Data-driven mode detection and stabilization of unknown switched linear systems

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Abstract—**This paper considers the stabilization of unknown switched linear systems using data. Instead of a full system model, we have access to a finite number of trajectories of each of the different modes prior to the online operation of the system. On the basis of informative enough measurements, formally characterized in terms of linear matrix inequalities, we design an online switched controller that alternates between a mode detection phase and a stabilization phase. Since the specific currently-active mode is unknown, the controller employs the most recent online measurements to determine it by implementing computationally efficient tests that check compatibility with the set of systems consistent with the pre-collected measurements. The stabilization phase applies the stabilizing feedback gain corresponding to the identified active mode and monitors the evolution of the associated Lyapunov function to detect switches. When a switch is detected, the controller returns to the mode-detection phase. Under average dwell- and activation-time assumptions on the switching signal, we show that the proposed controller guarantees an input-to-state-like stability property of the closed-loop switched system. Various simulations illustrate our results.**

I. Introduction

Switched linear systems have long been of interest to the systems and control community. Such systems consist of several modes and a logic rule which governs the switching between them. Many real-world applications are naturally modeled as switched systems, where the plant switches modes due to design specifications, recurrent environmental effects, human behavior, or a combination of thereof. From a dynamic perspective, switched linear systems exhibit far more complex behavior than linear systems and this makes the design of stabilizing controllers and their analysis challenging. Most design approaches build on the model-based paradigm, where a model of the system and each of its modes is available to solve the stabilization problem. In practice, this often requires considerable modeling effort through system identification. Motivated by these observations, we adopt an online approach based on data informativity to synthesize a controller that jointly deals with the (partial) identification and (robust) stabilization of the unknown switched system.

Literature review: The control of switched systems is a well-studied field, the complexity of which requires a wide range of stabilization and analysis techniques, see e.g., [2], [3]. These include system matrix-based methods [4], [5], common Lyapunov functions [6], [7], or methods based on multiple Lyapunov functions [8]–[10], to name a few. One relevant benefit of Lyapunov-based methods is that they provide simple criteria to detect unknown switches, as inspired by e.g., the literature on fault detection [11], [12] or state estimation [13] of switched systems. These works usually require a precise model of the modes of the switched system. In order to address uncertainties in the model, several works have employed robust and adaptive techniques for stabilization. For instance, [14], [15] develops a stabilizing controller of the switched system regardless of the realization of certain parameters within a given range. The work [16] proposes an alternative approach to the same problem based on adaptive controllers. Another relevant angle is the use of switched controllers in [17] for robustly stabilizing non-switched systems.

Robust and adaptive methods still rely on a nominal model and, as such, require some type of system identification. To circumvent this, and leverage the development of new computation and data acquisition methods, there is a growing adoption of data-driven techniques. A number of different types of switched systems, distinguished by the type of switching signal, have been the focus of such efforts. The works [18], [19] consider switched systems without external inputs where the controlling element is the switching signal itself. At the other side of the spectrum, [20]–[22] focus on finding feedback control laws that stabilize a switched linear system under arbitrary and unknown switching signals. This requires that the modes of the system can be simultaneously uniformly stabilized, a particularly strong assumption. Recent work [23], [24] has also explored the data-based design of switching controllers.

The aforementioned works are based on extending known model-based methods to the data-driven setup. An alternative approach that has recently gained traction is based on Willems' fundamental lemma [25]. Within this broader context, the work [26] proposes a stabilizing controller for an unknown switched system, which is found only on the basis of noiseless measurements of the currently active mode of the system. Our treatment here leverages the informativity approach to data-driven control, as introduced in [27] and recently extended in [28], [29]. In simple terms, this method characterizes the set of systems that are compatible with collected data, and produces a robust controller that stabilizes *all* the systems in this class. Within this framework, the work [30] considers the problem of data-driven stabilization in the situation where the switching signal is determined by the controller.

Statement of contributions: We consider the problem of

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stabilizing a switched system subject to an unknown switching signal and whose modes are unmodeled. Given that the switching signal is unknown, we require all modes to be stabilizable. Using the informativity framework, we first use pre-collected measurements to determine separate robust stabilizing feedback gains for each mode. Our online stabilizing controller design employs online data to switch between a *mode detection phase* and a *stabilization phase*. In the former, we use the most recent online measurements to uniquely determine the mode of the system currently active. We provide an algorithm that, for each time instance, applies bounded inputs to excite the system and evaluates if the new measurements obtained are compatible with the set of systems consistent with the pre-collected measurements of the assumed active mode. If this is not the case, then the assumed mode is not currently active. We ensure the computational efficiency of this test by providing conservative, but efficient, outer approximations to relax the testing of non-emptyness of the intersection of convex sets. Once the active mode is determined, the controller switches to the stabilization phase, which applies the corresponding stabilizing feedback gain and monitors the evolution of the associated Lyapunov function to detect switches. When a switch is detected, the controller returns to the mode-detection phase. We study the practical stability of the closed loop of the unknown system under the proposed online switched controller and show that, under average dwell- and activation-time assumptions on the switching signal, it enjoys an input-to-state-like stability property. In particular, when measurements are noiseless, the closed loop is asymptotically stable.

We have presented preliminary results of this work in the conference article [1], where we only considered the case of noiseless data and established practical stability of the closed-loop system. The consideration in the present treatment of noisy measurements requires significant extensions to all the ingredients of the approach, a complete generalization of the results for the initialization step, and an entirely new set of techniques for mode detection. These, along with refinements in the choice of bounded inputs to excite the system, lead to stronger practical stability results, yielding in the noiseless case asymptotic stability of the closed-loop system. The stronger stability results obtained here are illustrated in a novel set of simulation results.

II. Problem formulation

Consider[1] a discrete-time switched linear system with p modes of the form

$$x(t+1) = \hat{A}_{\sigma(t)}x(t) + \hat{B}_{\sigma(t)}u(t) + w(t), \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, and $w(t) \in \mathbb{R}^n$ is a noise signal. Here, $\sigma : \mathbb{N} \to \mathcal{P} := \{1, 2, \cdots, p\}$ is the switching signal. For all $i \in \mathcal{P}$, the matrices $\hat{A}_i$ and $\hat{B}_i$ are in $\mathbb{R}^{n\times n}$ and $\mathbb{R}^{n\times m}$, respectively.

[1]Throughout the paper, we use the following notation. We denote by $\mathbb{N}$ and $\mathbb{R}$ the sets of non-negative integer and real numbers, respectively. We let $\mathbb{R}^{n\times m}$ denote the space of $n \times m$ real matrices. For any $M \in \mathbb{R}^{n\times m}$, $\|M\|$ denotes the standard 2-norm. For $P \in \mathbb{R}^{n\times n}$, $P \succeq 0$ (resp. $P \succ 0$) denotes that P is positive semi-definite (resp. definite).

We are interested in finding a stabilizing controller for this system on the basis of measurements. To be precise, we assume that the dimensions n, m and the number of modes p are known, but that the precise dynamics of the modes are not available for design, that is, for each mode $i \in \mathcal{P}$, the matrices $\hat{A}_i$ and $\hat{B}_i$ are unknown. To offset this lack of knowledge, we have access to a finite set of measurements of the state and input trajectories. These measurements correspond to an unknown, but bounded noise signal. Specifically, we assume that, before the online operation of the system, we perform an *initialization* step, where we obtain measurements of each of the individual modes (but this data is not necessarily enough to identify the dynamics of each separate mode). After this, once the system is running, we have access to *online* measurements of the currently active mode. Our goal is formalized as follows.

Problem 1 (Online switched controller design)**.** Given initialization and online measurements, design a control law $u : \mathbb{N} \to \mathbb{R}^m$ such that the resulting interconnection (1) is guaranteed to satisfy the following *input-to-state stability* (ISS)-like property: there exist constants $c > 0, \zeta \in (0, 1)$ and $r \geqslant 0$ (depending on a bound on the noise) such that

$$\|x(t)\| \leqslant c\,\zeta^t \|x(0)\| + r, \tag{2}$$

for all initial states $x(0) \in \mathbb{R}^n$ and all time $t \in \mathbb{N}$.

We consider this problem in both the absence and presence of bounded noise. Moreover, we consider two different cases regarding the switching signal. At first, we consider the situation where the controller is aware *when* the systems switches modes, but not to which mode. After this, we explore the situation where the switching signal is completely unknown, but where we make certain regularity assumptions on the *dwell time*, that is, the frequency of switches.

In order to solve Problem 1, we employ the following multi-pronged approach. Since unique models for each of the modes cannot necessarily be determined, we employ the concept of *data informativity* to formulate conditions under which the initialization measurements guarantee the existence of a stabilizing feedback controller for each of the modes. Based on this, our switched controller operates in two phases. In the *mode detection phase*, the controller selects inputs that allow us to determine the active mode. In general this requires less measurements than fully identifying the system. In fact, in the absence of noise, a bound on the required number of steps can be guaranteed by a suitable choice of inputs. Once the active mode is identified, the controller switches to the *stabilization phase*, where the controller found in the initialization step corresponding to the mode is applied.

III. Initialization step

We begin our analysis with the initialization step, where we consider the problem of finding stabilizing controllers from pre-collected measurements of the system. To formalize the notion of informativity, we require some notation. For simplicity of exposition, we first consider a single linear system, then shift our focus to switched systems.

Consider measurements of the state and input signals x and u of a system

$$x(t+1) = \hat{A}x(t) + \hat{B}u(t) + w(t),$$

on the time interval $\{0,\dots,T\}$. We define matrices

$$\begin{aligned} X &:= \begin{bmatrix} x(0) & \cdots & x(T) \end{bmatrix}, \\ X_- &:= \begin{bmatrix} x(0) & \cdots & x(T-1) \end{bmatrix}, \\ X_+ &:= \begin{bmatrix} x(1) & \cdots & x(T) \end{bmatrix}, \\ U_- &:= \begin{bmatrix} u(0) & \cdots & u(T-1) \end{bmatrix}, \\ W_- &:= \begin{bmatrix} w(0) & \cdots & w(T-1) \end{bmatrix}. \end{aligned}$$

We assume that the state and input measurements (U_-, X) are known, but that the noise signal is unknown. However, the noise satisfies

$$\begin{bmatrix} I_n \\ W_-^\top \end{bmatrix}^\top \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix} \begin{bmatrix} I_n \\ W_-^\top \end{bmatrix} \geqslant 0, \tag{3}$$

where $\Pi_{11} \in \mathbb{R}^{n\times n}$ and $\Pi_{22} \in \mathbb{R}^{T\times T}$ are symmetric, and $\Pi_{12} = \Pi_{21}^\top \in \mathbb{R}^{n\times T}$. Such noise models arise in many applications and are amenable to technical analysis, see e.g. [28]. Throughout the paper, we assume $\Pi_{22} < 0$ and $\Pi_{11} - \Pi_{12}\Pi_{22}^{-1}\Pi_{21} \geqslant 0$. These assumptions guarantee that the set of matrices W_- for which (3) holds is nonempty and bounded [28, Theorem 3.2].

Remark III.1 (Noiseless measurements). A special case of the previous is the case where $\Pi_{22} = -I_T$, $\Pi_{12} = 0$, and $\Pi_{11} = 0$. In this case, the matrix W_- satisfies (3) if and only if $W_- = 0$, that is, the measurements are without noise. •

We can now define the set of systems *consistent* with the measurements, as

$$\Sigma(U_-, X) := \left\{(A,B) : X_+ = AX_- + BU_- + W_- \text{ w/ (3)}\right\}.$$

Clearly, since the measurements satisfy the noise model and are collected from the true system, we know that $(\hat{A},\hat{B})$ is contained in the set $\Sigma(U_-, X)$. If we now define the matrix

$$N := \begin{bmatrix} I_n & X_+ \\ 0 & -X_- \\ 0 & -U_- \end{bmatrix} \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix} \begin{bmatrix} I_n & X_+ \\ 0 & -X_- \\ 0 & -U_- \end{bmatrix}^\top, \tag{4}$$

it is straightforward to conclude that the set $\Sigma(U_-, X)$ can be equivalently represented as

$$\Sigma(U_-, X) = \left\{(A,B) \mid \begin{bmatrix} I_n \\ A^\top \\ B^\top \end{bmatrix}^\top N \begin{bmatrix} I_n \\ A^\top \\ B^\top \end{bmatrix} \geqslant 0\right\}. \tag{5}$$

We are interested in characterizing properties of the true system based on the measurements. However, the set $\Sigma(U_-, X)$ might contain other systems in addition to the true one. This means, for instance, that we can only conclude that a feedback gain K stabilizes the true system if this gain stabilizes any system whose system matrices are in $\Sigma(U_-, X)$. We are interested in determining when the available data is informative enough to allow us to accomplish this.

