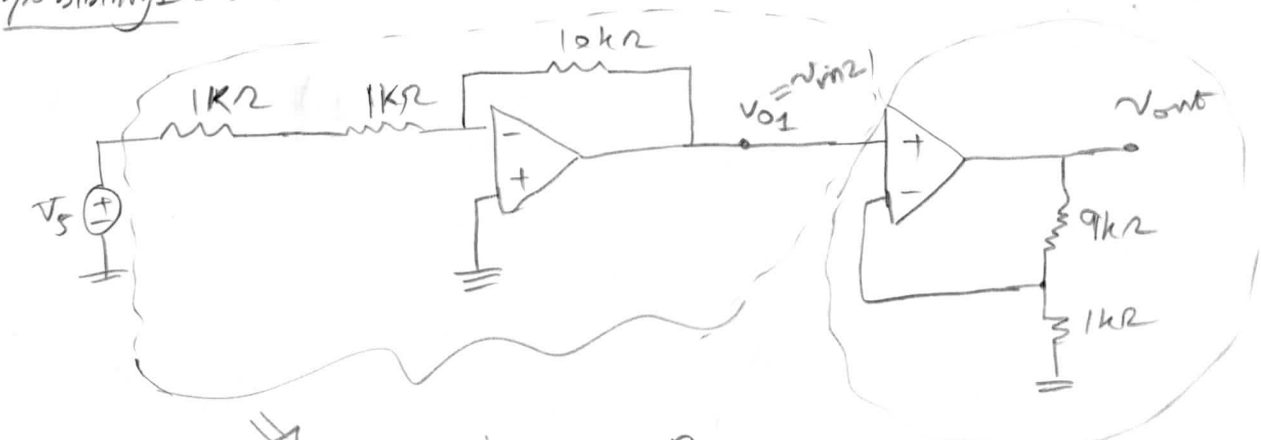
HW # 6: Solutions

Problem 4-20:

Goal: Achieve $v_{out} = -100 V_S$ by using circuits (1) & (2).

Possibilities: to either have [Source – circuit 2 – Circuit 1] or [Source – circuit 1 – Circuit 2]:

possibility 1: Source – Circuit 2 – Circuit 1:



acts as an inverting op-amp w/ $R_1 = 2k\Omega$ & $R_2 = 10k\Omega$

$$\Downarrow$$

$$v_{O1} = -\frac{10}{2} V_S = -5V_S$$

acts as an non-inverting op-amp w/ $R_1 = 9k\Omega$ & $R_2 = 1k\Omega$

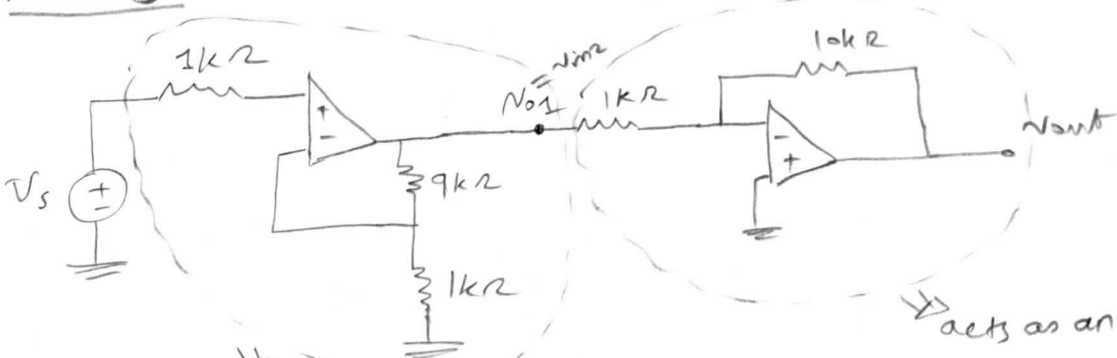
$$\Downarrow$$

$$v_{out} = \frac{9+1}{1} v_{in2} = 10 v_{in2}$$

$$v_{out} = -50 V_S$$

✗ it's not appropriate, we need -100 as our total gain

possibility 2: Source – Circuit 1 – circuit 2:



