

Lecture 4:  
***Introduction to discrete-time  
dynamics  
& the logistic map***

COSMOS - Making Robots and  
Making Robots *Intelligent*



# Tools: Plotting Iterated Maps and Stability Analysis

First example:  $x_{k+1} = \cos(x_k)$

Who cares?

Saw that  $x = 0.739\dots$  is a **stable fixed point**

Cruise control response for  $k = 0, 1, 2, \dots$

$$v_{k+1} = v_k + \frac{\Delta}{m} [-(b + K)v_k + K v_{des} + u_{hill}(t_k)]$$



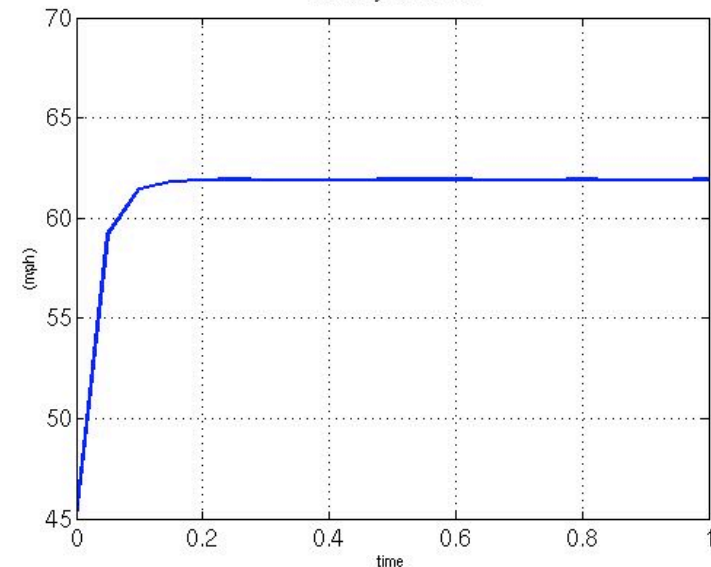
Plot velocity versus time

$$t_k = k * \Delta$$

Starts at 45mph, want 60mph in presence of random noise

$$\Delta = 0.05, m = 5$$

$$b = 1, K = 20$$



6/27/05

COSMOS - Making I  
Intelligent

# A Critical Piece of the Puzzle - The Model

General expression:

$$x_{k+1} = f(x_k)$$

“Bobs” response:

$$v_{k+1} = v_k + \frac{\Delta}{m} [-(b + K)v_k + K v_{des} + u_{hill}(t_k)]$$

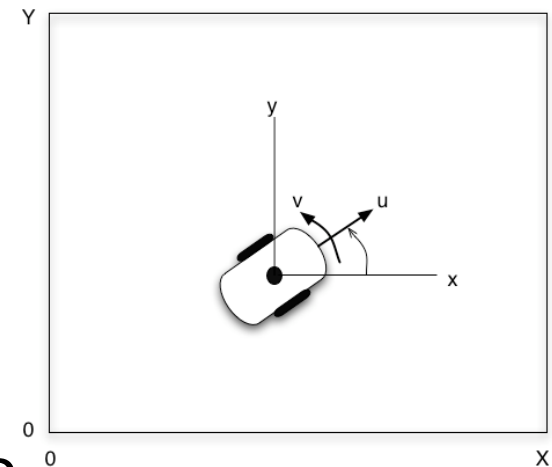


Robobrian Model:

$$x_{k+1} = x_k + \Delta u_k \cos(\theta_k)$$

$$y_{k+1} = y_k + \Delta u_k \sin(\theta_k)$$

$$\theta_{k+1} = \theta_k + \Delta v_k$$



Stability: Will **Robobrain** do what we want?

Yes, if feedback control  $(u, v)$  designed properly.

# Logistic Map - An Example of *Chaos*

$$x_{k+1} = rx_k(1 - x_k)$$

