, t_b) \leqslant N_0 + \frac{t_b - t_a}{\tau} \quad \forall t_a, t_b \in \mathbb{N}, t_a < t_b; \quad (8)$$

(iii) Let $M(t_a, t_b)$ be the total time spent in mode identification phases over the time interval $[t_a, t_b)$, that is, $M(t_a, t_b) := \sum_{t=t_a}^{t_b-1} \mathbf{1}(t)$, where

$$\mathbf{1}(t) := \begin{cases} 1 & \text{if } t \in [t_i^m, t_i^s) \text{ for some } i \in \mathbb{N}, \\ 0 & \text{otherwise.} \end{cases}$$

There exists $\eta \in [0, 1]$ and $T_0 \geqslant 0$ such that

$$M(t_a, t_b) \leqslant T_0 + \eta(t_b - t_a) \quad \forall t_a, t_b \in \mathbb{N}, t_a < t_b. \quad (9)$$

We next discuss the statements in Assumption 3. Statement (i) ensures that the mode detection phases and stabilization phases are alternating; in other words, the elements in $\mathbb{T}^m$ and $\mathbb{T}^s$ are ordered such that $t_1^m < t_1^s < t_2^m < t_2^s < \dots$. This implies that the time instances $t \in \{t_i^m, \cdots, t_i^s - 1\}$ are part of the i-th mode detection phase and that the time instances $t \in \{t_i^s, \cdots, t_{i+1}^m - 1\}$ are part of the i-th stabilization phase. From Corollary III.6, the time length of each mode detection phase is bounded above by $m + n$. In simulations, we have also observed that, in general, the mode detection phase is much shorter than this upper bound. Consequently, as long as the unknown switching signal does not switch too frequently, statement (i) holds.

Statement (ii) is an *average dwell-time* (ADT) condition, see e.g. [22], on the mode detection phase. Informally speaking, this statement implies that, on average, the controller is switched to mode detection phase no more than once per τ time instances. As such, with increasing τ, the frequency of switches decreases. Therefore, this assumption again holds if the system (1) switches relatively infrequently.

Finally, statement (iii) is an *average activation-time* (AAT) condition, see e.g. [23], on the mode detection phase. If the condition holds, the controller dwells in mode detection phase for at most a fraction η of the total time. This assumption holds if the mode detection phase is, on average, relatively short when compared to the stabilization phase.

Note that both the ADT and AAT conditions refer to the controller and not to the unknown switching signal σ. Nevertheless, they are related, as the switching of the controller will be infrequent if the system (1) switches slowly. This means that statements (ii) and (iii) in Assumption 3 can be considered as indirect assumptions on the switching frequency of the unknown signal σ.

The following result characterizes the stability properties of the closed-loop system.

Theorem IV.1 (Stability guarantee for the closed-loop system)**.** *Under Assumptions 1, 2, and 3, consider the switched system* (1) *under the data-driven switching feedback controller described by Algorithm 2. Let $\lambda_u, k > 0$ be such that*

$$\begin{bmatrix} P_i^{-1} & A_i & B_i \\ A_i^\top & \lambda_u P_i & 0 \\ B_i^\top & 0 & kI \end{bmatrix} \succeq 0, \quad \forall i \in \mathcal{P}, \quad (10)$$

and define

$$\mu := \max_{i,j \in \mathcal{P}} \|P_i P_j^{-1}\|. \quad (11)$$

If the following holds

$$\left(1 - \frac{\ln \lambda_u}{\ln \lambda}\right)\eta + \left(1 - \frac{\ln \mu}{\ln \lambda}\right)\frac{1}{\tau} < 1, \quad (12)$$

then for all initial states $x(0) \in \mathbb{R}^n$ and each time $t \in \mathbb{N}$, the solution of the closed-loop system satisfies

$$|x(t)| \leqslant \sqrt{\frac{\bar{\lambda}\lambda b}{\underline{\lambda} a}} a^{\frac{t}{2}} |x(0)| + u_{\max} \sqrt{\frac{bk}{\underline{\lambda}(1-a)}}, \quad (13)$$

where

$$a := \lambda \left(\frac{\mu}{\lambda}\right)^{\frac{1}{\tau}} \left(\frac{\lambda_u}{\lambda}\right)^{\eta} \in (\lambda, 1), \quad (14a)$$

$$b := \left(\frac{\mu}{\lambda}\right)^{\frac{1}{\tau} + N_0} \left(\frac{\lambda_u}{\lambda}\right)^{\eta + T_0}, \quad (14b)$$

and $\bar{\lambda}, \underline{\lambda}$ are the maximum of the largest eigenvalues and the minimum of the smallest eigenvalues among $\{P_i\}_{i \in \mathcal{P}}$ respectively.