acts as a non-inverting op-amp w/ $R_1 = 9k\Omega$ & $R_2 = 1k\Omega$

$$\Downarrow$$

$$v_{O1} = \frac{9+1}{1} V_S = 10 V_S$$

acts as an inverting op-amp w/ $R_1 = 1k\Omega$, $R_2 = 10k\Omega$

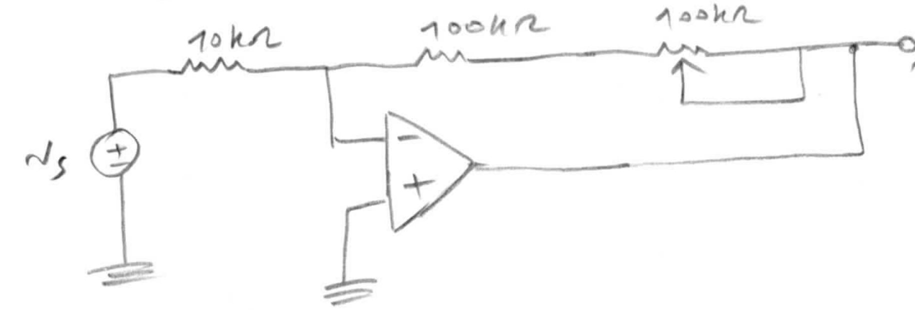
$$\Downarrow$$

$$v_{out} = -\frac{10}{1} v_{in2}$$

$$v_{out} = -100 V_S$$ ✓ it's appropriate design

4-22:

find range of gain v_o/v_s:



≡ inverting op-amp w/ $R_1 = 10k\Omega$

$R_2 = 100 + x \; k\Omega$

where $x \in [0, 100] \; k\Omega$

$$\Rightarrow \frac{v_o}{v_s} = -\left(\frac{100+x}{10}\right) = -10 - \frac{x}{10} \Rightarrow \begin{cases} \max\left(\frac{v_o}{v_s}\right) = -10 - \frac{x}{10}\Big|_{x=0} = -10 \\ \min\left(\frac{v_o}{v_s}\right) = -10 - \frac{x}{10}\Big|_{x=100} = -20 \end{cases}$$

$\Rightarrow \boxed{-20 \leq \frac{v_o}{v_s} \leq -10}$: range of gain

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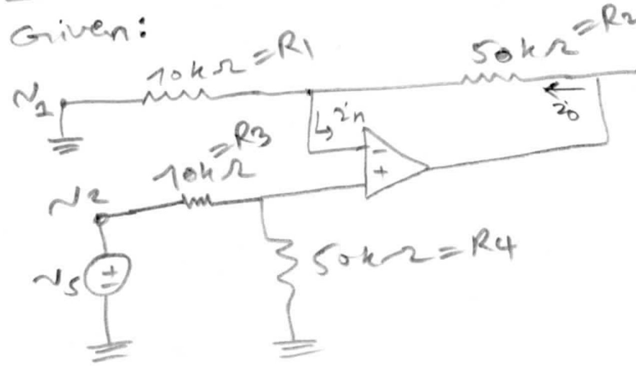
4-24: [one way]

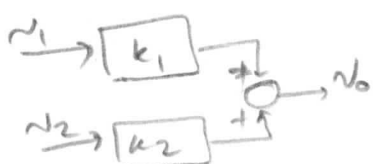
Given:



find: - v_o in terms of v_s

- find i_o for $v_s = 2V$

(a) we note that this circuit is a subtractor, so we have:



where: $k_1 = -\frac{R_2}{R_1} = -\frac{50}{10} = -5$

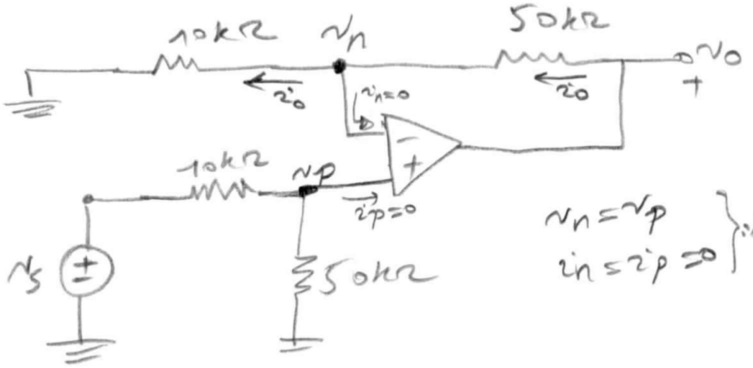
$$k_2 = \left(\frac{R_1+R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right) = \left(\frac{10+50}{10}\right)\left(\frac{50}{10+50}\right)$$

$\Rightarrow k_2 = 5$

$$\Rightarrow v_o = k_1 v_1 + k_2 v_2 = 5v_2 - 5v_1 \quad \text{where } \begin{cases} v_1 = 0 \\ v_2 = v_s \end{cases} \Rightarrow \boxed{v_o = 5v_s}$$

(b) $i_n = 0$ (basic op-amp property) $\Rightarrow i_o = \frac{v_o}{R_1+R_2} = \frac{v_o}{10+50} \xrightarrow{v_o = 5v_s} i_o = \frac{5}{60} v_s \xrightarrow{v_s = 2V} \boxed{i_o = \frac{1}{6} \, mA}$

4-24: [Alternative way]



$v_n = v_p$ } : basic properties of
$i_n = i_p = 0$ } op-amp

we find v_p by using voltage division trick:

$$v_p = \frac{50}{50+10} v_s = \frac{5}{6} v_s \Rightarrow \boxed{v_n = v_p = 5/6\, v_s} \quad (1)$$

Having found v_n, we can find i_o, noting that 10kΩ is grounded:

$$i_o = \frac{+v_n}{10} \xRightarrow{(1)} \boxed{i_o = +\frac{5}{60} v_s \ (mA)} \quad (2)$$

On the other hand voltage drop across 50kΩ:

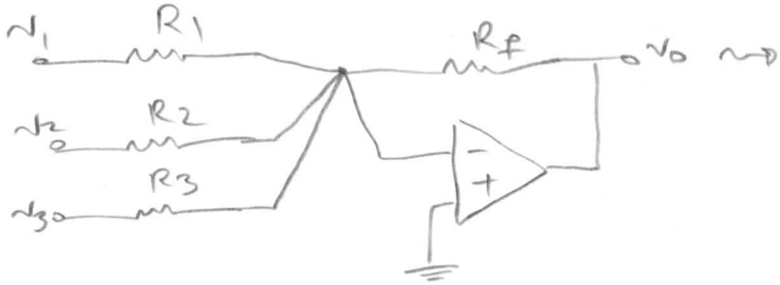
$$i_o \times 50 = v_o - v_n \Rightarrow v_o = v_n + 50\, i_o \xrightarrow[(2)]{(1)} 5/6\, v_s + 50 (5/60\, v_s)$$

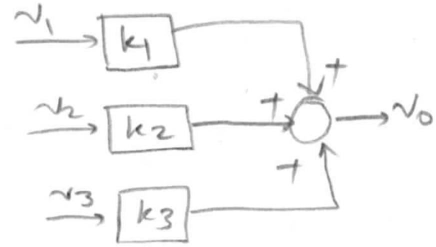
$$\Rightarrow v_o = (5/6 + 25/6) v_s \Rightarrow \boxed{v_o = 5 v_s}$$

Therefore, given $v_s = 2\,[V] \Rightarrow i_o = 5/60 \times 2 \Rightarrow \boxed{i_o = 1/6\ mA}$