Definition III.2 (Informativity for uniform stabilization)**.** The data (U_-, X) is *informative for uniform stabilization by state feedback with decay rate* $\lambda \in (0,1)$ if there exist $K \in \mathbb{R}^{m\times n}$ and $P \in \mathbb{R}^{n\times n}, P \succ 0$ such that

$$(A+BK)^\top P(A+BK) \prec \lambda P, \quad \forall (A,B) \in \Sigma(U_-, X). \tag{6}$$

Let $\tilde{V}(x) := x^\top P x$ and consider the closed loop of any system consistent with the data and the controller $u = Kx$. Then, if the data is informative,

$$\tilde{V}(x(t+1)) \leqslant \lambda x(t)^\top P x(t) = \lambda \tilde{V}(x(t)), \tag{7}$$

for all $t \in \mathbb{N}$. This means that, even if the matrices (A,B) can not be identified uniquely from the measurements, the feedback gain K stabilizes the system with Lyapunov function $\tilde{V}$ and decay rate λ.

To determine whether the data (U_-, X) is informative, we can exploit the fact that (5) and (6) are quadratic matrix inequalities in A and B. The following result extends [28, Theorem 5.1.(a)], which essentially considers the special case $\lambda = 1$, to reduce the problem to that of finding matrices K and P that satisfy a *linear matrix inequality* (LMI).

Theorem III.3 (Conditions for informativity for uniform stabilization)**.** *The data* (U_-, X) *is informative for uniform stabilization by state feedback with decay rate* λ *if and only if there exist* $Q \in \mathbb{R}^{n\times n}$*, with* $Q \succ 0$*,* $L \in \mathbb{R}^{m\times n}$ *and* $\beta > 0$ *such that*

$$\begin{bmatrix} \lambda Q - \beta I_n & 0 & 0 & 0 \\ 0 & 0 & 0 & Q \\ 0 & 0 & 0 & L \\ 0 & Q & L^\top & Q \end{bmatrix} - \begin{bmatrix} I_n & X_+ \\ 0 & -X_- \\ 0 & -U_- \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{bmatrix} \begin{bmatrix} I_n & X_+ \\ 0 & -X_- \\ 0 & -U_- \\ 0 & 0 \end{bmatrix}^\top \succeq 0. \tag{8}$$

Moreover, the matrices $K := LQ^{-1}$ *and* $P := Q^{-1}$ *satisfy* (6).

The proof of Theorem III.3 is given in the appendix. The characterization given in this result provides the backbone of the initialization step for our controller design. As noted in the problem formulation, we have access to measurements of each of the modes in the initialization step. We denote the measurements corresponding to mode $i \in \mathcal{P}$ by (U_-^i, X^i). We consider the corresponding matrices U_-^i, X^i and a noise model given by Π^i. Since we are interested in stabilizing the unknown switched system regardless of the switching signal, this implies that, each of the modes must be stabilizable. This motivates the following assumption.

Assumption 1 (Initialization step)**.** Let $\lambda \in (0,1)$. For each mode $i \in \mathcal{P}$, the data (U_-^i, X^i) is informative for uniform stabilization by state feedback with decay rate λ.

We denote by K_i the feedback corresponding to (U_-^i, X^i) obtained from Theorem III.3. The corresponding Lyapunov matrix is denoted by P_i. This means that we have (potentially) different feedback and Lyapunov matrices for each mode, but that the decay rate is uniform for all modes $i \in \mathcal{P}$. Note that the choice of a common decay rate across the modes does not introduce conservatism, since if (6) holds for λ, it also holds for any $\bar{\lambda}$ with $\lambda \leqslant \bar{\lambda} < 1$.

IV. Mode detection phase

After the initialization step, we consider the situation where the unknown system is in operation and additional *online*

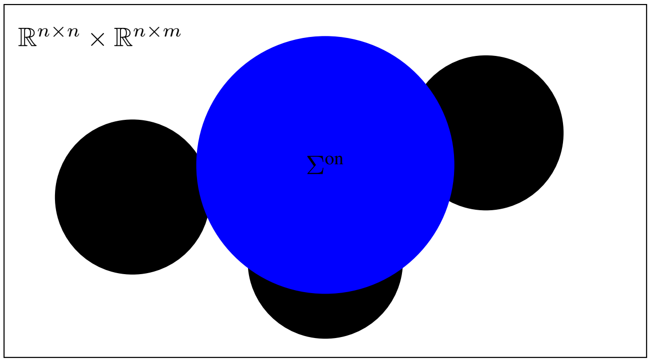


Figure 1: Graphical interpretation of the mode detection scheme. Initially, Σ^{on} intersects the sets corresponding to three different modes. As more data are collected, Σ^{on} decreases in size (cf. darker blue disks). When enough data are available, Σ^{on} eventually becomes compatible only with mode 2. Note that we do not need Σ^{on} to shrink into a singleton (system identification) before reaching unique compatibility.

measurements, denoted by $(U_-^{\text{on}}, X^{\text{on}})$, are collected. To be precise, we consider measurements that are collected sequentially from the system when it dwells in a single, unknown mode. As before, we let $\Sigma(U_-^{\text{on}}, X^{\text{on}})$ denote the set of all systems compatible with the online measurements. To simplify the exposition, we introduce for each $i \in \mathcal{P}$ the following shorthand notation,

$$\Sigma^i := \Sigma(U_-^i, X^i), \quad \Sigma^{\text{on}} := \Sigma(U_-^{\text{on}}, X^{\text{on}}).$$

We assume that the systems dwells in a single mode, and that we collect the online measurements sequentially. As such, after ℓ time instances, we can write

$$\Sigma^{\text{on}} = \bigcap_{t=0}^{\ell-1} \Sigma_t^{\text{on}},$$

where Σ_t^{on} is the set of systems consistent with the measurements collected at time instance t.

Our goal is to stabilize the switched system. To do this, we could simply test whether the online measurements $(U_-^{\text{on}}, X^{\text{on}})$ are informative for uniform stabilization with decay rate λ. However, the initialization data gives us additional information that can be exploited: we know that the true system is contained in $\Sigma^i \cap \Sigma^{\text{on}}$ for at least one $i \in \mathcal{P}$. If we could uniquely determine which of the p different systems has generated the online measurements $(U_-^{\text{on}}, X^{\text{on}})$, we can then apply the stabilizing controller found in the initialization step. The notion of data compatibility plays a key role in achieving this goal.

Definition IV.1 (Data compatibility)**.** The data pairs (U_-^1, X^1) and (U_-^2, X^2) are *compatible* if there exists a system that is consistent with both, that is, $\Sigma^1 \cap \Sigma^2 \neq \varnothing$.

We are interested in determining the active mode of the system from the online measurements.

Definition IV.2 (Informativity for mode detection)**.** Given initialization data $\{(U_-^i, X^i)\}_{i\in\mathcal{P}}$, the online measurements $(U_-^{\text{on}}, X^{\text{on}})$ are *informative for mode detection* if $(U_-^{\text{on}}, X^{\text{on}})$ and (U_-^i, X^i) are compatible for exactly one $i \in \mathcal{P}$.

In order for the mode detection phase to be successful, we assume that the initialization data are pairwise incompatible.

Assumption 2 (Initialization step –cont'd)**.** The data $\{(U_-^i, X^i)\}_{i\in\mathcal{P}}$ are such that (U_-^i, X^i) and (U_-^j, X^j) are incompatible for each pair $i \neq j \in \mathcal{P}$.

Since the online measurements are generated by precisely one of the modes $i \in \mathcal{P}$, there must be at least one $i \in \mathcal{P}$ such that $(U_-^{\text{on}}, X^{\text{on}})$ and (U_-^i, X^i) are compatible. By assuming that the initial data are pairwise incompatible, we can conclude that once Σ^{on} becomes sufficiently "small", the mode i corresponding to the data (U_-^i, X^i) which are compatible with $(U_-^{\text{on}}, X^{\text{on}})$ must be unique. Figure 1 provides a graphical illustration.

A. Mode detection for noiseless data

First, we consider the system without noise, that is, $w(t) = 0$. The following result characterizes compatibility in this case.

Lemma IV.3 (Conditions for noiseless data compatibility)**.** *Noiseless data (U_-^1, X^1) and (U_-^2, X^2) are compatible if and only if*

$$\ker \begin{bmatrix} X_-^1 & X_-^2 \\ U_-^1 & U_-^2 \end{bmatrix} \subseteq \ker \begin{bmatrix} X_+^1 & X_+^2 \end{bmatrix}. \tag{9}$$

Proof. Since $W_-^1 = W_-^2 = 0$, the data are compatible if and only if there exists A and B such that: $X_+^1 = AX_-^1 + BU_-^1$ and $X_+^2 = AX_-^2 + BU_-^2$. Equivalently, we have

$$\begin{bmatrix} X_+^1 & X_+^2 \end{bmatrix} = \begin{bmatrix} A & B \end{bmatrix} \begin{bmatrix} X_-^1 & X_-^2 \\ U_-^1 & U_-^2 \end{bmatrix}.$$

The existence of such $\begin{bmatrix} A & B \end{bmatrix}$ is equivalent to (9). □

This result provides a test to check whether the online measurements are compatible with the initialization data. At each step, the idea is to evaluate whether the online data are compatible with precisely one mode of the system. This iterative procedure raises the question of how to select the input to the unknown system appropriately at each step. In other words, we are interested in *generating* online data such that, after a bounded number of steps, $\Sigma(U_-^{\text{on}}, X^{\text{on}})$ becomes small enough for the data to be informative for mode detection. Formally, the problem is to find a time horizon T^{on} and inputs $u^{\text{on}}(0), \ldots, u^{\text{on}}(T^{\text{on}} - 1)$ such that the corresponding online data $(U_-^{\text{on}}, X^{\text{on}})$ are informative for mode detection.

To obtain such inputs, we adapt the experiment design method of [31]. When each of the modes $(\hat{A}_i, \hat{B}_i)$ of the system are controllable, [31, Theorem 1] gives a construction for inputs $u^{\text{on}}(0), \ldots, u^{\text{on}}(n+m-1)$ such that the corresponding set $\Sigma(U_-^{\text{on}}, X^{\text{on}})$ is a singleton. Assume

$$\operatorname{rank} \begin{bmatrix} X_-^{\text{on}} \\ U_-^{\text{on}} \end{bmatrix} \neq n + m.$$

Now, either $x(t) \notin \operatorname{im} X_-^{\text{on}}$, in which case, take $u(t)$ equal to 0. In the other case, [31, Theorem 1] shows that there exist η, ξ such that $\eta \neq 0$ and

$$\begin{bmatrix} \xi \\ \eta \end{bmatrix} \in \ker \begin{bmatrix} X_-^{\text{on}} \\ U_-^{\text{on}} \end{bmatrix}^\top.$$

Now, by taking any $u(t) \in \mathbb{R}^m$ such that $\xi^\top x(t)+\eta^\top u(t) \neq 0$, it can be shown that

$$\operatorname{rank} \begin{bmatrix} X_-^{\text{on}} \\ U_-^{\text{on}} \end{bmatrix} < \operatorname{rank} \begin{bmatrix} X_-^{\text{on}} & x(t) \\ U_-^{\text{on}} & u(t) \end{bmatrix}.$$

Note that, without loss of generality, we can choose $u(t)$ such that $\|u(t)\| \leqslant c\|x(t)\|$ for any $c > 0$.

Clearly, after $n+m$ repetitions, this procedure results in a data matrix which has full row rank. This implies that $\Sigma(U_-^{\text{on}}, X^{\text{on}})$ is a singleton. As such, under Assumption 2, the measurements up to $T^{\text{on}} = n+m$ are guaranteed to be informative for mode detection. This provides a worst-case bound as, in general, mode detection is achieved with far fewer measurements than those required for system identification.

Algorithm 1 Mode detection for noiseless data

Input: $\mathcal{P}_{\text{match}}, \{U_-^i, X^i\}_{i\in\mathcal{P}}, U_-^{\text{on}}, X^{\text{on}}, c$
Output: $\mathcal{P}_{\text{match}}, U_-^{\text{on}}, X^{\text{on}}$

1: **if** $U_-^{\text{on}} \neq []$ and $x(t) \in \operatorname{im} X_-^{\text{on}} \setminus \{0\}$ **then**
2: Pick $\begin{bmatrix} \xi \\ \eta \end{bmatrix} \in \ker \begin{bmatrix} X_-^{\text{on}} \\ U_-^{\text{on}} \end{bmatrix}^\top$ with $\eta \neq 0$
3: Let $u(t) \in \mathbb{R}^m$ be such that
$$\|u(t)\| \leqslant c\|x(t)\| \text{ and } \xi^\top x(t) + \eta^\top u(t) \neq 0$$
▷ Choose the next input
4: **else**
5: $u(t) \leftarrow 0$
6: **end if**
7: Get the next state $x(t+1)$
8: $U_-^{\text{on}} \leftarrow \begin{bmatrix} U_-^{\text{on}} & u(t) \end{bmatrix}$
9: $X^{\text{on}} \leftarrow \begin{bmatrix} X^{\text{on}} & x(t+1) \end{bmatrix}$ ▷ Append the online data
10: **if** $x(t+1) = 0$ **then** ▷ System stable for any feedback
11: $\mathcal{P}_{\text{match}} = \{1\}$
12: **else**
13: **for** $i \in \mathcal{P}_{\text{match}}$ **do**
14: **if** the inclusion (9) is violated **then**
▷ Data are incompatible with mode i
15: $\mathcal{P}_{\text{match}} = \mathcal{P}_{\text{match}} \backslash \{i\}$ ▷ Eliminate mode i
16: **end if**
17: **end for**
18: **end if**

Algorithm 1 formalizes the mode detection procedure for noiseless data. Note that if, at any point of the algorithm, the state satisfies $x(t) = 0$ (step 10), then choosing $u(t) = 0$ stabilizes any linear system, and therefore specifically all modes of the system. As such, we can apply any of the feedback gains K_i. We enforce this by setting $\mathcal{P}_{\text{match}}$ equal to any of the modes. Otherwise, the algorithm checks whether (9) is violated for each of the modes remaining in $\mathcal{P}_{\text{match}}$, in which case the corresponding mode is discarded. If $|\mathcal{P}_{\text{match}}| \neq 1$, the algorithm should be repeated and another online measurement collected. If $|\mathcal{P}_{\text{match}}| = 1$, the active mode detection is identified.

