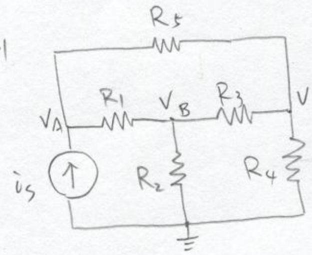
MAE 140 HW3 Solution

3-1

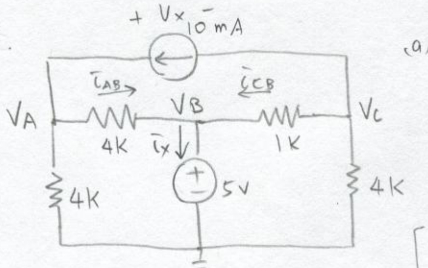


$G_1 = 1/R_1,\ G_2 = 1/R_2,\ G_3 = 1/R_3,\ G_4 = 1/R_4,\ G_5 = 1/R_5$

Formulate eqs by inspection:

$$\begin{bmatrix} G_1+G_5 & -G_1 & -G_5 \\ -G_1 & G_1+G_2+G_3 & -G_3 \\ -G_5 & -G_3 & G_3+G_4+G_5 \end{bmatrix} \cdot \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \\ 0 \end{bmatrix}$$

3-5.



(a) Use Method 2: carefully choose a ground.

$\underline{V_B = 5}$

Formulate eqs of nodes A and C by inspection:

$$\begin{bmatrix} \frac{1}{4k}+\frac{1}{4k} & \frac{-1}{4k} & 0 \\ 0 & \frac{-1}{1k} & \frac{1}{4k}+\frac{1}{1k} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 10\ mA \\ -10\ mA \end{bmatrix}$$

(b) Plug in $V_B = 5$ we get:

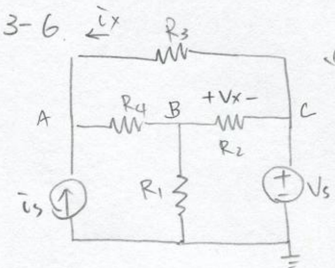
$$\begin{bmatrix} 0.5\times10^{-3} & 0 \\ 0 & 1.25\times10^{-3} \end{bmatrix} \begin{bmatrix} V_A \\ V_C \end{bmatrix} = \begin{bmatrix} 11.25\times10^{-3} \\ -5\times10^{-3} \end{bmatrix} \Rightarrow \underline{V_A = 22.5\,V} \quad \underline{V_C = -4V}$$

(c)

$V_x = V_A - V_C = 26.5\ V$.

$i_x = i_{AB} + i_{CB} = \frac{V_A - V_B}{4k} + \frac{V_C - V_B}{1k} = \underline{-4.625\ mA}$

3-6.



(a) Choose a ground as shown:

$\underline{V_C = V_S}$

Formulate eqs of nodes A an B by inspection

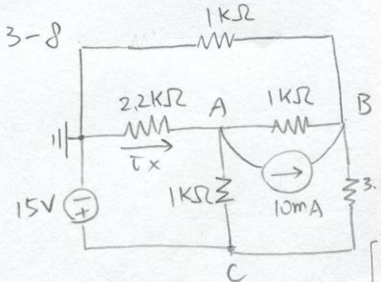
$$\begin{bmatrix} 1/R_3 + 1/R_4 & -1/R_4 & -1/R_3 \\ -1/R_4 & 1/R_1 + 1/R_2 + 1/R_4 & -1/R_2 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \end{bmatrix}$$

(b) Plug in $R_1 = R_2 = R_3 = R_4 = 10\ k$. $V_S = 25$, $i_s = 1mA$

$$\begin{bmatrix} 0.2\times10^{-3} & -0.1\times10^{-3} \\ -0.1\times10^{-3} & 0.3\times10^{-3} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 3.5\times10^{-3} \\ 2.5\times10^{-3} \end{bmatrix} \quad V_A = 26V,\ V_B = 17V.$$

3-6 (cont.) $V_x = V_B - V_C = -8\,V$, $i_x = \frac{V_C - V_A}{R_3} = -0.1\ mA$

3-8



(a) Choose a ground as shown:

$V_C = 15\ V$ #

Formulate eqs of nodes A and B by inspection

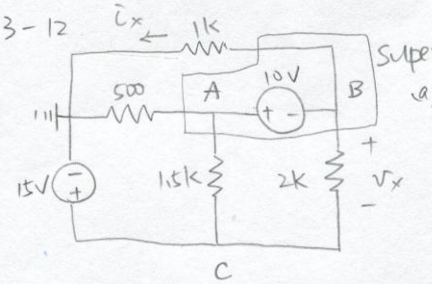
$$\begin{bmatrix} \frac{1}{2.2k}+\frac{1}{1k}+\frac{1}{1k} & \frac{-1}{1k} & \frac{-1}{1k} \\ \frac{-1}{1k} & \frac{1}{1k}+\frac{1}{1k}+\frac{1}{3.3k} & \frac{-1}{3.3k} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} -10\ mA \\ 10\ mA \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2.45 & -1 \\ -1 & 2.3 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 5 \\ 15 \end{bmatrix}$$