4-27

3 input inverting summer:





$$\Rightarrow v_o = k_1 v_1 + k_2 v_2 + k_3 v_3 = -\frac{R_f}{R_1} v_1 - \frac{R_f}{R_2} v_2 - \frac{R_f}{R_3} v_3$$

$$\Rightarrow v_o = -R_f \left(\frac{1}{R_1} v_1 + \frac{1}{R_2} v_2 + \frac{1}{R_3} v_3 \right)$$

given: $R_f = 100\,k\Omega$

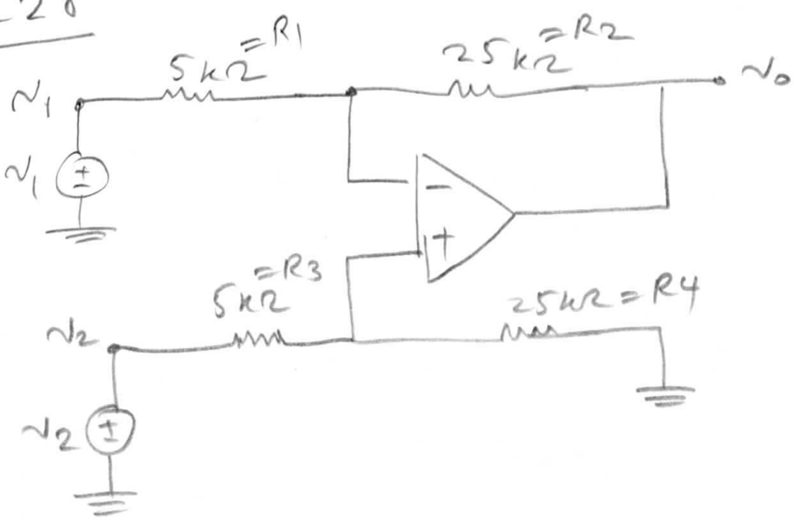
$v_o = -[v_1 + 10 v_2 + 100 v_3]$

$$\Rightarrow \frac{R_f}{R_1} = 1 \Rightarrow \boxed{R_1 = 100\,k\Omega}$$

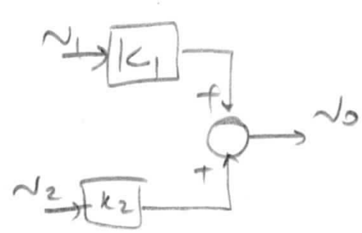
$$\frac{R_f}{R_2} = 10 \Rightarrow \boxed{R_2 = 10\,k\Omega}$$

$$\frac{R_f}{R_3} = 100 \Rightarrow \boxed{R_3 = 1\,k\Omega}$$

4-28



we note that it is a subtractor circuit, i.e., differential op-amp, so we get:



where $k_1 = -\frac{R_2}{R_1} = -\frac{25}{5} = -5$

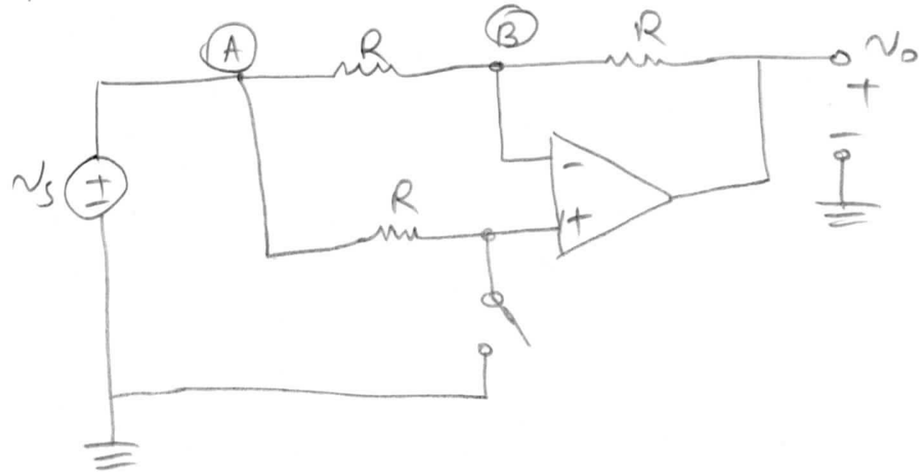
$$k_2 = \left(\frac{R_1 + R_2}{R_1}\right)\left(\frac{R_4}{R_3 + R_4}\right) = \left(\frac{5+25}{5}\right)\left(\frac{25}{5+25}\right)$$

$$\Rightarrow k_2 = 5$$

$$\Rightarrow \boxed{v_o = k_1 v_1 + k_2 v_2 = 5 v_2 - 5 v_1}$$

4-31:

The claim is false, as it is discussed bellow:



Switch is open:

basic op-amp properties: $v_n = v_p$, $i_p = i_n = 0$ $\Rightarrow \left\{ \begin{array}{l} v_A = v_B = v_s \\ \dfrac{v_B - v_o}{R} = 0 \Rightarrow v_B = v_o \end{array} \right\} \Rightarrow \boxed{v_o = v_s}$

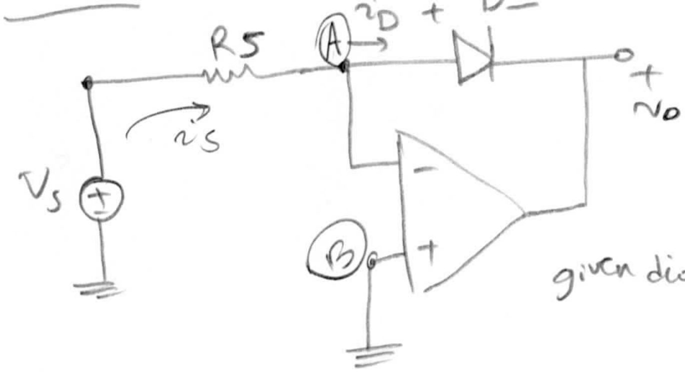
It disproves the 2nd statement of the claim

Switch is closed:

$$\left. \begin{array}{l} v_p = v_n = v_B = 0 \\ (i_n = 0 \Rightarrow \dfrac{v_A - v_B}{R} = \dfrac{v_B - v_o}{R} \\ v_A = v_s \end{array} \right\} \Rightarrow \boxed{v_o = -v_s}$$

→ It disproves the 1st statement of the claim

4-32:



$$\left\{ \begin{array}{l} i_n = i_p = 0 \Rightarrow i_s = i_D \\ v_p = v_n \Rightarrow v_A = v_B = 0 \end{array} \right.$$

$$\left. \begin{array}{l} i_s = \dfrac{v_s}{R_s} \\ \text{given diode eqn: } i_D = I_o\left(e^{v_D/v_T} - 1\right) \end{array} \right\} \Rightarrow \left. \begin{array}{l} \dfrac{v_s}{R_s} = I_o\left(e^{\frac{v_D}{v_T}} - 1\right) \\ v_D = v_A - v_o = 0 - v_o \end{array} \right\}$$

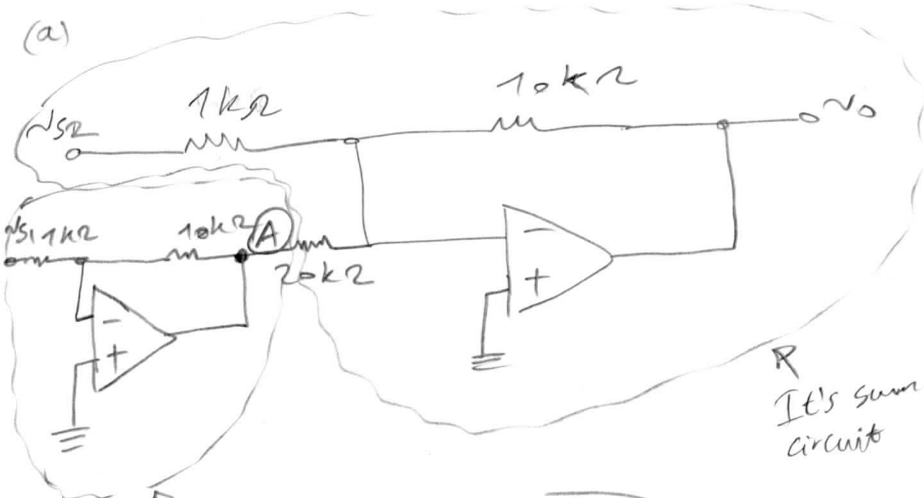
$$\boxed{\frac{v_s}{R_s} = I_o\left(e^{\frac{-v_o}{v_T}} - 1\right)} \quad (*)$$