Corollary IV.4 (Algorithm 1 terminates in a finite number of repetitions)**.** *Suppose that for each mode $i \in \mathcal{P}$ the matrix pair $(\hat{A}_i, \hat{B}_i)$ is controllable and that Assumptions 1 and 2 hold. Let $\mathcal{P}_{\text{match}} = \mathcal{P}$ and $X^{\text{on}} = [x(0)]$. Execute Algorithm 1 iteratively, updating $\mathcal{P}_{\text{match}}$ and $(U_-^{\text{on}}, X^{\text{on}})$ at every step. Then $|\mathcal{P}_{\text{match}}| = 1$ after at most $n+m$ iterations. Moreover, the online data $(U_-^{\text{on}}, X^{\text{on}})$ are either informative for mode detection or such that $x(T^{\text{on}}) = 0$.*

B. Mode detection for noisy data

As in the noiseless case, testing whether $\Sigma^i \cap \Sigma^{\text{on}} \neq \varnothing$ amounts to checking non-emptiness of the intersection of two convex sets. This needs to be done online, during the collection of measurements, and hence requires to be resolved in time with the evolution of the system. In the noiseless case, the fact that the convex sets under consideration were affine made solving the problem online feasible by employing Lemma IV.3. However, this is no longer the case in the presence of noise. Therefore, to be able to test for (in)compatibility in an online fashion, here we develop a number of conservative, but more computationally efficient, methods. Our exposition first shows how to over-approximate the sets of compatible systems by spheres. This allows us to provide simple tests for incompatibility after a single measurement. After this, we also propose methods to deal with sequential measurements.

Outer approximation of set of consistent systems on the basis of measurements: For simplicity of exposition, we first deal with a single true linear system and a single set of measurements (U_-, X). Moreover, to further ease the notation, we assume in the remainder of the paper that the noise models are given in the form of a bound on the energy of the noise, that is, $W_- W_-^\top \leqslant Q$, with $Q = Q^\top \geqslant 0$. In particular, this implies

$$\Pi := \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{12}^\top & \Pi_{22} \end{bmatrix} = \begin{bmatrix} Q & 0 \\ 0 & -I_T \end{bmatrix}. \tag{10}$$

Noise models of this form are very versatile. Depending on the choice of Q, we can model for instance, the assumption of a signal-to-noise ratio by taking $Q = \gamma X_- X_-^\top$. The noise model also captures the case of exact measurements ($Q = 0$).

Assuming that $\begin{bmatrix} X_- \\ U_- \end{bmatrix}$ has full row rank, we define the *center* Z of the set $\Sigma = \Sigma(U_-, X)$ of consistent systems, given as in (5), by

$$Z := X_+ \begin{bmatrix} X_- \\ U_- \end{bmatrix}^\dagger, \tag{11}$$

where $M^\dagger$ denotes the Moore-Penrose inverse of a matrix M. Let $\lambda_{\min}(M)$ and $\lambda_{\max}(M)$ denote the smallest and largest eigenvalue of M respectively, and define the *radius* r of Σ as

$$r := \sqrt{\frac{\lambda_{\max}(Q)}{\lambda_{\min}\left(\begin{bmatrix} X_- \\ U_- \end{bmatrix}\begin{bmatrix} X_- \\ U_- \end{bmatrix}^\top\right)}}. \tag{12}$$

The following result provides an outer approximation of the set Σ.

Lemma IV.5 (Outer approximation of set of consistent systems)**.** *Suppose that Π is given by* (10) *and $\begin{bmatrix} X_- \\ U_- \end{bmatrix}$ has full row rank. Then $\Sigma \subseteq B^r(Z) = \{(A, B) \mid \| \begin{bmatrix} A & B \end{bmatrix} - Z\| \leqslant r\}$, where Z is the center* (11) *and r is the radius* (12) *of Σ.*

Proof. By definition $(A,B)\in\Sigma$ if and only if

$$\left(X_+ - \begin{bmatrix}A & B\end{bmatrix}\begin{bmatrix}X_-\\U_-\end{bmatrix}\right)\left(X_+ - \begin{bmatrix}A & B\end{bmatrix}\begin{bmatrix}X_-\\U_-\end{bmatrix}\right)^\top \leqslant Q.$$

This implies that

$$\left(\begin{bmatrix}A & B\end{bmatrix} - Z\right)\begin{bmatrix}X_-\\U_-\end{bmatrix}\begin{bmatrix}X_-\\U_-\end{bmatrix}^\top\left(\begin{bmatrix}A & B\end{bmatrix} - Z\right)^\top \leqslant Q.$$

Since $\begin{bmatrix}X_-\\U_-\end{bmatrix}$ has full row rank, the matrix $\begin{bmatrix}X_-\\U_-\end{bmatrix}\begin{bmatrix}X_-\\U_-\end{bmatrix}^\top$ is positive definite. Moreover, for any matrix $M\in\mathbb{R}^{n\times n}$ such that $M\geqslant 0$, we have $\lambda_{\min}(M)I_n \leqslant M \leqslant \lambda_{\max}(M)I_n$. This implies that

$$\left(\begin{bmatrix}A & B\end{bmatrix} - Z\right)\left(\begin{bmatrix}A & B\end{bmatrix} - Z\right)^\top \leqslant r^2 I_n,$$

and hence we $\|\begin{bmatrix}A & B\end{bmatrix} - Z\|^2 \leqslant r^2$ for any $(A,B)\in\Sigma$. □

Conversely, if $\begin{bmatrix}X_-\\U_-\end{bmatrix}$ does not have full row rank, then the corresponding set Σ is unbounded, and there does not exist any r and Z such that $\Sigma\subseteq B^r(Z)$.

Conservative tests for incompatibility using outer approximations: Here we describe how to leverage the outer approximation on the set of systems consistent with some given measurements to determine data incompatibility. Given the initialization data, we define the distance between Σ^i and Σ^j for $i,j\in\mathcal{P}$ by

$$d_{ij} := \min_{\substack{(A_i,B_i)\in\Sigma^i\\(A_j,B_j)\in\Sigma^j}} \|\begin{bmatrix}A_i - A_j & B_i - B_j\end{bmatrix}\|.$$

Since the sets Σ^i are closed by definition, the measurements (U_-^i, X^i) and (U_-^j, X^j) are incompatible if and only if $d_{ij} > 0$. This distance can be determined from the initialization data and computed in the initialization step. We can give an alternative, more computationally efficient test under the additional assumption that the matrices $\begin{bmatrix}X_-^i\\U_-^i\end{bmatrix}$ have full row rank. In this case, we can use Lemma IV.5 to efficiently bound the distance between Σ^i and Σ^j from below. We denote by Z_i and r_i, respectively, the center and radius of Σ^i, for each $i\in\mathcal{P}$. Then, we have

$$d_{ij} \geqslant \|Z_i - Z_j\|^2 - r_i - r_j. \tag{13}$$

This can be used to formulate an efficient method of verifying Assumption 2 as follows.

Corollary IV.6 (Ensuring Assumption 2 holds)**.** *Suppose that $\Sigma^i\subseteq B^{r_i}(Z_i)$ for all $i\in\mathcal{P}$. If $\|Z_i - Z_j\|^2 > r_i + r_j$, for all $i\neq j$, then Assumption 2 holds.*

The same idea can be used for the case of online measurements $(U_-^{\text{on}}, X^{\text{on}})$. Recall that these are collected sequentially, giving rise to $\Sigma^{\text{on}} = \bigcap_{t=0}^{T^{\text{on}}-1}\Sigma_t^{\text{on}}$. Since the ultimate goal of checking for incompatibility is to determine the active mode of the switched system, after collecting each measurement, we face the dichotomy of already using the information or wait for the additional one provided by subsequent measurements. As a first step towards resolving this dichotomy, we develop a computationally efficient test to check for compatibility with a single set Σ_t^{on} determined by a *single* measurement.

In the single measurement case, the noise model (10) takes the form $w(t)w(t)^\top \leqslant q^2 I_n$, or equivalently, $\|w(t)\|\leqslant q$. Then,

$$\Sigma_t^{\text{on}} = \{(A,B) \mid \|\begin{bmatrix}A & B\end{bmatrix}\begin{pmatrix}x(t)\\u(t)\end{pmatrix} - x(t+1)\| \leqslant q\}. \tag{14}$$

The following result provides a sufficient condition for incompatibility with Σ_t^{on}.

Lemma IV.7 (Scalar test for incompatibility with single online measurement)**.** *Let $\Sigma\subseteq B^r(Z)$ for some r and Z. For Σ_t^{on} as in* (14)*, if*

$$\|Z\begin{pmatrix}x(t)\\u(t)\end{pmatrix} - x(t+1)\| > q + r\|\begin{pmatrix}x(t)\\u(t)\end{pmatrix}\|, \tag{15}$$

then $\Sigma\cap\Sigma_t^{\text{on}} = \varnothing$.

Proof. Suppose that (15) holds and let $\bar{Z}\in\Sigma$. We show that $\bar{Z}\notin\Sigma_t^{\text{on}}$. From the triangle inequality, we have

$$\begin{aligned}&\|\bar{Z}\begin{pmatrix}x(t)\\u(t)\end{pmatrix} - x(t+1)\|\\ &\quad\geqslant \|Z\begin{pmatrix}x(t)\\u(t)\end{pmatrix} - x(t+1)\| - \|(\bar{Z}-Z)\begin{pmatrix}x(t)\\u(t)\end{pmatrix}\|.\end{aligned}$$

Since, by assumption $\bar{Z}\in B^r(Z)$, we can write

$$\|(\bar{Z}-Z)\begin{pmatrix}x(t)\\u(t)\end{pmatrix}\| \leqslant r\|\begin{pmatrix}x(t)\\u(t)\end{pmatrix}\|.$$

Combining this with (15), we obtain

$$\|\bar{Z}\begin{pmatrix}x(t)\\u(t)\end{pmatrix} - x(t+1)\| > q,$$

and hence $\bar{Z}\notin\Sigma_t^{\text{on}}$ as claimed. □

This result allows to check incompatibility by means of a scalar condition, instead of finding the intersections of two quadratic sets in $\mathbb{R}^{n\times(n+m)}$. As a further benefit, we can apply the previous reasoning to efficiently determining when a *chosen* input is guaranteed to resolve the incompatibility problem.

Corollary IV.8 (Choosing inputs for incompatibility)**.** *Let Σ_t^{on} be as in* (14) *and suppose that $\Sigma^i\subseteq B^{r_i}(Z_i)$ for all $i\in\mathcal{P}$. Let $i\neq j\in\mathcal{P}$ and $x(t)\in\mathbb{R}^n$. If $u(t)$ is such that*

$$\|(Z_i - Z_j)\begin{pmatrix}x(t)\\u(t)\end{pmatrix}\| > (r_i + r_j)\|\begin{pmatrix}x(t)\\u(t)\end{pmatrix}\| + 2q, \tag{16}$$

then at most one of $\Sigma^i\cap\Sigma_t^{\text{on}}\neq\varnothing$ or $\Sigma^j\cap\Sigma_t^{\text{on}}\neq\varnothing$ holds.

Interestingly, the condition in Corollary IV.8 can be met wherever the effect of the input on the systems in Σ_i and Σ_j is ‘different enough’. Formally, if

$$\|(Z_i - Z_j)\begin{bmatrix}0\\I_m\end{bmatrix}\| > r_1 + r_2, \tag{17}$$

then, for any $x(t)$, there exists a large enough $u(t)$ for which (16) holds, allowing to discard at least one of the systems. Equipped with these results, one can generalize Algorithm 1 to noisy data by first outer approximating the sets of systems compatible with the initialization data $\{\Sigma_i\}_{i\in\mathcal{P}}$ using Lemma IV.5 and then (assuming that (17) holds for each pair) designing inputs guaranteed to eliminate at least one mode from consideration at each step, using Corollary IV.8. The corresponding mode detection scheme takes at most p steps. Instead of formalizing this procedure, we will take full advantage of the information provided by multiple measurements at once.

