MAE140 - Linear Circuits - Fall 13
Final, December 11

Instructions

(i) The exam is open book. You may use your class notes and textbook. You may use a hand calculator with no communication capabilities

(ii) You have 180 minutes

(iii) Do not forget to write your name and student number

(iv) On the questions for which the answers are given, please provide detailed derivations

(v) The exam has 5 questions for a total of 50 points and 3 bonus points

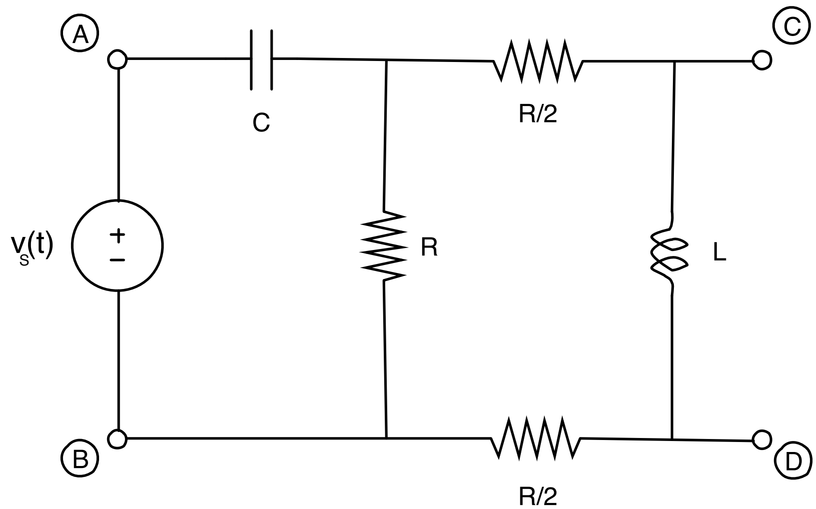

Figure 1: Circuit for Question 1.

1. **Equivalent Circuits**

Part I: [2 points] Assuming zero initial conditions, transform the circuit in Figure 1 into the s-domain.

Part II: [4 points] Find the impedance equivalent in the circuit obtained in Part I as seen from terminals A and B. The answer should be given as a ratio of two polynomials.

Part III: [4 points] Use source transformations to find the s-domain Thévenin equivalent of the circuit obtained in Part I as seen from terminals C and D.

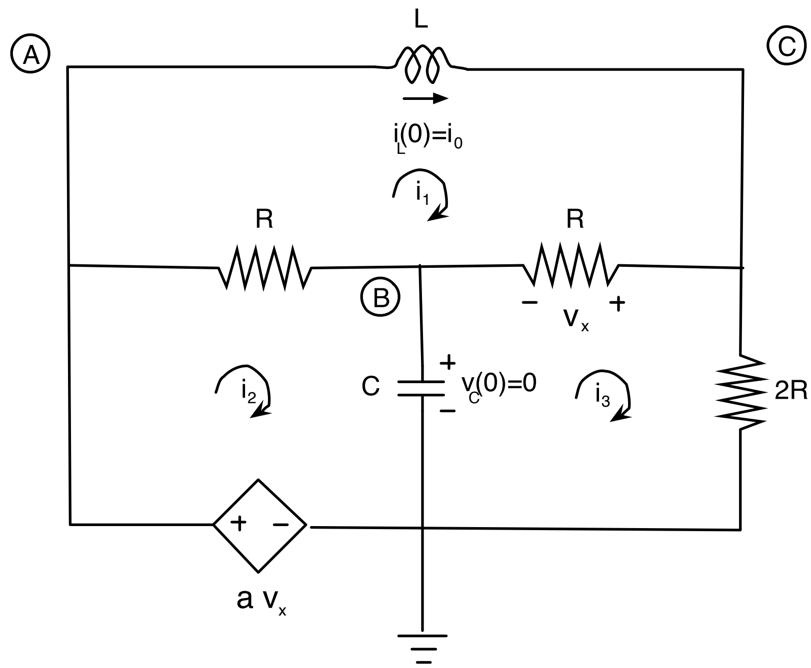

Figure 2: Nodal and Mesh Analysis Circuit

2. **Nodal and Mesh Analysis**

   Part I: [5 points] Formulate node-voltage equations in the s-domain for the circuit in Figure 2. Use the reference node and other labels as shown in the figure. Use the initial conditions indicated in the figure and transform them into current sources. Make sure your final answer has the same number of independent equations as unknown variables. No need to solve any equations!

   Part II: [5 points] Formulate mesh-current equations in the s-domain for the circuit in Figure 2. Use the currents shown in the figure. Use the initial conditions indicated in the figure and transform them into voltage sources. Make sure your final answer has the same number of independent equations as unknown variables. No need to solve any equations!