$V_A = 5.7V$, $V_B = 9\ V$

(b) $V_x = V_B = 9V$ # $i_x = \frac{0 - V_A}{2.2k} = -2.6\ mA$ #

3-12



supernode

(a) KCL for the supernode A & B

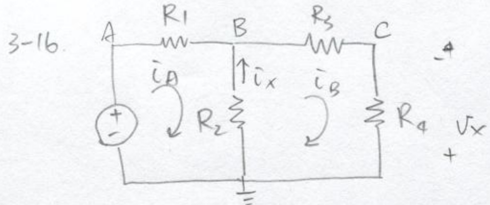
$$\frac{1}{1k}\cdot V_B + \frac{1}{500}\cdot V_A + \frac{1}{1.5k}(V_A - V_C) + \frac{1}{2k}(V_B - V_C) = 0$$

now use $V_C = 15$, $V_A = V_B + 10$

$\Rightarrow V_A = 7.8V$ $V_B = -2.2V$

(b) $V_x = V_B - V_C = -17.2V$ # $i_x = \frac{V_B}{1k} = -2.2\ mA$ #

3-16.



(a) $V_A = V_s$

Formulate eqs for nodes B and C.

$$\begin{bmatrix} \frac{-1}{R_1} & 1/R_1+1/R_2+1/R_3 & -1/R_3 \\ 0 & -1/R_3 & 1/R_3+1/R_4 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1/R_1+1/R_2+1/R_3 & -1/R_3 \\ -1/R_3 & 1/R_3+1/R_4 \end{bmatrix} \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} V_s/R_1 \\ 0 \end{bmatrix}$$

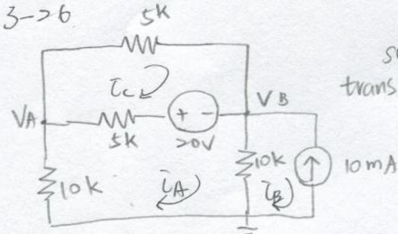
$$V_B = \frac{V_s R_2 (R_3+R_4)}{R_1R_2 + R_2R_3 + R_1R_3 + R_1R_4 + R_2R_4} \qquad V_c = \frac{V_s R_2 R_4}{R_1R_2 + R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4}$$

(b) set $Den = R_1R_2 + R_1R_3 + R_1R_4 + R_2R_3 + R_2R_4$

$$i_A = \frac{V_A - V_B}{R_1} = \frac{V_s(R_2+R_3+R_4)}{Den} \qquad i_B = \frac{V_c}{R_4} = \frac{V_s \cdot R_2}{Den}$$

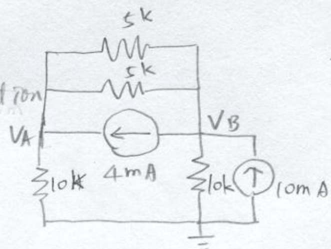
(c) $V_x = 0 - V_c = \dfrac{-V_s R_2 R_4}{Den}$ $\qquad i_x = -i_A + i_B = \dfrac{V_x(-R_3-R_4)}{Den}$

3-26



source transformation

$\Rightarrow$



$i = \frac{V}{R} = \frac{20}{5k} = 4\,mA$

Formulate eqs by inspection:

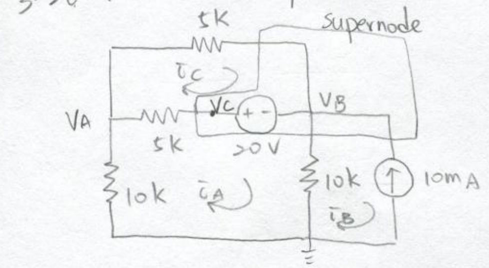
$$\begin{bmatrix} 1/5k+1/5k+1/10k & -1/5k \; -1/5k \\ -1/5k \; -1/5k & 1/5k+1/5k+1/10k \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 4\times10^{-3} \\ 10\times10^{-3} - 4\times10^{-3} \end{bmatrix} \Rightarrow \begin{matrix} V_A = 48.9\ V \\ V_B = 51.1\ V \end{matrix}$$

$$i_A = \frac{-V_A}{10\,A} = -4.89\ mA$$

$$i_B = -10\ mA$$

$$i_c = \frac{V_A - V_B}{5\ k} = -0.44\ mA$$

3-26 (Another way)



Formulate eqs by using a supernode B&C

$V_C - V_B = 20$

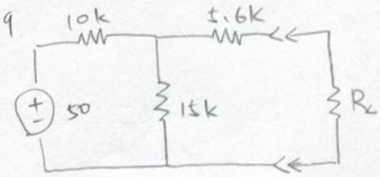
$\frac{1}{1k}(V_B - V_A) + \frac{1}{1k}(V_C - V_A) + \frac{1}{10k}(V_B) = 10\ mA$