Considering incremental measurements: Treating online measurements separately may prove restrictive. In fact, the true system is not just contained in each Σ_t^{on}, but in their intersection $\Sigma^{\text{on}} = \bigcap_{t=0}^{T^{\text{on}}-1} \Sigma_t^{\text{on}}$. To address this, we develop here results dealing with intersections of such sets. In order to circumvent the computational complexity associated with considering increasing numbers of intersections of quadratic sets, we overestimate them with a single set, which leads to conservative, more computationally efficient tests. We employ a set representation of the form (5), that is, we have for each $t = 0, \dots, T^{\text{on}} - 1$,

$$\Sigma_t^{\text{on}} = \left\{ (A, B) \mid \begin{bmatrix} I_n \\ A^\top \\ B^\top \end{bmatrix}^\top N_t^{\text{on}} \begin{bmatrix} I_n \\ A^\top \\ B^\top \end{bmatrix} \geqslant 0 \right\},$$

for some N_t^{on} given in the form of (4), with a corresponding noise model (10). Given $\alpha_0, \dots, \alpha_{T^{\text{on}}-1} \geqslant 0$, we denote the vector $\alpha := (\alpha_0 \cdots \alpha_{T^{\text{on}}-1})^\top$ and define $N^{\text{on}}(\alpha) := \sum_{t=0}^{T^{\text{on}}-1} \alpha_t N_t^{\text{on}}$. Similarly, we let N^i be the matrices corresponding to the initialization sets Σ^i, and define $N^{i,\text{on}}(\alpha) := N^i + N^{\text{on}}(\alpha)$, for each $i \in \mathcal{P}$. Consider the sets

$$\Sigma^{\text{on}}(\alpha) := \left\{ (A, B) \mid \begin{bmatrix} I_n \\ A^\top \\ B^\top \end{bmatrix}^\top N^{\text{on}}(\alpha) \begin{bmatrix} I_n \\ A^\top \\ B^\top \end{bmatrix} \geqslant 0 \right\},$$

$$\Sigma^{i,\text{on}}(\alpha) := \left\{ (A, B) \mid \begin{bmatrix} I_n \\ A^\top \\ B^\top \end{bmatrix}^\top N^{i,\text{on}}(\alpha) \begin{bmatrix} I_n \\ A^\top \\ B^\top \end{bmatrix} \geqslant 0 \right\}.$$

Note that, if a number of quadratic matrix inequalities are satisfied, then so is any nonnegative combination of them. Therefore, for any $\alpha_0, \dots, \alpha_{T^{\text{on}}-1} \geqslant 0$,

$$\Sigma^{\text{on}} \subseteq \Sigma^{\text{on}}(\alpha) \text{ and } \Sigma^i \cap \Sigma^{\text{on}} \subseteq \Sigma^{i,\text{on}}(\alpha).$$

This provides over-approximations of Σ^{on} and $\Sigma^i \cap \Sigma^{\text{on}}$ in terms of such nonnegative combinations, allowing us to state the following result.

Lemma IV.9 (Parameterized condition for incompatibility with online measurements)**.** *Given online measurements* $(U_-^{\text{on}}, X^{\text{on}})$ *and* $i \in \mathcal{P}$, *if there exist* $\alpha_0, \dots, \alpha_{T^{\text{on}}-1} \geqslant 0$ *such that either*

(i) $\Sigma^i \cap \Sigma^{\text{on}}(\alpha) = \varnothing$ *or*
(ii) $\Sigma^{i,\text{on}}(\alpha) = \varnothing$,

then $\Sigma^i \cap \Sigma^{\text{on}} = \varnothing$.

This result allows us to test intersections of multiple quadratic sets for emptiness in terms of a parametrized intersection of either two such sets using Lemma IV.9(i), or even one, using Lemma IV.9(ii). The following result provides a way to efficiently test for the latter.

Lemma IV.10 (Spectral test for incompatibility with online measurements)**.** *Given initialization data* $\{(U_-^i, X^i)\}_{i \in \mathcal{P}}$ *and online measurements* $(U_-^{\text{on}}, X^{\text{on}})$, *suppose* $\begin{bmatrix} X_-^i \\ U_-^i \end{bmatrix}$ *has full row rank for* $i \in \mathcal{P}$. *For any* i *in*$\mathcal{P}$ *and* $\alpha_0, \dots, \alpha_{T^{\text{on}}-1} \geqslant 0$, $\Sigma^{i,\text{on}}(\alpha) \neq \varnothing$ *if and only if* $N^{i,\text{on}}(\alpha)$ *has precisely* $n + m$ *negative eigenvalues. Equivalently, given*

$$N^{i,\text{on}}(\alpha) = \begin{bmatrix} \bar{N}_{11} & \bar{N}_{12} \\ \bar{N}_{21} & \bar{N}_{22} \end{bmatrix},$$

then $\Sigma^{i,\text{on}}(\alpha) \neq \varnothing$ *if and only if* $\bar{N}_{11} - \bar{N}_{12}\bar{N}_{22}^{-1}\bar{N}_{21} \geqslant 0$.

Proof. Given that $\bar{N}_{22} < 0$ by construction, the statement follows from [28, Thm. 3.2]. ☐

One can use either criteria in Lemma IV.9 to generalize Algorithm 1 to noisy data in a way that integrates the information provided by multiple measurements at once. Next, we formalize this using Lemma IV.9(i), where we take $\alpha = \quad \in \mathbb{R}^{T^{\text{on}}}$, the vector of all ones. Recall that, during online operation, we collect single measurements, with each noise sample satisfying $\|w(t)\| \leqslant q$, for some $q > 0$. The set Σ_t^{on} is then described by (14). Consider

$$N^{\text{on}}(\quad) = \begin{bmatrix} I_n & X_+ \\ 0 & -X_- \\ 0 & -U_- \end{bmatrix} \begin{bmatrix} q^2 T^{\text{on}} I_n & 0 \\ 0 & -I_{T^{\text{on}}} \end{bmatrix} \begin{bmatrix} I_n & X_+ \\ 0 & -X_- \\ 0 & -U_- \end{bmatrix}^\top.$$

Using the Schur complement, we can conclude that $(A, B) \in \Sigma^{\text{on}}(\quad)$ if and only if

$$\begin{bmatrix} q^2 T^{\text{on}} I & X_+^{\text{on}} - AX_-^{\text{on}} - BU_-^{\text{on}} \\ (X_+^{\text{on}} - AX_-^{\text{on}} - BU_-^{\text{on}})^\top & I \end{bmatrix} \geqslant 0.$$

A similar LMI can be obtained to check for whether $(A, B) \in \Sigma^i$, leading to the following result.

Corollary IV.11 (LMI test for incompatibility with online measurements)**.** *Given online single measurements obtained sequentially, with each noise sample satisfying* $\|w(t)\| \leqslant q$, *for some* $q > 0$, *let* $i \in \mathcal{P}$ *and assume the initialization data* (U_-^i, X^i) *has noise model* (10) *with* $Q^i = q^2 T_i I$. *If there are no matrices* A *and* B *that satisfy simultaneously*

$$\begin{bmatrix} q^2 T^i I_n & X_+^i - AX_-^i - BU_-^i \\ (X_+^i - AX_-^i - BU_-^i)^\top & I_{T^i} \end{bmatrix} \geqslant 0, \tag{18a}$$

$$\begin{bmatrix} q^2 T^{\text{on}} I_n & X_+^{\text{on}} - AX_-^{\text{on}} - BU_-^{\text{on}} \\ (X_+^{\text{on}} - AX_-^{\text{on}} - BU_-^{\text{on}})^\top & I_{T^{\text{on}}} \end{bmatrix} \geqslant 0, \tag{18b}$$

then the data are incompatible, that is, $\Sigma^i \cap \Sigma^{\text{on}} = \varnothing$.

Algorithm 2 presents a mode detection procedure that employs these LMI conditions (instead of the kernel condition (9) in Algorithm 1) to check for incompatibility. Also, the inputs at each step of the algorithm are chosen randomly.

V. Stabilization phase and detecting switches

Having solved the mode detection problem, here we turn our attention to the stabilization phase. At the start of this phase, we know the mode the system is operating in. Therefore, we simply apply the controller computed in the initialization step, see Section III,

$$u = K_i x,$$

Algorithm 2 Mode detection for noisy data

Input: $\mathcal{P}_{\text{match}}, \{U_-^i, X^i\}_{i\in\mathcal{P}}, U_-^{\text{on}}, X^{\text{on}}, c, q$
Output: $\mathcal{P}_{\text{match}}, U_-^{\text{on}}, X^{\text{on}}$

Pick a random $u(t) \in \mathbb{R}^m$ such that $\|u(t)\| \leqslant c\|x(t)\|$
Get the next state $x(t+1)$
$U_-^{\text{on}} \leftarrow \begin{bmatrix} U_-^{\text{on}} & u(t) \end{bmatrix}$
$X^{\text{on}} \leftarrow \begin{bmatrix} X^{\text{on}} & x(t+1) \end{bmatrix}$ ▷ Append the data
for $i \in \mathcal{P}_{\text{match}}$ **do**
  if the pair of LMIs (18) is infeasible **then** ▷ Data are incompatible with mode i
    $\mathcal{P}_{\text{match}} = \mathcal{P}_{\text{match}} \backslash \{i\}$ ▷ Eliminate mode i
  end if
end for

where $i \in \mathcal{P}$ is the currently active mode. The controller remains in the stabilization phase until a switch in the mode of the system is detected. As mentioned in the problem formulation, in Section II, we consider two scenarios:

- the switching signal is partially known: the controller is aware of when the system switches modes, yet it does not know which mode the system has switched into;
- the switching signal is completely unknown: this means that the controller needs to implement a mechanism to detect whether a switch has occurred.

In either case, once a switch has been detected, to determine the current active mode, the controller will switch back to the mode detection phase described in Section IV.

Here we describe a procedure to detect switches in the system mode in the second scenario above. This consists of monitoring the evolution of the Lyapunov function, $V_i(x) = x^\top P_i x$, computed in the initialization step, cf. Section III, associated with the currently active mode $i \in \mathcal{P}$. From the triangle inequality and (6), when the system is operating in mode i with the feedback control $u = K_i x$, and under the noise bound $\|w(t)\| \leqslant q$, we have

$$\begin{aligned} V_i(x(t+1)) &= \|P_i^{\frac{1}{2}}((\hat{A}_i + \hat{B}_i K_i)x(t) + w(t))\| \\ &\leqslant \sqrt{\lambda} V_i(x(t)) + \lambda_{\max}(P_i^{\frac{1}{2}})q. \end{aligned} \tag{19}$$

The contrapositive of this statement reads as follows: if the inequality does *not* hold, then the system cannot be operating in mode i. We employ it to detect when the system has switched between modes, that is, the controller switches to the mode detection phase whenever

$$V_i(x(t+1)) > \sqrt{\lambda} V_i(x(t)) + \lambda_{\max}(P_i^{\frac{1}{2}})q. \tag{20}$$

Note that the system (1) may switch between modes while the inequality (19) is still preserved. In this case, the controller applies the controller corresponding to the previously detected mode, and the mode of the system and the mode of the controller become desynchronized. As our ensuing technical analysis shows, this does not affect the stability properties of the closed-loop system.

VI. Stabilization via online switched controller

In this section, we describe our online switched controller design and establish the asymptotic convergence properties of the resulting closed-loop switched system.

A. Switched controller design

Our controller combines the mode detection and the stabilization phases into a single design, formalized in Algorithm 3, for the case of noisy data and an unknown switching signal. Next, we provide an intuitive description of the pseudocode language:

> *Informal description:* The algorithm assumes an initialization step satisfying Assumptions 1 and 2 has been performed. The algorithm starts in the mode detection phase, indicated by the variable S_{phase}. During this phase, Algorithm 2 is executed for each iteration until mode detection is successful (signaled by $f_{\text{done}} = 1$). The variable σ_d is then taken to be the active mode of the system. Next, the algorithm switches to the stabilization phase (signaled by $S_{\text{phase}} = 1$). During this phase, the control input $u(t) = K_{\sigma_d} x(t)$ is applied until (20) holds. This marks the detection of a mode switch, which leads the controller to go back to the mode detection phase (by toggling S_{phase} to 0). In the meantime, U_-^{on} and X^{on} are reset to record the new online data.