   Part III: [1 bonus point] Express the transform $I_L(s)$ of the inductor current in terms of your unknown variables of Part I and also in terms of your unknown variables of Part II.

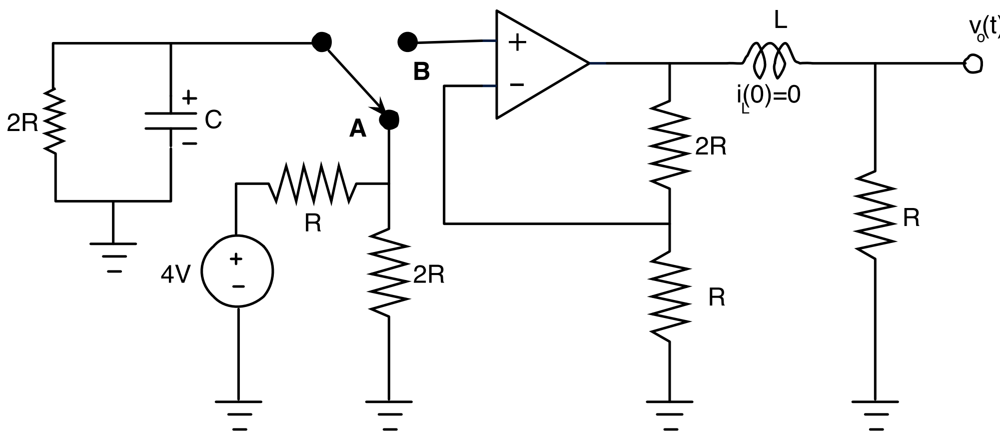

Figure 3: RC circuit for Laplace Analysis

3. **Laplace Domain Circuit Analysis**

Part I: [2 points] Consider the circuit depicted in Figure 3. The voltage source is constant. The switch is kept in position **A** for a very long time. At $t = 0$ it is moved to position **B**. Show that the initial capacitor voltage is given by

$$v_C(0^-) = 2V.$$

[Show your work]

Part II: [3 points] Use this initial condition to transform the circuit into the s-domain for $t \geq 0$. Use an equivalent model for the capacitor in which the initial condition appears as a voltage source. Find the transfer function of the circuit.
[Show your work]

Part III: [5 points] Use domain circuit analysis and inverse Laplace transforms to show that the output voltage $v_o(t)$ when $C = \frac{1}{6}F$, $L = 1H$, and $R = 3\Omega$ is

$$v_o(t) = 9(e^{-t} - e^{-3t})u(t).$$

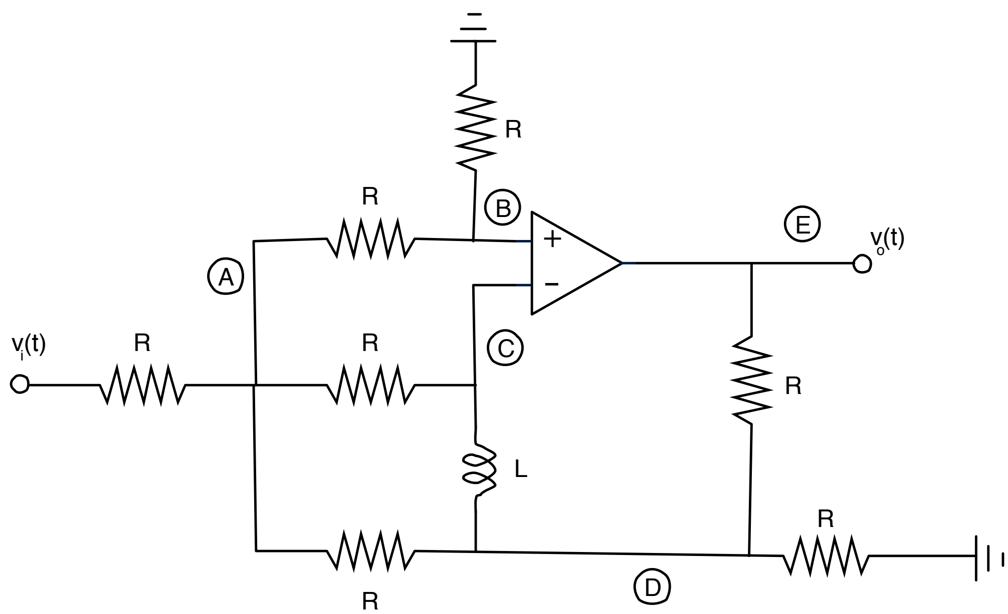

Figure 4: Frequency Response Analysis.

4. **Frequency Response Analysis**

Part I: [1 point] Assuming zero initial conditions, transform the circuit in Figure 4 into the s-domain.

