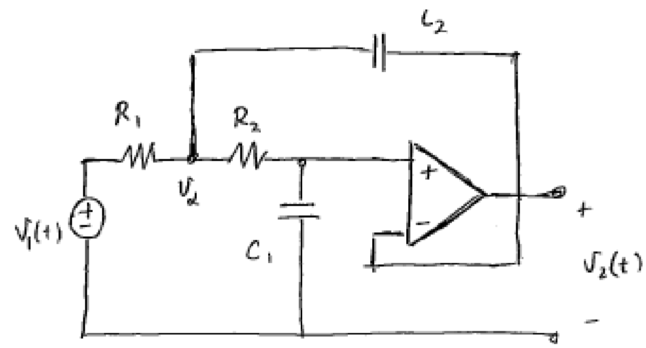
10/55



$$V_\alpha(s) = \frac{R_2 + \frac{1}{sC_1}}{\frac{1}{sC_1}} V_2(s)$$

$$\Rightarrow V_\alpha(s) = (sC_1R_2 + 1) V_2(s)$$

KCL V_α:

$$V_\alpha\left(\frac{1}{R_1} + \frac{1}{R_2} + sC_2\right) = \frac{V_1(s)}{R_1} + V_2(s)\left(sC_2 + \frac{1}{R_2}\right)$$

$$V_2(s)\,(sC_1R_2+1)\left(\frac{1}{R_1} + \frac{1}{R_2} + sC_2\right) - V_2(s)\left(sC_2 + \frac{1}{R_2}\right) = \frac{V_1(s)}{R_1}$$

$$V_2(s)\left(\frac{sC_1R_2}{R_1} + sC_1 + s^2C_1C_2R_2 + \frac{1}{R_1} + \cancel{\frac{1}{R_2}} + \cancel{sC_2} - \cancel{sC_2} - \cancel{\frac{1}{R_2}}\right) = \frac{V_1(s)}{R_1}$$

$$\therefore \quad V_1(s) = V_2(s)\,(sC_1R_2 + sC_1R_1 + s^2C_1C_2R_1R_2 + 1)$$

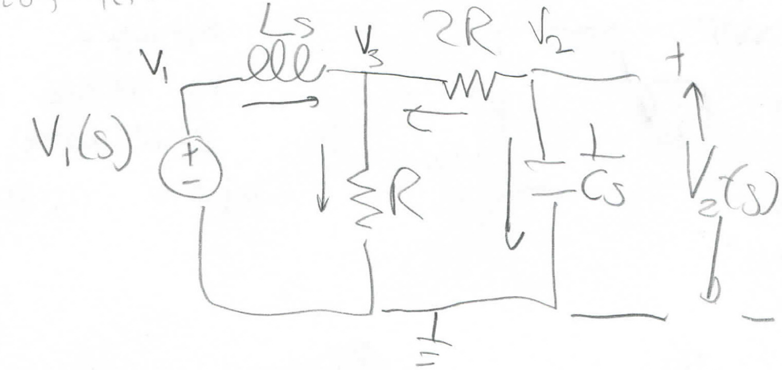
and

$$\frac{V_2(s)}{V_1(s)} = \boxed{T(s) = \frac{(C_1C_2R_1R_2)^{-1}}{s^2 + s[(C_2R_1)^{-1} + (C_2R_2)^{-1}] + (C_1C_2R_1R_2)^{-1}}}$$

$$\left(\text{equivalently} \quad T(s) = \frac{1}{C_1C_2R_1R_2s^2 + (R_2+R_1)Cs + 1}\right)$$