Algorithm 3 Data-driven online switched feedback controller for noisy data with unknown switching signal

Input: $\mathcal{P}, \{U_-^i, X^i, K_i, P_i\}_{i\in\mathcal{P}}, \lambda, c, q$

$\mathcal{P}_{\text{match}} \leftarrow \mathcal{P}$
$S_{\text{phase}} \leftarrow 0$ ▷ Initialize to mode detection phase
$X^{\text{on}} \leftarrow x(0)$
$U_-^{\text{on}} \leftarrow []$ ▷ Initialize online data
while the system (1) is running **do**
  if $S_{\text{phase}} = 0$ **then** ▷ Mode detection phase
    Run Algorithm 2 to update U_-^{on}, X^{on}, and $\mathcal{P}_{\text{match}}$
    if $|\mathcal{P}_{\text{match}}| = 1$ **then**
      Pick $\sigma_d \in \mathcal{P}_{\text{match}}$ ▷ Set the controller mode
      $S_{\text{phase}} \leftarrow 1$ ▷ Change controller to stabilization phase
    end if
  else ▷ Stabilization phase
    Apply control $u(t) = K_{\sigma_d} x(t)$
    Obtain the next state $x(t+1)$
    if (20) holds **then** ▷ Quit stabilization phase
      $U_-^{\text{on}} \leftarrow u(t)$ ▷ Reset the online data
      $X^{\text{on}} \leftarrow \begin{bmatrix} x(t) & x(t+1) \end{bmatrix}$
      $\mathcal{P}_{\text{match}} \leftarrow \mathcal{P}$ ▷ Reset $\mathcal{P}_{\text{match}}$
      $S_{\text{phase}} \leftarrow 0$ ▷ Change controller to mode detection phase
    end if
  end if
  $t \leftarrow t+1$ ▷ Update the time
end while

Note that the closed-loop system is stable when the controller is in its stabilization phase. However, during the mode detection phase, the effect of the controller on the system might be destabilizing because of the potential mismatch with the system mode. Therefore, determining overall stability properties of the form (2) relies critically on the switching behavior, both of the system and the controller.

Remark VI.1 (Data-driven online switched feedback controller for noiseless data and known switching signal). Algorithm 3 requires minor modifications in (i) the noiseless case or (ii) when the switching signal is known. For (i), one replaces Algorithm 2 in Step 7 with Algorithm 1 and takes $q = 0$ when evaluating (20) in Step 15. For (ii), one replaces the check in Step 15 by the condition $\sigma(t+1) \neq \sigma(t)$. •

Remark VI.2 (Appending the online data to initialization measurements). Once the mode detection phase is successful, we know that the gathered online measurements were generated by the active mode $i \in \mathcal{P}_{\text{match}}$. This raises the possibility of incorporating such online data to the initialization measurements corresponding to the mode $i \in \mathcal{P}_{\text{match}}$. Intuitively, this would decrease the size of the set Σ^i, which means that future mode detection phases would require fewer online measurements at the cost of having the controller keep previous measurements in its memory. •

B. Average dwell- and activation-time of switching signal

In this section, we introduce some assumptions on the switching frequency of the signal σ in order to establish stability guarantees for the closed-loop system.

Assumption 3 (Properties of the switching signal of the controller)**.** Let $\mathbb{T}^m := \{t_1^m, t_2^m, \cdots\}$ be the ordered set consisting of the initial time instants of each mode detection phase. Similarly, let $\mathbb{T}^s := \{t_1^s, t_2^s, \cdots\}$ correspond to the time instants when the stabilization phase starts. Assume

(i) the system does not switch while the controller is in the mode detection phase;

(ii) let $N(t_a, t_b)$ be the total number of mode detection phases over the time interval $[t_a, t_b)$, that is, $N(t_a, t_b) := |[t_a, t_b) \cap \mathbb{T}^m|$. There exists τ and $N_0 \geqslant 1$ such that

$$N(t_a, t_b) \leqslant N_0 + \frac{t_b - t_a}{\tau}, \quad \forall t_a, t_b \in \mathbb{N}, t_a < t_b; \tag{21}$$

(iii) let $M(t_a, t_b)$ be the total time spent in mode detection phases over the time interval $[t_a, t_b)$, that is, $M(t_a, t_b) := \sum_{t=t_a}^{t_b-1} \mathbf{1}(t)$, where

$$\mathbf{1}(t) := \begin{cases} 1 & \text{if } t \in [t_i^m, t_i^s) \text{ for some } i \in \mathbb{N}, \\ 0 & \text{otherwise.} \end{cases}$$

Then, there exists $\eta \in [0, 1]$ and $T_0 \geqslant 0$ such that

$$M(t_a, t_b) \leqslant T_0 + \eta(t_b - t_a) \quad \forall t_a, t_b \in \mathbb{N}, t_a < t_b. \tag{22}$$

Recall that by construction, the controller alternates between the mode detection phase and the stabilization phase. In other words, the elements in $\mathbb{T}^m$ and $\mathbb{T}^s$ are ordered such that

$$t_1^m < t_1^s < t_2^m < t_2^s < \dots$$

Now, statement (i) ensures that the collected online measurements correspond to a single active mode. From Corollary IV.4, the time length of each mode detection phase for noiseless data is bounded above by $n+m$. In simulations, we have observed that regardless of whether the data are noisy or noiseless, the mode detection phase is much shorter than this upper bound. This means that, as long as the unknown switching signal does not switch too frequently, statement (i) holds in general.

Statement (ii) is an *average dwell-time* (ADT) condition [32] on the mode detection phase. Its interpretation is that, on average, the controller is switched to the mode detection phase no more than once per τ time instances. Clearly, this condition holds if τ is relatively large, or equivalently, if the controller switches infrequently.

Finally, statement (iii) is an *average activation-time* (AAT) condition [33] on the mode detection phase. Its interpretation is that, on average, the controller dwells in the mode detection phase for at most a fraction η of the total time. Thus statement (iii) holds if the mode detection phase is, on average, relatively short when compared to the stabilization phase. This holds if either the mode detection phase is short or the system switches infrequently.

Remark VI.3 (Relationship with switching of the system). Note that Assumption 3(ii) and (iii) are formulated for the switching signal σ_d associated to the controller. However, this is directly related to the switching signal σ of the system. In fact, let

$$\mathbb{T} := \{t \in \mathbb{N} \backslash \{0\} : \sigma(t) \neq \sigma(t-1)\},$$

denote the set of time instances at which a system switch occurs. If the controller has access to these time instances, then $\mathbb{T} = \mathbb{T}^m$. Instead, if the switching signal σ is unknown, this will not be the case in general. Nevertheless, by definition, a switch in the controller mode σ_d can occur only if a switch in σ has occurred. (and in fact, switches in σ that do not lead to a violation of the condition (19) do not give rise to a switch in σ_d). This means that, as long as statement (i) holds, the switching of the controller is infrequent if the system (1) switches infrequently. •

C. Stability analysis of the closed-loop system

The following result characterizes the stability properties of the closed-loop system.

Theorem VI.4 (Stability guarantee for the closed-loop system)**.** *Consider initialization data (U_-^i, X^i) with, for each $i \in \mathcal{P}$, noise model of the form* (10), $Q^i = q^2 T_i I$, *for some $q > 0$, and satisfying Assumptions 1 and 2. Let online measurements be collected sequentially, with $\|w(t)\| \leqslant q$ at each time t. Assume that the switching signal of the controller satisfies Assumption 3. Let*

$$\mu := \max_{i,j \in \mathcal{P}} \|P_i^{\frac{1}{2}} P_j^{-\frac{1}{2}}\|^2, \tag{23}$$

and $\lambda_u \geqslant 1$ *be such that for any* $i \in \mathcal{P}$*, there exists* $k_i \geqslant 0$ *such that*

$$\begin{bmatrix} \lambda_u P_i - k_i c^2 I_n & 0 & \hat{A}_i^\top \\ 0 & k_i I_m & \hat{B}_i^\top \\ \hat{A}_i & \hat{B}_i & P_i^{-1} \end{bmatrix} \geqslant 0. \tag{24}$$

If the following holds

$$\left(1 - \frac{\ln \lambda_u}{\ln \lambda}\right)\eta + \left(1 - \frac{\ln \mu}{\ln \lambda}\right)\frac{1}{\tau} < 1, \tag{25}$$

then, for all initial states $x(0) \in \mathbb{R}^n$ *and each time* $t \in \mathbb{N}$*, the solution of the closed-loop system* (1) *with the data-driven switching feedback controller described by Algorithm 3 satisfies*

$$\|x(t)\| \leqslant \frac{p_{\max}}{p_{\min}} a^t b \|x(0)\| + \frac{p_{\max}}{p_{\min}} \frac{ab}{(1-a)\sqrt{\lambda}} q, \tag{26}$$

where

$$a := \sqrt{\lambda \left(\frac{\mu}{\lambda}\right)^{\frac{1}{\tau}} \left(\frac{\lambda_u}{\lambda}\right)^{\eta}} \in (\sqrt{\lambda}, 1), \tag{27a}$$

$$b := \sqrt{\left(\frac{\mu}{\lambda}\right)^{N_0} \left(\frac{\lambda_u}{\lambda}\right)^{T_0}}, \tag{27b}$$

and $p_{\max} = \max_{i \in \mathcal{P}} \lambda_{\max}(P_i^{\frac{1}{2}})$*,* $p_{\min} = \min_{i \in \mathcal{P}} \lambda_{\min}(P_i^{\frac{1}{2}})$*.*

The proof of Theorem VI.4 is provided in the Appendix. Note that, for each $i \in \mathcal{P}$, the LMI (24) holds for sufficiently large λ_u and k_i. Given Assumption 1, $\lambda < 1$ is an upper bound on the decay rate during the stabilization phase. Instead, $\lambda_u \geq 1$ is an upper bound on the growth rate during the mode detection phase. The parameter μ corresponds to the destabilizing effect introduced by each switching. The interpretation of condition (25) is as follows: the first term quantifies the combined effect on the growth rate of the state of the mode detection and stabilization phases. The second term quantifies the destabilizing effect of mode switches, which happen on average every τ time instances. For a given switched system, the constants μ, λ, and λ_u are fixed. Hence, the condition (25) is always satisfied if τ is sufficiently large and η is sufficiently small. This means that the ISS-like property (26) holds as long as the system switches sufficiently infrequently and the mode detection phases are sufficiently short.

VII. Simulation results

We illustrate the performance of the proposed data-driven switching feedback controller through 4 simulation experiments of an unknown switched linear system with $n = 5$ states, $m = 3$ inputs, and $p = 5$ modes. The system matrices $\hat{A}_i, \hat{B}_i$ are randomly selected and given by

$$\hat{A}_1 = \begin{bmatrix} -0.73 & -0.68 & -0.03 & -1.34 & -0.20 \\ -0.26 & 0.27 & 0.18 & -0.02 & 0.79 \\ -0.00 & -0.15 & -0.09 & -0.13 & 0.46 \\ 0.55 & 0.01 & 0.47 & 0.85 & 0.42 \\ -0.24 & 0.38 & -0.17 & 0.81 & 0.05 \end{bmatrix} \quad \hat{B}_1 = \begin{bmatrix} -0.38 & 0.56 & 0.70 \\ -0.36 & -0.91 & -0.81 \\ 0.42 & -1.15 & 0.57 \\ 0.54 & -0.52 & 1.69 \\ 0.38 & -0.80 & -0.88 \end{bmatrix}$$

$$\hat{A}_2 = \begin{bmatrix} 0.26 & -0.03 & 0.67 & 0.77 & -0.00 \\ -0.02 & 0.46 & 0.10 & 0.48 & 0.54 \\ -0.15 & -0.07 & -0.17 & 0.51 & -0.15 \\ 0.43 & -0.37 & -1.01 & -0.36 & 0.32 \\ 0.00 & 0.21 & 0.90 & 0.11 & 0.12 \end{bmatrix} \quad \hat{B}_2 = \begin{bmatrix} 1.59 & 1.26 & -1.04 \\ 0.73 & 1.62 & -0.42 \\ 0.96 & -0.54 & 0.73 \\ 0.85 & 0.90 & 1.51 \\ -0.12 & -0.78 & 0.00 \end{bmatrix}$$

$$\hat{A}_3 = \begin{bmatrix} -0.25 & 0.52 & 0.39 & 1.15 & -0.29 \\ 0.29 & 0.28 & -0.21 & 0.18 & 0.36 \\ -0.19 & -0.40 & -0.16 & 1.19 & -0.08 \\ -0.03 & 0.72 & -0.80 & 0.21 & -0.66 \\ 0.19 & 0.09 & -0.08 & 0.00 & 0.22 \end{bmatrix} \quad \hat{B}_3 = \begin{bmatrix} 2.04 & -0.29 & 0.10 \\ 0.20 & -2.00 & -0.19 \\ -0.12 & -2.62 & 0.50 \\ 0.76 & 0.71 & 0.34 \\ -2.52 & 0.01 & 0.94 \end{bmatrix}$$

$$\hat{A}_4 = \begin{bmatrix} 0.18 & 0.22 & 0.16 & -0.20 & 0.64 \\ -0.07 & 0.58 & 0.99 & -0.12 & 0.66 \\ 0.41 & 0.27 & 0.24 & 0.40 & -0.36 \\ 0.36 & -0.33 & 0.80 & -0.05 & 0.35 \\ 0.04 & 0.09 & -0.83 & -0.21 & 0.04 \end{bmatrix} \quad \hat{B}_4 = \begin{bmatrix} 0.47 & 0.95 & -0.41 \\ -1.07 & 0.37 & 0.53 \\ -0.50 & 1.14 & 1.77 \\ -0.61 & 0.12 & -1.93 \\ -0.99 & 1.88 & 2.02 \end{bmatrix}$$

$$\hat{A}_5 = \begin{bmatrix} 0.27 & 0.09 & 0.87 & 0.28 & -0.10 \\ -0.54 & 0.44 & 0.25 & 0.35 & 0.04 \\ 0.53 & -0.37 & 0.00 & 0.36 & -0.49 \\ 0.16 & 0.20 & 0.00 & 0.67 & 0.26 \\ -0.10 & -0.42 & -0.08 & -0.48 & -0.14 \end{bmatrix} \quad \hat{B}_5 = \begin{bmatrix} -0.86 & 0.58 & -1.38 \\ 1.52 & -0.54 & -2.46 \\ -0.74 & -1.00 & 0.16 \\ -2.31 & 1.47 & -0.55 \\ 0.44 & -1.91 & 0.16 \end{bmatrix}$$

For all simulations, we select the same switching signal σ, shown as the blue curve in Figure 2(a) through Figure 5(a). On average, this signal has 1 switch per 20 units of time. Each of the 4 simulations has as initial condition $x(0) = \begin{bmatrix} 1000 & 0 & 0 & 0 & 0 \end{bmatrix}^\top$.

