Lecture 7:
Feedback Control of an Inverted Pendulum

COSMOS - Making Robots and
Making Robots *Intelligent*

Engineering Equilibrium = Choice of Feedback

Model of Pendulum:

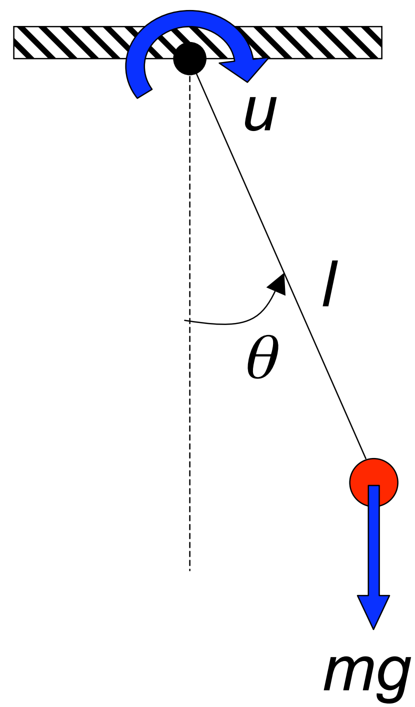
$$ml^2 \frac{d^2\theta(t)}{dt^2} = mgl \left[u(t) - \sin(\theta(t))\right]$$

Discrete time approximation:

$$\theta_{k+1} = 2\theta_k - \theta_{k-1} + \frac{g\Delta^2}{l}\left[u_k - \sin(\theta_k)\right]$$

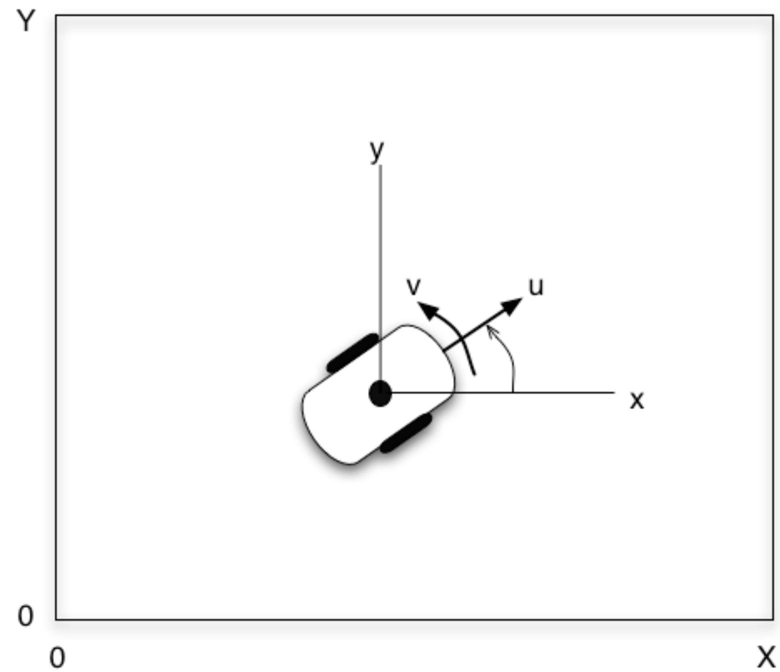
Equilibrium / Fixed Points:

$$\theta_{\mathrm{eq}} = 0, \pm\pi, \pm 2\pi, \ldots$$



Next Step is Using Torque Control ***u*** to make the *Inverted* position Stable! … Anyone think we can do that ?

Next Step: Robobrain Control Design



States: (x, y, θ) - position and orientation

Inputs (Controls): (u, v) - forward and turning velocity

Dynamics:

$$x_{k+1} = x_k + \Delta u_k \cos(\theta_k)$$

$$y_{k+1} = y_k + \Delta u_k \sin(\theta_k)$$

$$\theta_{k+1} = \theta_k + \Delta v_k$$

Preview: $v_k = k_p (x_k - d_{\text{sep}}) + k_d \frac{x_k - x_{k-1}}{\Delta}$

Objective

• Examining the dynamics for a pendulum model (fixed points and stability).

• Stabilizing an *engineered* fixed-point/equilibrium of the pendulum - the upright or inverted position.

• Will need these techniques for ***Robobrain****!*