From eq'n (*): $\frac{v_s}{I_0 R_S} = e^{-v_o/v_T} - 1 \Rightarrow -\frac{v_o}{v_T} = \ln\left(\frac{v_s}{I_0 R_S}+1\right)$

$$\Rightarrow \boxed{v_o = -v_T \ln\left(\frac{v_s}{R_S I_0}+1\right)}$$

4_36:

(a)



It's summer circuit $\Rightarrow v_o = \left(-\frac{10}{1}\right)v_{S2} + \left(\frac{-10}{20}\right)v_A$

$$= -10v_{S2} - \frac{1}{2}v_A$$

$$\Downarrow v_A = -10v_{S1}$$

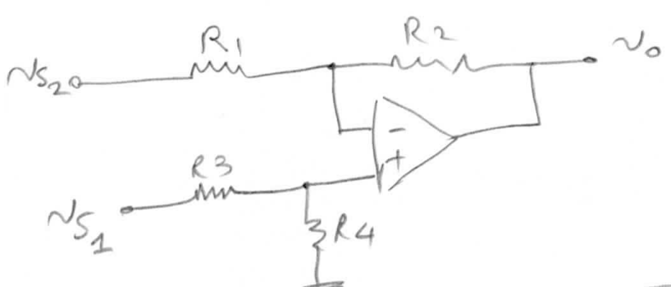
$$v_o = -10v_{S2} - \frac{1}{2}(-10v_{S1})$$

$$\Downarrow$$

$$\boxed{v_o = 5v_{S1} - 10v_{S2}} \quad (*)$$

It's an inverting op-amp $\Rightarrow \boxed{v_A = -\frac{10}{1}v_{S1}}$

(b) As an alternative way, we can use subtractor circuit:



for which we have: $\boxed{v_o = \left(\frac{R_1+R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right)v_{S1} - \left(\frac{R_2}{R_1}\right)v_{S2}}$ (**)

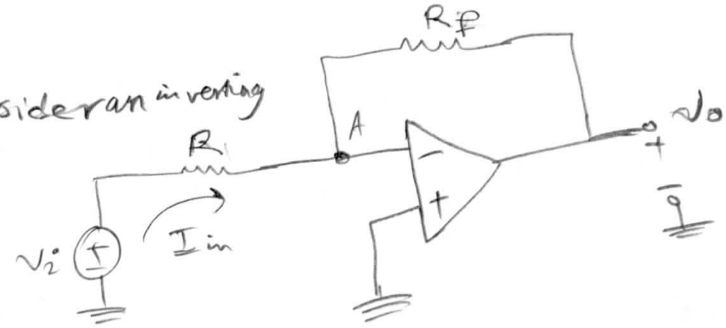
Comparing eq'ns (*) & (**):

$$\left.\begin{array}{l} 5=\left(\frac{R_1+R_2}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right) \\ -10=-\frac{R_2}{R_1} \Rightarrow \boxed{R_2=10R_1} \ (\star) \end{array}\right\} \Rightarrow 5=\left(\frac{R_1+10R_1}{R_1}\right)\left(\frac{R_4}{R_3+R_4}\right) \Rightarrow \boxed{\frac{R_3}{R_4}=\frac{6}{5}} \ (\star\star)$$

therefore, subtractor circuit w/ $R_1 = 1_{k\Omega}$; $R_2 = 10_{k\Omega}$; $R_3 = 6_{k\Omega}$; $R_4 = 5_{k\Omega}$, would yield the same result. [these values are some typical values, so any values which satisfy ($\star$) & ($\star\star$) are correct]