Part II: [3 points] Show that the transfer function from $V_i(s)$ to $V_o(s)$ is given by

$$T(s) = \frac{V_o(s)}{V_i(s)} = \frac{-3Ls}{5R + Ls}.$$

[Show your work]
Hint: use node voltage analysis

Part III [4 points] Let $R = 100\ \text{m}\Omega$, $L = 10\ \text{mH}$. Compute the gain and phase functions of $T(s)$. What are the DC gain and the ∞-freq gain? What is the cut-off frequency ω_c? Use these values to sketch the magnitude of the frequency response of the circuit. Is this circuit a low-pass, high-pass, or band-pass filter?
[Explain your answer]

Part IV [2 points] Using what you know about frequency response, compute the steady state response $v_o^{SS}(t)$ of this circuit when $v_i(t) = -2\cos(50t - \frac{\pi}{4})$ using the same values of R and L as in Part III.

Part V: [2 bonus points] Design an inverting OpAmp circuit that has transfer function $T(s)$. What design would you recommend, your design or the one in Figure 4? Why?

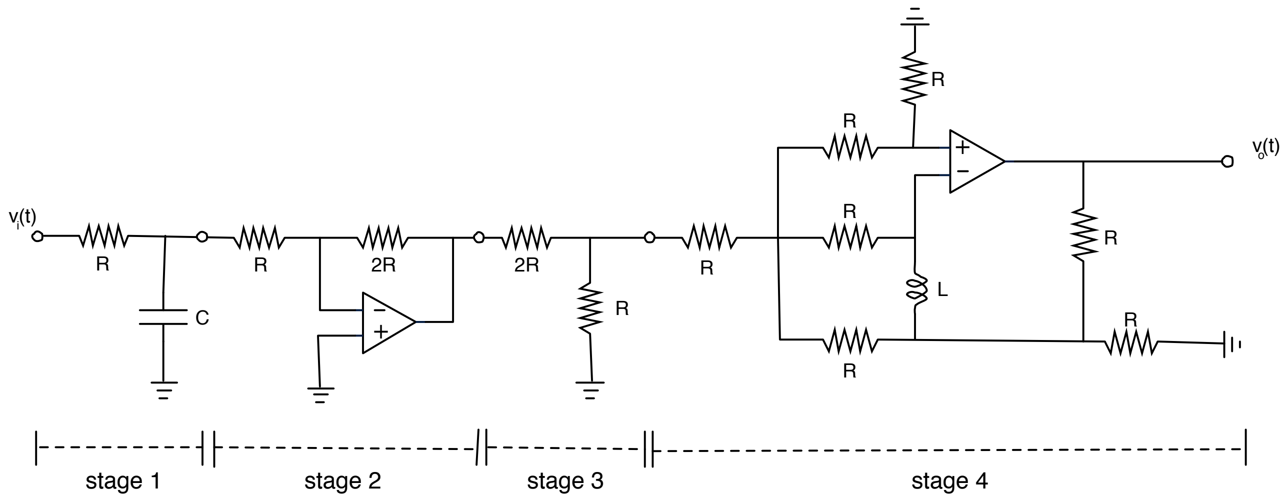

Figure 5: Circuit for Question 5. Stage 4 is the circuit of Question 4.

5. **Chain Rule and Circuit Design**

   Consider the circuit in Figure 5 (note that stage 4 is the circuit of Question 4). You can assume zero initial conditions.

   Part I: [3 points] Redraw the circuit of Figure 5 in the s-domain and compute the transfer functions $T_1(s)$, $T_2(s)$, $T_3(s)$, $T_4(s)$ of each one of the stages.

   Part II: [2 points] Somebody with a rusty recollection of linear circuits analyzed the circuit in Figure 5 and concluded that the transfer function $T(s)$ from $V_i(s)$ to $V_o(s)$ is equal to the product of the transfer functions

$$\widetilde{T}(s) = T_1(s) \times T_2(s) \times T_3(s) \times T_4(s) = \frac{1}{RCs+1}(-2)\frac{1}{3}\left(\frac{-3sL}{5R+sL}\right)$$
$$= \frac{2sL}{RCLs^2 + (5R^2C + L)s + 5R}$$

   of the 4 stages depicted in the plot. Identify two problems that invalidate this conclusion.

   Part III: [2 points] Modify Figure 5, keeping all 4 stages but possibly re-ordering them, so that the resulting circuit does have transfer function $\widetilde{T}(s)$ by adding at most 1 OpAmp.
   [Justify your answer]

   Part IV: [3 points] Use stage 1, a noninverting OpAmp, and a voltage divider to design a circuit whose transfer function is $\widetilde{T}(s)$.
   [Provide reasons that justify how you arrived at your design]