In the first two simulations, a noiseless data pair (U_-^i, X^i) with $T^i = 7$ is collected in the initialization step for each mode $i \in \mathcal{P} = \{1, \ldots, 5\}$. Note that $n + m = 8$, and hence it is impossible to uniquely determine the dynamics of the modes from these 7 measurements. We set $\lambda = 0.8$ as the desired decay rate for each mode and note that both Assumptions 1 and 2 are satisfied on the initial data. Moreover, with $c = 0.1$, it can be computed that $\mu = 1.26$, (see (23)), and the LMI (24) holds with $\lambda_u = 5.86$. We apply our proposed data-driven controller: in the first simulation, shown in Figure 2, the controller is aware of when the systems switches and in the second simulation, shown in Figure 3, the switching signal is unknown. In both cases, we run the mode detection algorithm for noiseless data, that is Algorithm 1.

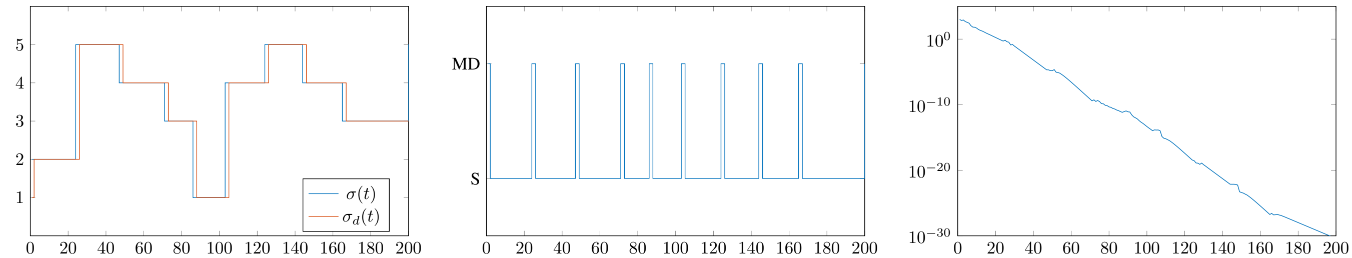
Comparing Figure 2(a) and Figure 3(a), we observe that the switching signals σ_d of the controller generated in the two different scenarios are almost the same, meaning that the condition (19) with $q = 0$ is effective in detecting switches. When $\sigma_d(t)$ is not equal to $\sigma(t)$, the controller is operating in the mode detection phase, as seen in Figure 2(b) and Figure 3(b). One can see that for both simulations, σ_d switches at the same rate as σ; in particular, when σ is unknown in the second simulation, switches are always immediately detected, implying that the controller changes to the mode detection phase whenever σ changes value. Moreover, each instance of the mode detection phase takes 2 units of time.

This also implies that Assumption 3 holds with parameters $\tau = 20$ and $\eta = 0.1$. As a result,

$$\left(1 - \frac{\ln \lambda_u}{\ln \lambda}\right)\eta + \left(1 - \frac{\ln \mu}{\ln \lambda}\right)\frac{1}{\tau} = 0.9942 < 1.$$

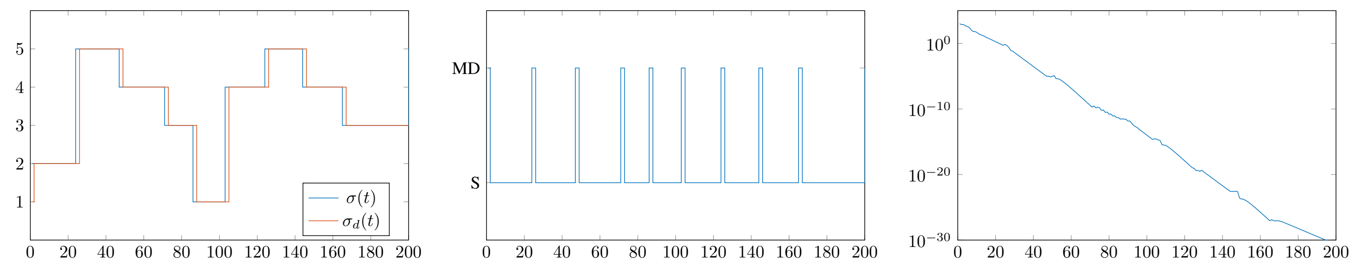
Therefore, the condition (25) is met and hence, by Theorem VI.4, the closed-loop system (1) satisfies (26). Since the data is noiseless ($q = 0$), the system is actually asymptotically stable. This is indeed reflected by the plots of the norms of the trajectories, as shown in Figures 2(c) and 3(c).

In the third and fourth simulations, we assume the measurements are corrupted with noise of magnitude $q = 0.01$. During the initialization step, a noisy data pair (U_-^i, X^i) with $T^i = 9$ is collected for each mode. The desired decay rate is again $\lambda = 0.8$ and it can be verified that both Assumptions 1



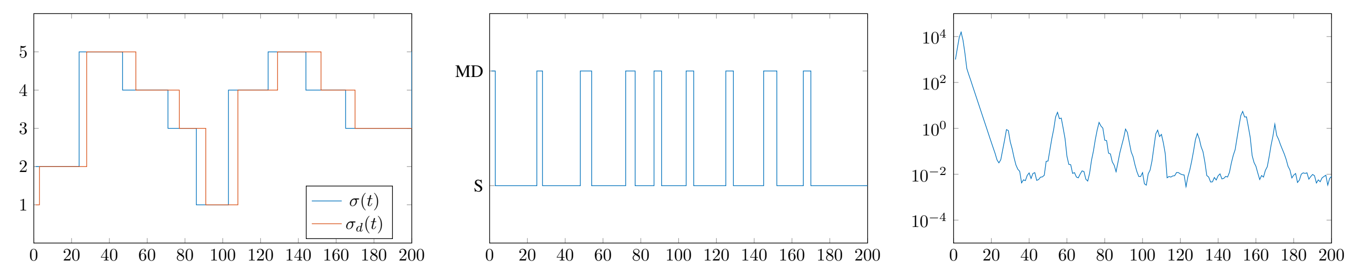
(a) (b) (c)

Figure 2: Unknown switched linear system with 5 states, 3 inputs, and 5 modes evolving under the data-driven online switched feedback controller of Algorithm 3. Simulations correspond to the case with noiseless data and the controller knowing when the system switches between modes. Plot (a) shows the switching signals of the system (σ) and of the controller (σ_d). Meanwhile, (b) plots the active (mode detection or stabilization) phase of the controller across time. Lastly, (c) shows the semi-logarithmic plot of the norm of the state as a function of time.



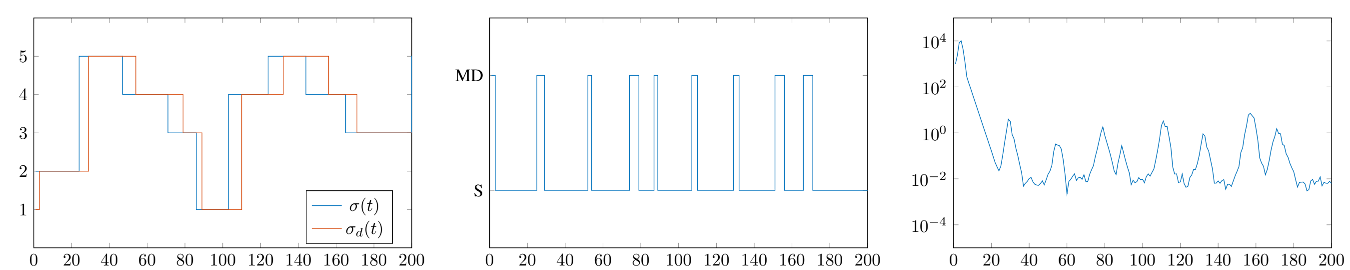
(a) (b) (c)

Figure 3: Simulation results for the system considered in Figure 2 with the same noiseless data, but with a switching signal completely unknown to the controller.



(a) (b) (c)

Figure 4: Unknown switched linear system with 5 states, 3 inputs, and 5 modes evolving under the data-driven online switched feedback controller in Algorithm 3. Simulations correspond to the case with noisy data (with measurement noise bounded by $q = 0.01$) and the controller knowing when the system switches modes. Plot (a) shows the switching signals of the system (σ) and of the controller (σ_d). In (b) we plot the active (mode detection or stabilization) phase of the controller across time. Lastly, (c) shows the semi-logarithmic plot of the state norm as a function of time.



(a) (b) (c)

Figure 5: Simulation results for the system considered in Figure 4 with the same noisy data, but with a switching signal completely unknown to the controller.

and 2 are satisfied on the initial data. Moreover, with $c = 1$, it can be computed that $\mu = 2.14$, and the LMI (24) holds with $\lambda_u = 264$. We remark that a value of c larger than in the noiseless simulations is needed in order to distinguish the inputs from the noise. Similar to the previous two examples, we study the performance of the data-driven controller in the scenarios with knowledge of when the system mode switches, shown in Figure 4, and when the switching signal is unknown, shown in Figure 5. As suggested by Figures 4(b) and 5(b), the presence of noise increases the duration of the mode detection phase. In both simulations, we obtain $\eta = 0.22$.

Comparing Figures 5(a) and 5(b), one can see that the delay between σ and σ_d is not equal to the length of the mode detection phase. This difference precisely corresponds to the time required to detect the switch (note that instantaneous convergence of the states is still guaranteed by (19)). In this case, we have

$$\left(1 - \frac{\ln \lambda_u}{\ln \lambda}\right)\eta + \left(1 - \frac{\ln \mu}{\ln \lambda}\right)\frac{1}{\tau} = 5.94 > 1,$$

i.e., the condition (25) is not met. Nevertheless, the behavior displayed by the trajectories in Figures 4(c) and 5(c), with oscillations (that are bounded) in the stabilization phase due to the noise, suggests ISS-like stability with respect to q.

VIII. Conclusions

We have considered the problem of stabilizing an unknown switched linear system on the basis of measured data. Prior to the online operation of the switched system, we have access to measurements of each of the individual modes. We have derived conditions in terms of linear matrix inequalities under which this pre-collected data is informative enough for finding a uniformly stabilizing controller for each mode. Once the system is running, we have access to online measurements of the currently active mode. Our online switched controller design alternates between a phase that performs mode detection on the basis of the online measurements and a stabilization phase that exploits this identification. Under average dwell- and activation-time assumptions on the switching signal, the proposed controller guarantees an input-to-state-like stability property of the closed-loop switched system. Our technical exposition has dealt with both noiseless and noisy measurements, and the cases when the controller knows the switching times of the system or has complete lack of knowledge about them. Future research will investigate methods of input design in the presence of noise to accelerate mode detection, develop measures of informativity of online measurements to keep the most useful data stored and integrate it in the mode detection phase, and exploit recently collected online measurements with those collected before to detect switches even during the mode detection phase.

References

[1] J. Eising, S. Liu, S. Martínez, and J. Cortés, "Using data informativity for online stabilization of unknown switched linear systems," in *IEEE Conf. on Decision and Control*, Cancun, Mexico, Dec. 2022, pp. 8–13.

[2] D. Liberzon, *Switching in Systems and Control*, ser. Systems & Control: Foundations & Applications. Birkhäuser, 2003.

[3] H. Lin and P. J. Antsaklis, "Stability and stabilizability of switched linear systems: A survey of recent results," *IEEE Transactions on Automatic Control*, vol. 54, no. 2, pp. 308–322, 2009.

[4] D. Cheng, L. Guo, Y. Lin, and Y. Wang, "Stabilization of switched linear systems," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 661–666, 2005.

[5] Y. Sun, "Stabilization of switched systems with nonlinear impulse effects and disturbances," *IEEE Transactions on Automatic Control*, vol. 56, no. 11, pp. 2739–2743, 2011.