11.5 , 12.8 , 12.5

11.5



$$\frac{V_1 - V_3}{Ls} - \frac{V_3}{R} + \frac{V_2 - V_3}{2R} = 0$$

① $$\frac{V_2}{2R} - \left(\frac{1}{Ls} + \frac{1}{R} + \frac{1}{2R}\right)V_3 = \frac{-V_1}{Ls}$$

$$-\frac{V_2 - V_3}{2R} - \frac{V_2}{1}(Cs) = 0$$

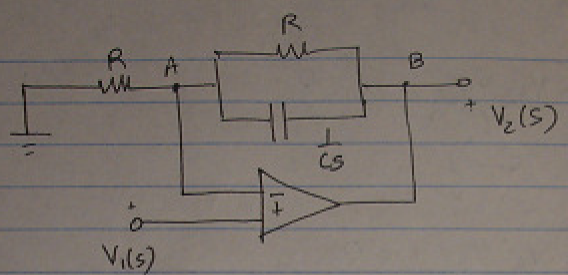
② $$-\left(\frac{1}{2R} + Cs\right)V_2 + \frac{V_3}{2R} = 0$$

$$\begin{bmatrix} \frac{1}{2R} & -\left(\frac{1}{Ls} + \frac{1}{R} + \frac{1}{2R}\right) \\ -\left(\frac{1}{2R} + Cs\right) & \frac{1}{2R} \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} \frac{-V_1}{Ls} \\ 0 \end{bmatrix}$$

using Matlab, $V_2 = \dfrac{R \cdot V_1}{3CLRs^2 + (2CR^2+L)s + R}$

$$\boxed{\frac{V_2}{V_1} = \frac{R}{3CLRs^2 + (2CR^2+L)s + R}}$$

11.7



$V_B = V_2(s)$

$V_+ = V_-$

$V_A = V_1(s)$

KCL At A $\quad \frac{V_A}{R} + \frac{V_A - V_B}{R} + \frac{V_A - V_B}{\frac{1}{Cs}} = 0$

$$\frac{V_A}{R} + \frac{V_A - V_B}{\frac{R\left(\frac{1}{Cs}\right)}{R + \frac{1}{Cs}}} = 0$$

$$\frac{V_A}{R} + \frac{V_A - V_B}{\frac{R}{RCs+1}} = 0$$

$$\frac{V_A}{R} + \frac{RCs+1}{R} V_A = \frac{RCs+1}{R} V_B$$

$$V_1(s)\left[\frac{1}{R} + \frac{RCs+1}{R}\right] = \frac{RCs+1}{R} V_B$$

$$V_1(s)\left[\frac{RCs+2}{R}\right] = \frac{RCs+1}{R} V_2(s)$$

$$\frac{V_2(s)}{V_1(s)} = \frac{(RCs+2)\cancel{R}}{\cancel{R}(RCs+1)} = \frac{RCs+2}{RCs+1}$$

pole at $-1k$ rad/s

$$RCs+1=0 \implies s = \frac{-1}{RC} = -1k$$

$$\boxed{\begin{array}{l} R = 1k \\ C = 1\mu F \end{array}}$$

11/8 $T_I(s) = \frac{I_2(s)}{I_1(s)} = \frac{R}{3R + sL}$ (immediate from source transformation)

$$\boxed{T_I(s) = \frac{R/L}{s + 3R/L}}$$

e.g. $\boxed{R = \frac{500}{3}\ \Omega, \quad L = 1\ H}$

This answer is <u>non-unique</u>

11-28

$R_1 = 100\,\Omega$
$R_2 = 400\,\Omega$
$L = 100 \times 10^{-3}\,H$
repeat for $i_1(t) = 10\cos(5000t)\cdot 10^{-3}A$

$\mathcal{L}(\cos\beta t) = \frac{s}{s^2+\beta^2}$

$i_1(s) = \frac{10s}{s^2+\beta^2}\cdot 10^{-3}$

$\beta = 500$

$-R_2 i_2 - Ls i_2 - R_1(i_2 - i_1) = 0$

$R_1 i_1 - (R_1 + R_2 + Ls) i_2 = 0$

$$\begin{bmatrix} 1 & 0 \\ R_1 & -(R_1+R_2+Ls) \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \frac{10s}{s^2+\beta^2} \\ 0 \end{bmatrix}$$

for $\beta = 500$ $\quad i_2(s) = \frac{10^{-2} R_1 s}{Ls^3 + (R_1+R_2)s^2 + \beta^2 Ls + \beta^2(R_1+R_2)}$