Note that the LMIs in (10) always hold by picking λ_u and k sufficiently large. With λ given as in Assumption 1, we know that the decay rate during the stabilization phase is faster than λ, and λ_u gives an upper bound on the possible growth rate during the mode detection phase. The parameter μ corresponds to the destabilizing effect introduced by each switching. For a given switched system, the constants μ, λ, and λ_u are fixed. Hence the condition (12) is always satisfied if τ is sufficiently large and η is sufficiently small. This means as long as the system switches sufficiently infrequently and the mode detection phases are sufficiently short, then (13) holds.

V. Simulation results

We illustrate the performance of the proposed data-driven switching feedback controller. We consider an unknown switched linear system with parameters $n = 5$, $m = 3$, and $p = 5$. In order to conserve space, we refrain from presenting the state and input matrices here. A data pair (U_-^i, X^i) with $T = 7$ is collected in the initialization phase for each mode $i \in \mathcal{P} = \{1, \dots, 5\}$. We set $\lambda = 0.8$ as the desired decay rate for each mode. Both Assumptions 1 and 2 are satisfied on the initial data. While on average, we let the system switch once per 8 units of time as shown by the blue curve in Figure 2(a), the mode-to-go is set to be random. In the mode detection algorithm, we choose $u_{\max} = 1$ as the bound on the input. Figure 2 shows the result of implementing the data-driven switched controller on the system. Recall that the switching signal $\sigma_d(t)$ is the controller's determination of the active mode and the control $u(t) = K_{\sigma_d(t)} x(t)$ is applied to the system during the stabilization phase. The comparison between $\sigma(t)$ and $\sigma_d(t)$ in Figure 2(a) shows that the "lag" is almost always 3 time instants. During the mismatch between $\sigma_d(t)$ and $\sigma(t)$, the controller switches to the mode detection phase, cf. Figure 2(b). From this plot, one can see that Assumption 3 holds in general with parameters $\tau = 8$, the

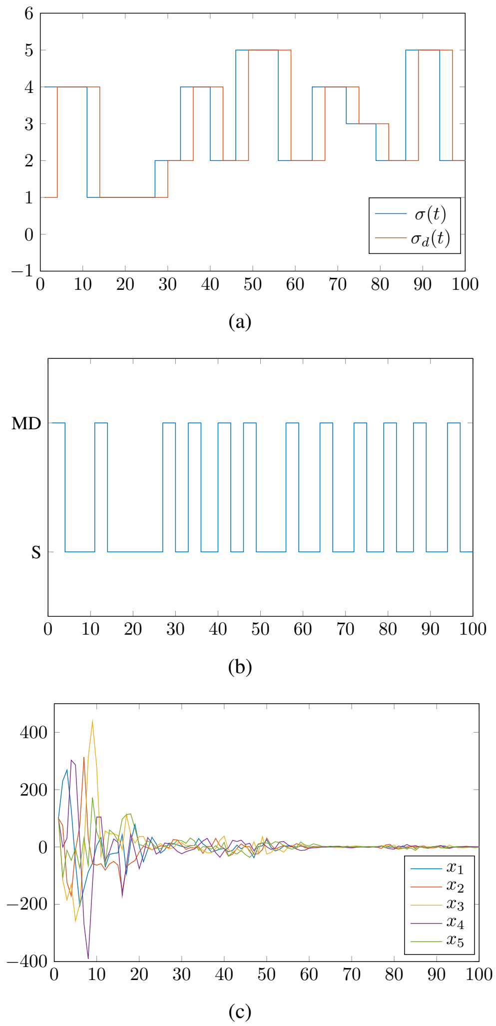

Figure 2: The results of the example of Section V. (a) shows the switching signals of the system ($\sigma(t)$) and of the controller ($\sigma_d(t)$), (b) shows the active phase of the controller, which is either mode detection or stabilization, and (c) shows the resulting state trajectories.

average dwell time of $\sigma(t)$, and $\eta = 3/8$. Although the switching is frequent enough such that the theoretical stability-guaranteeing inequality (12) is not met for this system, the resulting solution trajectory turns out to converge towards the origin, cf. Figure 2(c). We also observe some minor oscillations of the solution around the origin, which are caused by the persistent mode detection input during each mode detection phase.

VI. Conclusions

We have addressed the data-driven stabilization of switched linear systems in scenarios where no knowledge of the system matrices of each operating mode is available and the switching signal is also unknown. Instead, we have access to a number of system measurements for each mode. We have used ideas from the data informativity framework to develop a controller that alternates between mode detection and stabilization phases to guarantee an ISS-like property for a wide range of unknown switching signals. Future work will address the extension of our results to scenarios where multiple modes are compatible with the data available in the initialization phase and to the case of noisy measurements.

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