[6] M. Hou, F. Fu, and G. Duan, "Global stabilization of switched stochastic nonlinear systems in strict-feedback form under arbitrary switchings," *Automatica*, vol. 49, no. 8, pp. 2571–2575, 2013.

[7] D. Zhai, L. An, J. Dong, and Q. Zhang, "Switched adaptive fuzzy tracking control for a class of switched nonlinear systems under arbitrary switching," *IEEE Transactions on Fuzzy Systems*, vol. 26, no. 2, pp. 585–597, 2018.

[8] M. S. Branicky, "Multiple Lyapunov functions and other analysis tools for switched and hybrid systems," *IEEE Transactions on Automatic Control*, vol. 43, no. 4, pp. 475–482, 1998.

[9] J. C. Geromel and P. Colaneri, "Stability and stabilization of continuous-time switched linear systems," *SIAM Journal on Control and Optimization*, vol. 45, no. 5, pp. 1915–1930, 2006.

[10] P. Colaneri, J. C. Geromel, and A. Astolfi, "Stabilization of continuous-time switched nonlinear systems," *Systems & Control Letters*, vol. 57, no. 1, pp. 95–103, 2008.

[11] M. T. Raza, A. Q. Khan, G. Mustafa, and M. Abid, "Design of fault detection and isolation filter for switched control systems under asynchronous switching," *IEEE Transactions on Control Systems Technology*, vol. 24, no. 1, pp. 13–23, 2016.

[12] L. Li, S. X. Ding, Y. Na, and J. Qiu, "An asynchronized observer based fault detection approach for uncertain switching systems with mode estimation," *IEEE Transactions on Circuits and Systems II: Express Briefs*, vol. 69, no. 2, 2022.

[13] X. Zhao, H. Liu, J. Zhang, and H. Li, "Multiple-mode observer design for a class of switched linear systems," *IEEE Transactions on Automation Science and Engineering*, vol. 12, no. 1, pp. 272–280, 2015.

[14] H. Lin and P. J. Antsaklis, "Switching stabilizability for continuous-time uncertain switched linear systems," *IEEE Transactions on Automatic Control*, vol. 52, no. 4, pp. 633–646, 2007.

[15] L. I. Allerhand and U. Shaked, "Robust control of linear systems via switching," *IEEE Transactions on Automatic Control*, vol. 58, no. 2, pp. 506–512, 2013.

[16] S. Yuan, B. de Schutter, and S. Baldi, "Adaptive asymptotic tracking control of uncertain time-driven switched linear systems," *IEEE Transactions on Automatic Control*, vol. 62, no. 11, pp. 5802–5807, 2017.

[17] B. D. O. Anderson, T. S. Brinsmead, F. De Bruyne, J. Hespanha, D. Liberzon, and A. S. Morse, "Multiple model adaptive control. Part 1: Finite controller coverings," *International Journal of Robust and Nonlinear Control*, vol. 10, no. 11-12, pp. 909–929, 2000.

[18] C. Zhang, M. Gan, and J. Zhao, "Data-driven optimal control of switched linear autonomous systems," *International Journal of Systems Science*, vol. 50, no. 6, pp. 1275–1289, 2019.

[19] A. Kundu, "Data-driven switching logic design for switched linear systems," *arXiv preprint arXiv:2003.05774*, 2020.

[20] T. Dai and M. Sznaier, "Data driven robust superstable control of switched systems," *IFAC-PapersOnLine*, vol. 51, no. 25, pp. 402–408, 2018, 9th IFAC Symposium on Robust Control Design ROCOND 2018.

[21] ——, "A moments based approach to designing MIMO data driven controllers for switched systems," in *IEEE Conf. on Decision and Control*, Miami, FL, 2018, pp. 5652–5657.

[22] Z. Wang, G. O. Berger, and R. M. Jungers, "Data-driven feedback stabilization of switched linear systems with probabilistic stability guarantees," in *IEEE Conf. on Decision and Control*, Austin, TX, 2021.

[23] V. Breschi and S. Formentin, "Direct data-driven design of switching controllers," *International Journal on Robust and Nonlinear Control*, vol. 30, no. 15, pp. 6042–6072, 2020.

[24] X. Wang, J. Sun, G. Wang, F. Allgöwer, and J. Chen, "Data-driven control of distributed event-triggered network systems," *IEEE/CAA J. Autom. Sinica*, vol. 10, no. 2, pp. 351–364, 2023.

[25] J. C. Willems, P. Rapisarda, I. Markovsky, and B. L. M. De Moor, "A note on persistency of excitation," *Systems & Control Letters*, vol. 54, no. 4, pp. 325–329, 2005.

[26] M. Rotulo, C. De Persis, and P. Tesi, "Online learning of data-driven controllers for unknown switched linear systems," *Automatica*, vol. 145, p. 110519, 2022.

[27] H. J. van Waarde, J. Eising, H. L. Trentelman, and M. K. Camlibel, "Data informativity: a new perspective on data-driven analysis and control," *IEEE Transactions on Automatic Control*, vol. 65, no. 11, pp. 4753–4768, 2020.

[28] H. J. van Waarde, M. K. Camlibel, J. Eising, and H. L. Trentelman, "Quadratic matrix inequalities with applications to data-based control," *arXiv preprint arXiv:2203.12959*, 2022.

[29] J. Eising and J. Cortés, "When sampling works in data-driven control: informativity for stabilization in continuous time," *IEEE Transactions on Automatic Control*, 2023, submitted. Available at https://arxiv.org/abs/2301.10873.

[30] M. Bianchi, S. Grammatico, and J. Cortés, "Data-driven stabilization of switched and constrained linear systems," *IEEE*

Transactions on Automatic Control, 2022, submitted. Available at https://arxiv.org/abs/2208.11392.
[31] H. J. van Waarde, "Beyond persistent excitation: Online experiment design for data-driven modeling and control," *IEEE Control Systems Letters*, vol. 6, pp. 319–324, 2022.
[32] J. P. Hespanha and A. S. Morse, "Stability of switched systems with average dwell-time," in *IEEE Conf. on Decision and Control*, Shangai, China, Dec. 1999, pp. 2655–2660.
[33] M. A. Müller and D. Liberzon, "Input/output-to-state stability and state-norm estimators for switched nonlinear systems," *Automatica*, vol. 48, no. 9, pp. 2029–2039, 2012.
[34] G. Yang and D. Liberzon, "A Lyapunov-based small-gain theorem for interconnected switched systems," *Systems & Control Letters*, vol. 78, pp. 47–54, 2015.
[35] J. I. Poveda and A. R. Teel, "A framework for a class of hybrid extremum seeking controllers with dynamic inclusions," *Automatica*, vol. 76, pp. 113–126, 2017.
[36] H. Khalil, *Nonlinear Systems, 3rd ed.* Englewood Cliffs, NJ: Prentice Hall, 2002.

Appendix

Proof of Theorem III.3

We prove the result following the arguments in the proof of [28, Theorem 5.1.(a)] with the necessary adjustments. We are interested in characterizing when (6) holds. Applying the Schur complement, we see that $(A+BK)^\top P(A+BK) \prec \lambda P$ is equivalent to

$$\begin{bmatrix} \lambda P & (A+BK)^\top \\ (A+BK) & P^{-1} \end{bmatrix} \succ 0.$$

Defining $Q = P^{-1}$ and using the Schur complement again this holds if and only if

$$\lambda Q - (A+BK)Q(A+BK)^\top \succ 0.$$

Define then the matrix

$$M := \left[\begin{array}{c:cc} \lambda Q & 0 & 0 \\ \hdashline 0 & -Q & -QK^\top \\ 0 & -KQ & -KQK^\top \end{array}\right],$$

Given the above reasoning and the description of $\Sigma(U_-, X)$ in (5), one has that (6) is equivalent to

$$\begin{bmatrix} I_n \\ A^\top \\ B^\top \end{bmatrix}^\top N \begin{bmatrix} I_n \\ A^\top \\ B^\top \end{bmatrix} = 0 \implies \begin{bmatrix} I_n \\ A^\top \\ B^\top \end{bmatrix}^\top M \begin{bmatrix} I_n \\ A^\top \\ B^\top \end{bmatrix} \succ 0,$$

where N is given by (4). Using [28, Cor. 4.13], this holds if and only if there exists $\alpha \geqslant 0$ and $\beta > 0$ such that

$$M - \alpha N \succeq \begin{bmatrix} \beta I_n & 0 \\ 0 & 0 \end{bmatrix}.$$

Let $K := LQ^{-1}$. Again using a Schur complement argument, this is equivalent to

$$\begin{bmatrix} \lambda Q - \beta I_n & 0 & 0 & 0 \\ 0 & 0 & 0 & Q \\ 0 & 0 & 0 & L \\ 0 & Q & L^\top & Q \end{bmatrix} - \alpha \begin{bmatrix} N & 0 \\ 0 & 0 \end{bmatrix} \succeq 0.$$

For this to hold, it must be that $\alpha \neq 0$. As such, we can scale Q and β to arrive at (8), and this concludes the proof.

Proof of Theorem VI.4

To prove Theorem VI.4, we construct two discrete timers to deal with the ADT and AAT conditions. Using timers for handling these conditions is a common approach in the study of switched systems and in fact similar timers can also be found in [34], [35]. For the ADT condition, the timer τ_d provides a mechanism for keeping track of to what extent the system behavior is in line with the assumption that we have $1/\tau$ switches per unit time. Similarly, for the AAT condition, the timer τ_a provides a mechanism for keeping track of to what extent the system behavior is in line with the assumption spending a fraction η of the total time in the mode detection phase.

Lemma A.1 (Discrete timers for ADT and AAT conditions)**.** *Assume the system does not switch while the controller is in the mode detection phase. Then, Assumption 3(ii) implies that there exists a timer $\tau_d : \mathbb{N} \mapsto [0, N_0]$ such that*

$$\tau_d(t+1) = \tau_d(t) + \frac{1}{\tau} - 1 \qquad \textit{if } t \in \mathbb{T}^m, \tag{28a}$$

$$\tau_d(t+1) \in \big[\tau_d(t), \tau_d(t) + \frac{1}{\tau}\big] \qquad \textit{otherwise}. \tag{28b}$$

Similarly Assumption 3(iii) implies that there is a timer $\tau_a : \mathbb{N} \mapsto [0, N_0]$ such that

$$\tau_a(t+1) = \tau_a(t) + \eta - 1 \qquad \textit{if } t \in \{t_i^m, \cdots, t_i^s - 1\}, i \in \mathbb{N}, \tag{29a}$$

$$\tau_a(t+1) \in \big[\tau_a(t), \tau_a(t) + \eta\big] \qquad \textit{otherwise}. \tag{29b}$$

Proof. We study the ADT condition first. Let

$$n_d(t) := \min_{s=0,\dots,t} \{N(0,s) - \frac{s}{\tau}\}. \tag{30}$$

By definition $n_d(t)$ is non-increasing. We claim that

$$\tau_d(t) := N_0 + n_d(t) - (N(0,t) - \frac{t}{\tau})$$

satisfies (28). Clearly, it follows from the definition of $n_d(t)$ that $\tau_d(t) \leqslant N_0$. Also, for $s = 0, \dots, t$, we have

$$\begin{aligned} N_0 + (N(0,s) - \frac{s}{\tau}) - (N(0,t) - \frac{t}{\tau}) \\ = N(0,s) + \big(\frac{t-s}{\tau} + N_0\big) - N(0,t) \\ \geqslant N(0,s) + N(s,t) - N(0,t) = 0. \end{aligned}$$

Hence the range of τ_d is $[0, N_0]$. To verify that (28a) holds, let $t \in \mathbb{T}^m$. In this case, we have $N(0,t+1) = N(0,t) + 1$. Since $\tau \geqslant 1$,

$$N(0,t+1) - \frac{t+1}{\tau} \geqslant N(0,t) - \frac{t}{\tau} \geqslant n_d(t).$$

This implies that $n_d(t+1) = n_d(t)$, and hence

$$\begin{aligned} &\tau_d(t+1) - \tau_d(t) \\ &= (n_d(t{+}1) - n_d(t)) - (N(0,t{+}1) - N(0,t) - \frac{1}{\tau}) = \frac{1}{\tau} - 1, \end{aligned}$$

concluding (28a). We now consider the case where $t \notin \mathbb{T}^m$. Note that by definition $N(0,t{+}1) = N(0,t)$. Direct inspection yields

$$\tau_d(t+1) - \tau_d(t) = (n_d(t+1) - n_d(t)) + \frac{1}{\tau} \leqslant \frac{1}{\tau}. \tag{31}$$

Meanwhile, note that $n_d(t+1) < n_d(t)$ if and only if $N(0,t+1) - \frac{t+1}{\tau} < n_d(t)$, in which case

$$\begin{aligned} n_d(t+1) - n_d(t) &= N(0,t+1) - \frac{t+1}{\tau} - n_d(t) \\ &\geqslant (N(0,t+1) - \frac{t+1}{\tau}) - (N(0,t) - \frac{t}{\tau}) = -\frac{1}{\tau}, \end{aligned}$$

and consequently,

$$\tau_d(t+1) - \tau_d(t) \geqslant 0. \tag{32}$$

Therefore, (28b) follows from (31) and (32).