4-41

let's consider an inverting op-amp:



for this op-amp we have:
$$K=\frac{v_o}{v_i}=-\frac{R_f}{R}.$$

The restrictions of the problem are:
$$\begin{cases} R > 10\,k\Omega \\ R, R_f: \text{5\% resistance values} < 300\,k\Omega \\ \text{voltage gain: } K=-R_f/R=-10 \end{cases}$$

therefore, we can pick:

$R_f = 200\,k\Omega < 300\,k\Omega$

and so, $R = 20\,k\Omega$, which is a 5% resistance value $> 10\,k\Omega$ which also satisfies $R_f/R = 10$.

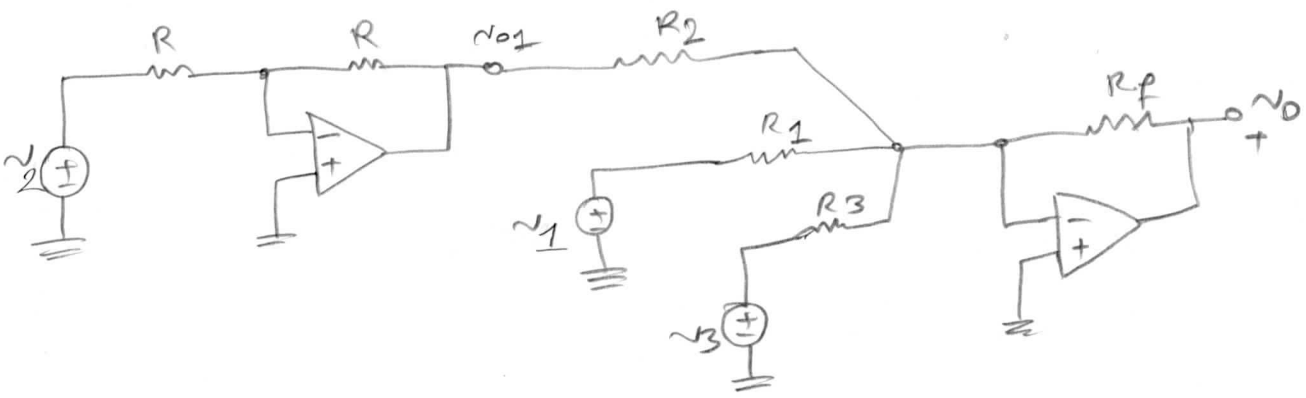
[In order to pick 5% resistor value, refer to standard values table at the end of book].

[The values for R & Rf are not unique, so long as the requirements are met, it is fine]

4-45:

goal: $\begin{cases} \text{using 2 op-amps w/ inputs } v_1, v_2, v_3: \\ \text{to get: } v_o = -3v_1 + 2v_2 - 5v_3 \end{cases}$

one possibility is to put one inverting & one summer in series:



$$v_{o1} = -\frac{R}{R} v_2 = -v_2$$

$$v_o = \left(-\frac{R_f}{R_2}\right) v_{o1} + \left(-\frac{R_f}{R_1}\right) v_1 + \left(-\frac{R_f}{R_3}\right) v_3$$

$\Rightarrow v_o = +\frac{R_f}{R_2} v_2 - \frac{R_f}{R_1} v_1 - \frac{R_f}{R_3} v_3$

Need: $v_o = 2v_2 - 3v_1 - 5v_3$

$$\Rightarrow \frac{R_f}{R_2} = 2; \quad \frac{R_f}{R_1} = 3; \quad \frac{R_f}{R_3} = 5$$

So, we can pick:

$$\begin{cases} R = 1\ k\Omega \\ R_f = 30\ k\Omega \\ R_2 = 15\ k\Omega \\ R_1 = 10\ k\Omega \\ R_3 = 6\ k\Omega \end{cases}$$

[these are some typical values, so long as the fractions are satisfied, it would be fine]