$i_2(s) = \frac{s}{.1s^3 + 500s^2 + 25000s + 1.25\times 10^8}$

$i_2(s) = \frac{-.002}{s+5000} + \frac{.001 - .0001i}{s - 500i} + \frac{.001 + .0001i}{s + 500i}$

$i_2(t) - .002e^{-5000t} + (.001 - .0001i)e^{500it} + (.001 + .0001i)e^{-500it}$

decays to zero $+ (.001 - .0001i)(\cos 500t + i\sin 500t) + (.001 + .0001i)(\cos 500t - i\sin 500t)$

$\boxed{i_2(t) = -2\left(\cos 500t + \frac{\sin 500t}{10}\right) mA} \approx \boxed{-2\cos(500t - .0997)}$

for $\beta = 5000$, $\quad i_2(s) = \frac{s}{.1s^3 + 500s^2 + 2.5\times 10^6 + 1.25\times 10^{10}} = \frac{-.001}{s+5000} + \frac{+5\times 10^{-4} + 5\times 10^{-4}i}{s - 5000i} + \frac{5\times 10^{-4} - 5\times 10^{-4}i}{s + 5000i}$

$i_2(t) = -.001e^{-5000t} + (5\times 10^{-4} + 5\times 10^{-4}i)e^{5000it} + (5\times 10^{-4} - 5\times 10^{-4}i)e^{-5000it}$

decays to zero

$\boxed{i_{2ss}(t) = -(\cos 5000t + \sin 5000t)\ mA} = \boxed{-\sqrt{2}\cos(5000t - \frac{\pi}{4})}$

11.61) $T_V(S) = \pm \dfrac{2\times 10^5}{(S+100)(S+10000)}$

Case i)

$$T_V(S) = \frac{k_1}{(S+100)} \; k_2 \; \frac{k_3}{S+10000}$$

$$\frac{k_1}{S+100} = \frac{k_1/S}{1+100/S} = \frac{Z_2(S)}{Z_1(S)+Z_2(S)}$$

$$Z_2(S) = \frac{k_1}{S} \qquad Z_1(S) = 1 + \frac{100}{S} - \frac{k_1}{S}$$

$$\boxed{Z_2 = \frac{100}{S} \qquad Z_1(S) = 1} \qquad = 1 + \frac{(100-k_1)}{S}$$

$k_1 = 100 = 10^2$

$$\frac{k_3}{S+10000} = \frac{Z_2(S)}{Z_1(S)+Z_2(S)} = \frac{k_3/S}{1+10000/S}$$

$$Z_2(S) = \frac{k_3}{S} \qquad Z_1(S) = 1 + \frac{10000-k_3}{S}$$

$k_3 = 1000 = 10^3$

$$\boxed{Z_2(S) = \frac{10^3}{S} \qquad Z_1(S) = 1 + \frac{9000}{S}}$$

Case (i): $k_1 k_2 k_3 = +2\times 10^5$

$C < 1\mu F = 10^{-6}$

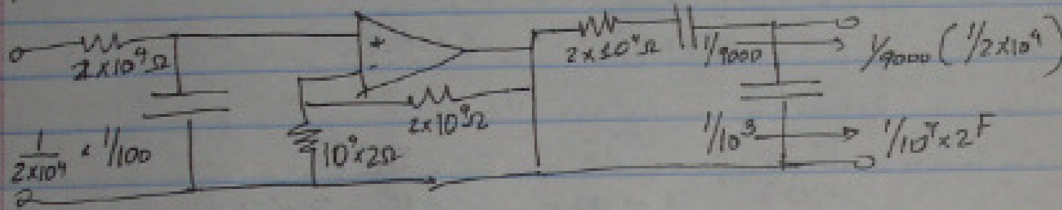
$R > 10K = 10^4$

$10^5 k_2 = 2 \times 10^5$

$k_2 = \frac{2}{\cancel{1}} = 2 = \frac{Z_1 + Z_2}{Z_2}$ $\quad Z_1 + Z_2 = 2Z_2$