$\frac{1}{1k}(V_A - V_B) + \frac{1}{1k}(V_A - V_C) + \frac{1}{10k}(V_A) = 0$

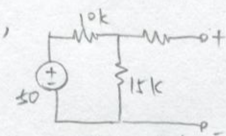
Replace V_C by $V_C = V_B + 20$

$$\Rightarrow \begin{bmatrix} \frac{1}{1k} + \frac{1}{1k} + \frac{1}{10k} & -\frac{1}{1k} - \frac{1}{1k} \\ -\frac{1}{1k} - \frac{1}{1k} & \frac{1}{1k} + \frac{1}{1k} + \frac{1}{10k} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 4 \times 10^{-3} \\ 6 \times 10^{-3} \end{bmatrix} \Rightarrow \begin{matrix} V_A = 48.9\ V \\ \underline{V_B = 51.1\ V} \end{matrix}$$

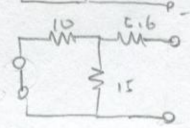
3-49



(a)



$V_{oc} = 50 \cdot \frac{15}{15+10} = \underline{30\ V}$

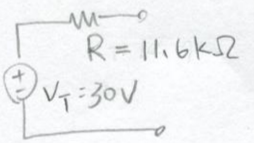


$R_{eq} = 10K // 15K + 5.6\ k$

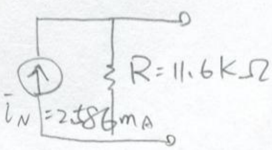
$= 6K + 5.6\ k = \underline{11.6\ k}$

$i_N = \frac{V_{oc}}{R_{eq}} = \underline{2.586\ mA}$

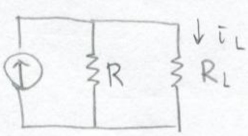
Thévenin :



Norton :

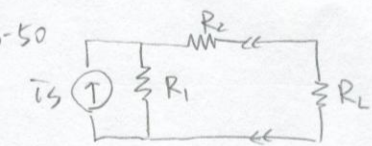


(b) Use Norton equivalent :



$i_L = i_N \cdot \frac{1/R_L}{1/R + 1/R_L} = \underline{0.893\ mA}$

3-50



use source transformation

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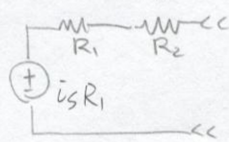
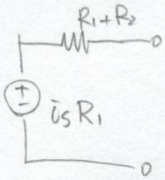


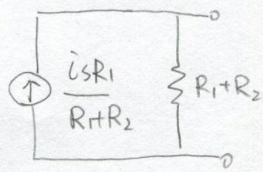
P4

3-50 (cont.)

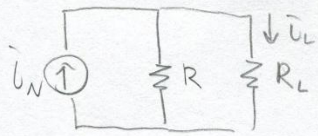
Thévenin :



Norton :



(b) Use Norton equivalent

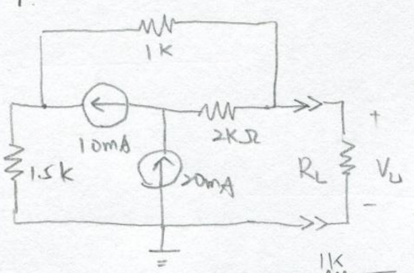


$$i_L = i_N \cdot \frac{1/R_L}{1/R_L + 1/R} = \frac{i_S R_1}{R_1+R_2+R_L}$$ #

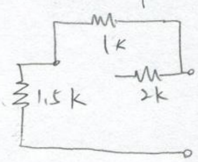
(c) current division :

$$i_L = i_S \cdot \frac{1/(R_2+R_L)}{1/R_1 + 1/(R_2+R_L)} = \frac{i_S R_1}{R_1+R_2+R_L}$$ #

3-57.

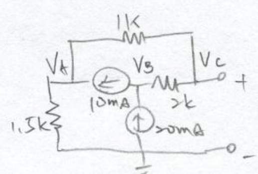


1. find Req



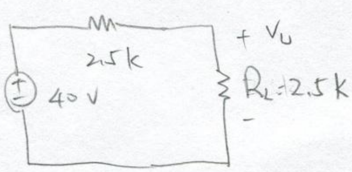
Req = 2.5k

2. find V_T :



$$\begin{bmatrix} \frac{1}{1.5k}+\frac{1}{1k} & 0 & \frac{-1}{1k} \\ 0 & \frac{1}{2k} & \frac{-1}{2k} \\ \frac{-1}{1k} & \frac{-1}{2k} & \frac{1}{1k}+\frac{1}{2k} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 10\ mA \\ 10\ mA \\ 0 \end{bmatrix} \Rightarrow \begin{matrix} V_A = 30\ V \\ V_B = 60\ V \\ V_C = 40\ V \end{matrix}, \quad V_T = V_C = 40V$$

$\Rightarrow$



$$\Rightarrow V_U = 40 \cdot \frac{2.5}{2.5+2.5} = 20\ V$$ #