To study the AAT condition, we define

$$n_a(t) := \min_{s=0,\dots,t} \{M(0,s) - \eta s\}. \tag{33}$$

By definition $n_a(t)$ is non-increasing. We claim that

$$\tau_a(t) := T_0 + n_a(t) - (M(0,t) - \eta t)$$

satisfies (29). Clearly it follows from the definition of $n_a(t)$ that $\tau_a(t) \leqslant T_0$. Moreover, for $s = 0, \dots, t$,

$$\begin{aligned} &T_0 + (M(0,s) - \eta s) - (M(0,t) - \eta t) \\ &\quad = M(0,s) + (\eta(t-s) + T_0) - M(0,t) \\ &\quad\quad \geqslant M(0,s) + M(s,t) - N(0,t) = 0. \end{aligned}$$

Hence the range of τ_a is indeed $[0, T_0]$. To verify (29a), we first assume $t \in \{t_i^m, \cdots, t_i^s - 1\}$ for some $i \in \mathbb{N}$. In this case we have $M(0,t+1) = M(0,t) + 1$. Since $\eta \in [0,1]$, we have that $M(0,t+1) - \eta(t+1) \geqslant M(0,t) - \eta t$. In turn, this implies that $n_a(t+1) = n_a(t)$. Therefore we can conclude

$$\begin{aligned} &\tau_a(t+1) - \tau_a(t) \\ &= (n_a(t+1) - n_a(t)) - (M(0,t+1)) - M(0,t) - \eta) = \eta - 1, \end{aligned}$$

proving (29a). We move our attention to the case where $t \in \{t_i^s, \cdots, t_{i+1}^m - 1\}$ for some $i \in \mathbb{N}$. We have $N(0,t+1) = N(0,t)$ and hence

$$\tau_a(t+1) - \tau_a(t) = (n_a(t+1) - n_a(t)) + \eta \leqslant \eta. \tag{34}$$

On the other hand, note that $n_a(t+1) < n_a(t)$ only if $M(0,t+1) - \eta(t+1) < n_a(t)$. Therefore, we can conclude

$$\begin{aligned} n_a(t+1) - n_a(t) &= M(0,t+1) - \eta(t+1) - n_a(t) \\ &\geqslant (M(0,t+1) - \eta(t+1)) - (M(0,t) - \eta t) = -\eta. \end{aligned}$$

In turn, this implies that

$$\tau_a(t+1) - \tau_a(t) \geqslant 0. \tag{35}$$

We can now conclude (29b) by combining (34) and (35). □

We rely on Lemma A.1 to prove Theorem VI.4 next.

Proof of Theorem VI.4. Construct two timers $\tau_d : \mathbb{N} \mapsto [0, N_0]$, $\tau_a : \mathbb{N} \mapsto [0, T_0]$ as in Lemma A.1. Denote the extended state at time t,

$$\xi(t) := \begin{pmatrix} x(t) \\ \sigma_d(t) \\ \tau_d(t) \\ \tau_a(t) \end{pmatrix} \in \mathbb{R}^n \times \mathcal{P} \times [0, N_0] \times [0, T_0] =: \mathcal{X}. \tag{36}$$

Further define functions $U, V, W : \mathcal{X} \mapsto \mathbb{R}_{\geqslant 0}$ by

$$\begin{aligned} U(\xi) &:= \sqrt{\left(\frac{\mu}{\lambda}\right)^{\tau_d} \left(\frac{\lambda_u}{\lambda}\right)^{\tau_a}}, \\ V(\xi) &:= V_{\sigma_d}(x) = \|P_{\sigma_d}^{\frac{1}{2}} x\|, \\ W(\xi) &:= U(\xi) V(\xi). \end{aligned}$$

Note that since $\mu \geqslant 1$, $\lambda_u \geqslant 1$ and $0 < \lambda < 1$, we have that U is increasing with respect to both τ_d and τ_a, and $a \geqslant \sqrt{\lambda}$, where a is defined in (27a). In addition, since $\tau_d \in [0, N_0], \tau_a \in [0, T_0]$, we see that

$$U(\xi) \in \left[1, \sqrt{\left(\frac{\mu}{\lambda}\right)^{N_0} \left(\frac{\lambda_u}{\lambda}\right)^{T_0}}\right] = [1, b],$$

where b is defined in (27b). It follows from (25) that

$$\begin{aligned} \ln a &= \frac{1}{2} \left(\ln \lambda + (\ln \mu - \ln \lambda) \frac{1}{\tau} + (\ln \lambda_u - \ln \lambda) \eta \right) \\ &= \frac{\ln \lambda}{2} \left(1 - \big(1 - \frac{\ln \mu}{\ln \lambda}\big) \frac{1}{\tau} - \big(1 - \frac{\ln \lambda_u}{\ln \lambda}\big) \eta \right) < 0. \end{aligned}$$

Therefore we have that $a < 1$. We now investigate the one-step change of the function $t \mapsto W(\xi(t))$ in three separate cases.

1) The case $t \in \mathbb{T}^m$; that is, a switch is detected at time t. In this case, the controller has switched to the mode detection phase. We apply the Schur complement on (24) to deduce that

$$\begin{bmatrix} \lambda_u P_i & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \hat{A}_i & \hat{B}_i \end{bmatrix}^\top P_i \begin{bmatrix} \hat{A}_i & \hat{B}_i \end{bmatrix} - k_i \begin{bmatrix} c^2 I_n & 0 \\ 0 & -I_n \end{bmatrix} \geqslant 0.$$

Since $k_i \geqslant 0$, for any $x \in \mathbb{R}^n, u \in \mathbb{R}^m$ such that

$$\begin{bmatrix} x \\ u \end{bmatrix}^\top \begin{bmatrix} c^2 I & 0 \\ 0 & -I \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \geqslant 0, \tag{37}$$

it must be that

$$\begin{bmatrix} x \\ u \end{bmatrix}^\top \left(\begin{bmatrix} \lambda_u P_i & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} \hat{A}_i & \hat{B}_i \end{bmatrix}^\top P_i \begin{bmatrix} \hat{A}_i & \hat{B}_i \end{bmatrix} \right) \begin{bmatrix} x \\ u \end{bmatrix} \geqslant 0. \tag{38}$$

Notice that (37) is equivalent to $\|u\| \leqslant c\|x\|$, which is the condition on the control used during the mode detection phase. On the other hand, (38) is equivalent to

$$(\hat{A}_i x + \hat{B}_i u)^\top P_i (\hat{A}_i x + \hat{B}_i u) \leqslant \lambda_u x^\top P_i x. \tag{39}$$

Denote $\sigma_d(t) = j, \sigma_d(t+1) = k$, which are different because of the phase switch at $t \in \mathbb{T}^m$. It follows from the triangle inequality that

$$\begin{aligned} V_{\sigma_d(t+1)}(x(t+1)) &= V_k(x(t+1)) \\ &= \|P_k^{\frac{1}{2}} (\hat{A}_k x(t) + \hat{B}_k u(t) + w(t))\| \\ &\leqslant \sqrt{\lambda_u} \|P_k^{\frac{1}{2}} x(t)\| + p_{\max} q \\ &\leqslant \sqrt{\lambda_u} \|P_k^{\frac{1}{2}} P_j^{-\frac{1}{2}}\| \|P_j^{\frac{1}{2}} x(t)\| + p_{\max} q \\ &\leqslant \sqrt{\lambda_u \mu} V_{\sigma(t)}(x(t)) + p_{\max} q \end{aligned}$$

This can be equivalently written as $V(\xi(t+1)) \leqslant \sqrt{\lambda_u \mu} V(\xi(t)) + p_{\max} q$. On the other hand, it follows from (28a) and (29a) that

$$U(\xi(t+1)) = \sqrt{\left(\frac{\mu}{\lambda}\right)^{\tau_d(t)+\frac{1}{\tau}-1}\left(\frac{\lambda_u}{\lambda}\right)^{\tau_a(t)+\eta-1}}$$
$$= \sqrt{\left(\frac{\mu}{\lambda}\right)^{\frac{1}{\tau}-1}\left(\frac{\lambda_u}{\lambda}\right)^{\eta-1}}U(\xi(t)) = \sqrt{\frac{\lambda}{\lambda_u\mu}}aU(\xi(t)).$$

Therefore,

$$W(\xi(t+1)) = U(\xi(t+1))V(\xi(t+1))$$
$$\leqslant \sqrt{\lambda}aU(\xi(t))V(\xi(t)) + \sqrt{\frac{\lambda}{\lambda_u\mu}}U(\xi(t))ap_{\max}q$$
$$\leqslant \sqrt{\lambda}aW(\xi(t)) + \sqrt{\frac{\lambda}{\lambda_u\mu}}abp_{\max}q$$

and we conclude

$$W(\xi(t+1)) \leqslant aW(\xi(t)) + \tilde{b}q, \tag{40}$$

where $\tilde{b} := \frac{ab}{\sqrt{\lambda}}p_{\max}$.

2) The case where $t \in \{t_i^m+1, \cdots, t_i^s-1\}$ for some $i \in \mathbb{N}$. Since the controller is in the mode detection phase and σ_d does not switch at $t+1$, we can similarly conclude from (39) as in the previous case that

$$V(\xi(t+1)) \leqslant \sqrt{\lambda_u}V(\xi(t)) + p_{\max}q.$$

Moreover, from (28b) and (29a) we can conclude that

$$U(\xi(t+1)) \leqslant \sqrt{\left(\frac{\mu}{\lambda}\right)^{\tau_d(t)+\frac{1}{\tau}}\left(\frac{\lambda_u}{\lambda}\right)^{\tau_a(t)+\eta-1}}$$
$$= \sqrt{\left(\frac{\mu}{\lambda}\right)^{\frac{1}{\tau}}\left(\frac{\lambda_u}{\lambda}\right)^{\eta-1}}U(\xi(t)) = \frac{a}{\sqrt{\lambda_u}}U(\xi(t)).$$

Therefore

$$W(\xi(t+1)) = U(\xi(t+1))V(\xi(t+1))$$
$$\leqslant aU(\xi(t))V(\xi(t)) + \frac{a}{\sqrt{\lambda_u}}U(\xi(t))p_{\max}q$$
$$\leqslant aW(\xi(t)) + \frac{ab}{\sqrt{\lambda_u}}p_{\max}q,$$

and again we conclude that (40) holds.

3) The case where $t \in \{t_i^s, \cdots, t_{i+1}^m - 1\}$ for some $i \in \mathbb{N}$; that is, when the controller is in the stabilization phase. In this case, it follows from (19) that

$$V(\xi(t+1)) \leqslant \sqrt{\lambda}V(\xi(t)) + p_{\max}q.$$

In turn, we can conclude from (28b) and (29b) that

$$U(\xi(t+1)) \leqslant \sqrt{\left(\frac{\mu}{\lambda}\right)^{\tau_d(t)+\frac{1}{\tau}}\left(\frac{\lambda_u}{\lambda}\right)^{\tau_a(t)+\eta}}$$
$$= \sqrt{\left(\frac{\mu}{\lambda}\right)^{\frac{1}{\tau}}\left(\frac{\lambda_u}{\lambda}\right)^{\eta}}U(\xi(t)) = \frac{a}{\sqrt{\lambda}}U(\xi(t)).$$

Therefore

$$W(\xi(t+1)) = U(\xi(t+1))V(\xi(t+1))$$
$$\leqslant aU(\xi(t))V(\xi(t)) + \frac{a}{\sqrt{\lambda}}U(\xi(t))p_{\max}q$$
$$\leqslant aW(\xi(t)) + \frac{ab}{\sqrt{\lambda}}p_{\max}q,$$

and we can conclude the inequality (40) again.

As a result of the above reasoning, we conclude that (40) holds for all $t \in \mathbb{N}$. By the comparison principle [36, Lem. 3.4], we conclude from (40) that

$$W(\xi(t)) \leqslant a^t W(\xi(0)) + \frac{1-a^t}{1-a}\tilde{b}q \leqslant a^t W(\xi(0)) + \frac{\tilde{b}}{1-a}q.$$

Recall the definition of the extended state ξ and the fact that $p_{\min}\|x\| \leqslant V(\xi) \leqslant p_{\max}\|x\|$ for any $\xi \in \mathcal{X}$. We have

$$p_{\min}\|x(t)\| \leqslant V(\xi) = \frac{W(\xi(t))}{U(\xi(t))} \leqslant W(\xi(t))$$
$$\leqslant a^t W(\xi(0)) + \frac{\tilde{b}}{1-a}q$$
$$= a^t U(\xi(0))V(\xi(0)) + \frac{\tilde{b}}{1-a}q$$
$$\leqslant a^t b p_{\max}\|x_0\| + \frac{\tilde{b}}{1-a}q.$$

Dividing both sides by $p_{\min}$ yields (26), proving the result. □

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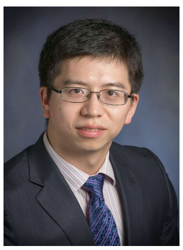

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