$Z_1 = Z_2$

using scaling factor $k_m = 10^4 \times 2$

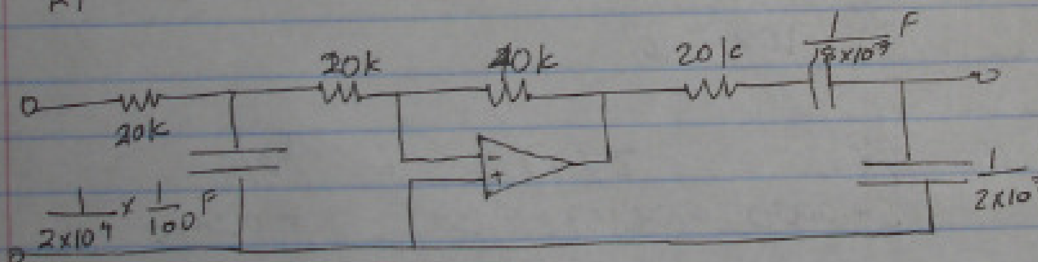


Case (ii) $k_1 k_2 k_3 = -2 \times 10^5$

$10^5 k_2 = -2 \times 10^5$

$\Rightarrow k_2 = -2$

$\frac{-R_2}{R_1} = -2 \quad \Rightarrow R_2 = 2R_1$



11/58a cct 1: $V_1(s) = V_p(s)$

$$V_p(s) \cdot \left(\frac{1}{10k} + \frac{1}{10k} + (100n)s\right) = V_2(s)\left(\frac{1}{10k} + (100n)s\right)$$

$$V_1(s) \times \left(2 \times 10^{-4} + 10^{-7}s\right) = V_2(s)\left(10^{-4} + 10^{-7}s\right)$$

$$\frac{V_2(s)}{V_1(s)} = T_V(s) = \left(\frac{s + 2000}{s + 1000}\right) \quad \blacksquare$$

cct 2:
$$V_2(s) = \frac{10k + (50n \cdot s)^{-1}}{20k + (50n \cdot s)^{-1}} V_1(s)$$

$$\frac{V_2(s)}{V_1(s)} = T_V(s) = \frac{(10k)(50n)s + 1}{(20k)(50n)s + 1}$$

$$= \frac{\frac{1}{2}s + 1000}{s + 1000} \times \frac{2}{2}$$

$$T_V(s) = \frac{1}{2}\left(\frac{s + 2000}{s + 1000}\right) \quad \blacksquare$$

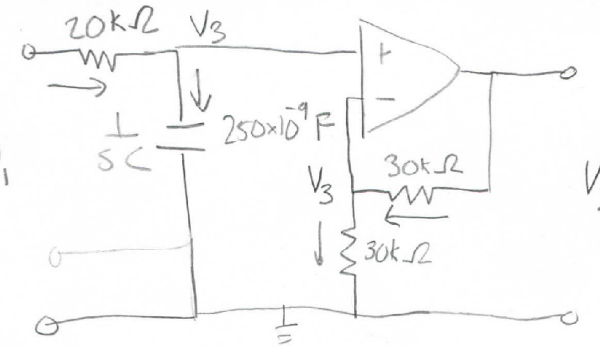
11/58b Since loading, especially for impedances of less than 10 kΩ, changes the V_1 to V_2 characteristic of circuit 2, circuit 1 is clearly superior.

11/58c An input impedance changes the V_1 to V_2 characteristic of circuit 2, so circuit 1 is superior; however, one may argue that the 100 Ω input impedance has negligible influence on the transfer function of circuit 2 (considering tolerances of practical components), so, because circuit 2 uses fewer & cheaper parts, it is superior.

11/58d This assertion is true. Note that the transfer function of circuit 1 is unaffected by both input and output impedances. The transfer function of circuit 2 may be affected by input or output impedances; however, circuit 1 introduces neither (this may be verified by a straightforward argument using Thevenin's Theorem ... but only if you're feeling really anal). Hence the circuits may be connected in either order.

12.3

$\frac{400}{s+200}$



$\frac{V_1 - V_3}{20k} - V_3 sC = 0$

$\frac{V_2 - V_3}{30k} - \frac{V_3}{30k} = 0$

$\frac{1}{20k}V_1 + 0V_2 - \left(\frac{1}{20k} + sC\right)V_3 = 0$

$0V_1 + \frac{1}{30k}V_2 - \left(\frac{1}{15k}\right)V_3 = 0$

$\frac{1}{20k} \cdot \left(\frac{V_1}{\frac{1}{20k} + sC}\right) = V_3$

$V_2 = 2V_3$

$V_2 = \frac{2}{20k} \cdot \frac{V_1}{\frac{1}{20k}+sC} \cdot \frac{20k}{20k} = \frac{2}{1+20ksC} = \frac{V_2}{V_1}$

a) $\boxed{\frac{V_2}{V_1} = \frac{400}{s+200}}$

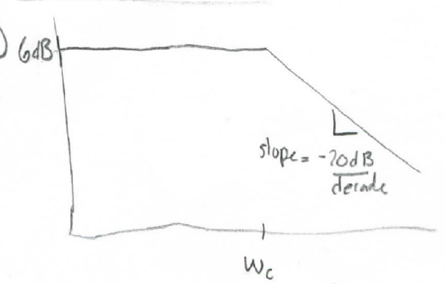
$\boxed{\text{DC gain: } 2}$

$\boxed{\text{infinite frequency gain: } 0}$

$\frac{400}{\sqrt{\omega^2+200^2}} = \frac{2}{\sqrt{2}} \Rightarrow \frac{400^2}{2} = \omega^2 + 200^2$ $\boxed{\omega_c = 200 \text{ rad/s}}$

$\boxed{\text{low pass filter}}$

b)



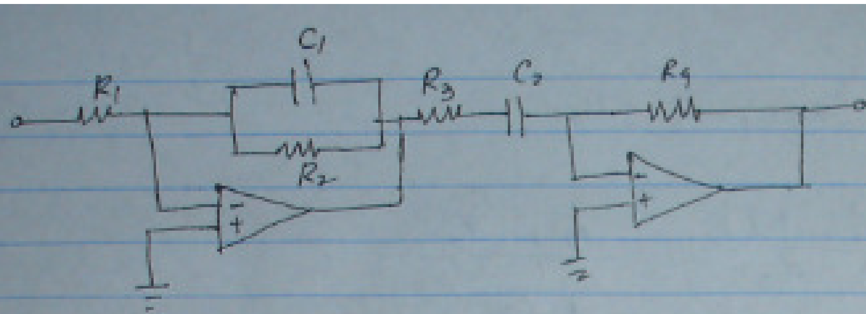
c) gain at $.5\omega_c$: $\frac{400}{\sqrt{100^2+200^2}} = \boxed{1.789}$

gain at ω_c: $\boxed{1.414}$

gain at $2\omega_c$: $\boxed{.8944}$

e) increase the horizontal 30kΩ resistor to a 270kΩ resistor

12.15



Bandpass gain = 30

$\omega_{c1} = 100 \text{ rad/s} \qquad \omega_{c2} = 2500 \text{ rad/s}$

$R \geq 10k \qquad C \leq 1\mu F$

Stage 1: Inverter

$$K_1 = \frac{-\cancel{Z_2}}{\cancel{Z_1}} = \frac{-\left(\dfrac{\frac{1}{C_1 s}\cdot R_2}{R_2 + \frac{1}{C_1 s}}\right)}{R_1} = \frac{-R_2}{(R_2 C_1 s + 1)R_1}$$

Stage 2:

$$k_2 = \frac{-Z_2}{Z_1} = -\frac{R_4}{R_3 + \frac{1}{C_2 s}} = \frac{-R_4 C_2 s}{R_3 C_2 s + 1}$$

$$T_v(s) = \frac{-R_2}{R_1(R_2 C_1 s + 1)} \cdot \frac{-R_4 C_2 s}{(R_3 C_2 s + 1)}$$

$$T(s) = \left(\frac{R_2 / R_1 R_2 C_1}{s + \frac{R_1}{R_1 R_2 C_1}}\right) \cdot \left(\frac{\frac{R_4 C_2 s}{R_3 C_2}}{s + \frac{1}{R_3 C_2}}\right)$$

$\omega_{c2} = \dfrac{1}{R_2 C_1} \qquad \omega_{c1} = \dfrac{1}{R_3 C_2} \qquad k_1 = \dfrac{1}{R_1 C_1}$

$k_2 = \dfrac{R_4}{R_3}$

Band pass filter

$$\frac{|k_1 k_2|}{\alpha_2} = \frac{\frac{1}{R_1 C_1} \cdot \frac{R_4}{\cancel{R_3}}}{\frac{1}{\cancel{R_3} C_2}}$$

$$\Rightarrow \frac{R_4 C_2}{R_1 C_1} = 30$$

$$\frac{1}{R_3 C_2} = 100 \text{ rad/s} \Rightarrow$$

$$\frac{1}{R_2 C_1} = 2500$$

$$C_1 = \frac{1}{2500 R_2} \qquad C_2 = \frac{1}{100 R_3}$$

$R_2 = 10k$

$C_1 = 4 \times 10^{-9} F$

$R_3 = 10k$

$C_2 = 10^{-7} F$

$$\frac{R_4}{R_1} \cdot \frac{10^{-7}}{4 \times 10^{-9}} = 30$$

$$\frac{R_4}{R_1} = \frac{6}{5} \Rightarrow R_4 = 60k$$

$R_1 = 50k$

12/57 | 1^{st} cct: filter characteristics

$$v_p(s) = \frac{1k}{100m + \frac{1}{400ns} + 1k} V_1(s) = \frac{10k}{100k} V_2(s)$$

$$\frac{V_2(s)}{V_1(s)} = T_1(s) = \frac{10k}{s100m + \frac{1}{s400n} + 1k}$$

$$= \frac{(10k)(400n)s}{s^2(100m)(400n) + (1k)(400n)s + 1}$$

$$= \frac{10^5 s}{s^2 + 10^4 s + 25 \times 10^6}$$

$$= \frac{10^5 s}{(s+5000)(s+5000)}$$

The so-called pass-band gain is attained at roughly $\boxed{\omega_0 = 5000 \text{ rad/s}}$ (the center frequency)

$$|T(5000j)| = \left| \frac{10^5 (\cancel{5000} j)}{2(\cancel{5000})(5000)} \right| = |10j| = 10$$

$$\Rightarrow \boxed{|T(\omega_0 j)| = 10} \quad \text{(pass-band gain)}$$

The cut-off frequencies are

$$\omega_{c1} = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_{c2} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} =$$

$$B = \omega_{c2} - \omega_{c1} = \boxed{10 \text{ krad/s} = B} \quad \text{(bandwidth)}$$

Filter 1 meets specs almost exactly. ~~Then~~
Because it is also a superior design when comparing part count & use of standard components, there is no reason to investigate the filter specs of circuit 2 (though there is nothing wrong with doing so.)

Hence, filter 1 is the better design

(Should be worth half of points)