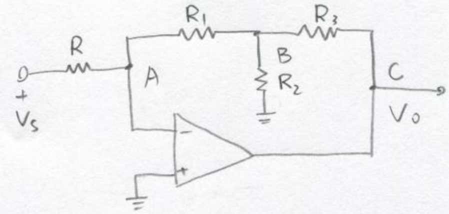
MAE 140. Homework 6 Solution

4-41.



(a) This is neither an inverting nor a non-inverting OpAmp. We need to use node voltage analysis to solve the problem

KCL @ A. $\dfrac{V_A - V_s}{R} + \dfrac{V_A - V_B}{R_1} = 0$

KCL @ B: $\dfrac{V_B - V_A}{R_1} + \dfrac{V_B - V_C}{R_3} + \dfrac{V_B}{R_2} = 0$

Equation of an ideal Op Amp: $V_A = 0$

3 equations and 3 unknowns solve the equations we have:

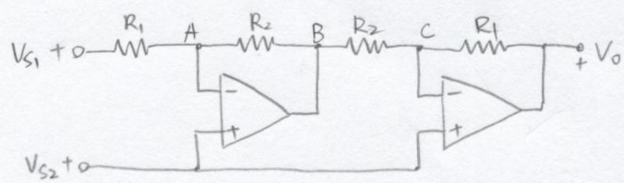
$V_A = 0$, $V_B = V_S \cdot \dfrac{-R_1}{R}$, $V_C = V_S \cdot \dfrac{-R_1R_2 - R_1R_3 - R_2R_3}{RR_2}$

$\Rightarrow K = \dfrac{-R_1R_2 - R_2R_3 - R_3R_1}{RR_2}$

(b) $K = -6$, We can choose: $R_1 = R_2 = R_3 = 10\ k\Omega$, $R = 5\ k\Omega$

(multiple answers are possible)

4-44



From equations of an ideal OpAmp: $V_P = V_N \Rightarrow V_A = V_{S2}$, $V_C = V_{S2}$

KCL @ A: $\dfrac{V_A - V_{S1}}{R_1} + \dfrac{V_A - V_B}{R_2} = 0$

KCL @ C: $\dfrac{V_C - V_B}{R_2} + \dfrac{V_C - V_O}{R_1} = 0$

4 equations and 4 unknowns. solve the equations we get:

$V_o = V_{S1}$

9-2. $f(t) = 20 \sin(377t)\, u(t)$

$\Rightarrow F(S) = 20 \cdot \dfrac{377}{S^2 + 377^2}$ poles @ $\pm 377j$ no zeros

9-3:

$f(t) = -10\delta(t) + 10u(t) \Rightarrow F(s) = -10 + \frac{10}{s} = \frac{-10s+10}{s}$ poles @ 0, zero @ 1

9-6. $f(t) = 0.005\left[10 - 10\cos(1000t)\right]u(t)$

$\Rightarrow F(s) = 0.005\left[\frac{10}{s} - 10 \cdot \frac{s}{s^2+10^6}\right] = 0.05 \cdot \frac{10^6}{s(s^2+10^6)}$

poles @ 0, $\pm 1000j$, No zeros

9-8. $f(t) = \delta(t) - 200e^{-20t}\cos(200t)\,u(t)$

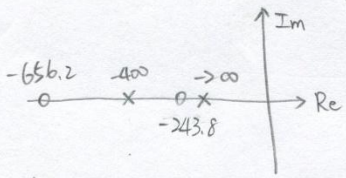
$F(s) = 1 - 200 \cdot \frac{s+20}{(s+20)^2+(200)^2} = \frac{s^2 - 160s + 36400}{(s+20)^2 + 200^2}$

poles @ $-20 \pm 200j$, zeros @ $80 \pm 173j$

9-11 (a) $f_1(t) = 2\delta(t) + \left[200e^{-200t} + 400e^{-400t}\right]u(t)$

$F(s) = 2 + \frac{200}{s+200} + \frac{400}{s+400} = \frac{2s^2 + 1800s + 320000}{(s+200)(s+400)}$ poles @ -200, -400

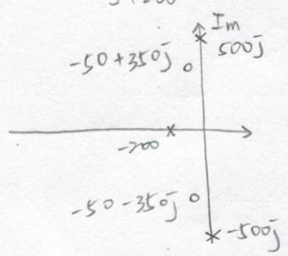
zeros @ -656.2, -243.8



(b) $f_2(t) = \left[15e^{-200t} + 15\cos 500t\right]u(t)$

$F(s) = \frac{15}{s+200} + \frac{15 \cdot s}{s^2+500^2} = \frac{15\,(2s^2 + 200s + 250000)}{(s+200)(s^2+500^2)}$ poles @ $-200, \pm 500j$

zeros @ $-50 \pm 350j$



9-21 (a) $F_1(s) = \frac{s+30}{s(s+40)} = \frac{C_1}{s} + \frac{C_2}{s+40}$

$C_1 = \lim_{s\to 0} F_1(s) \cdot s = 3/4$ $C_2 = \lim_{s \to -40} F_1(s)\cdot(s+40) = 1/4$

$\underline{f_1(t) = \left[3/4 + 1/4 \cdot e^{-40t}\right]\cdot u(t)}$ *

9-21 (b) $F_2(s) = \dfrac{(s+10)(s+20)}{s(s+50)(s+100)} = \dfrac{C_1}{s} + \dfrac{C_2}{s+50} + \dfrac{C_3}{s+100}$

$C_1 = \lim\limits_{s \to 0} F_2(s) \cdot s = 0.04$, $C_2 = \lim\limits_{s \to -50} F_2(s) \cdot (s+50) = -0.48$

$C_3 = \lim\limits_{s \to -100} F_2(s)(s+100) = 1.44$

$f_2(t) = \left[0.04 - 0.48 e^{-50t} + 1.44 e^{-100t} \right] u(t)$ ※

9-22 (a) $F_1(s) = \dfrac{5000(s+1000)}{(s+500)(s+5000)} = \dfrac{C_1}{s+500} + \dfrac{C_2}{s+5000}$

$C_1 = \lim\limits_{s \to -500} F_1(s) \cdot (s+500) = 555.6$

$C_2 = \lim\limits_{s \to -5000} F_1(s)(s+5000) = 4444$

$f_1(t) = \left[555.6 \cdot e^{-500t} + 4444 \cdot e^{-5000t} \right] u(t)$ ※

(b) $F_2(s) = \dfrac{5s^2}{(s+100)(s+500)} = 5 - \dfrac{3000s + 250000}{(s+100)(s+500)} = 5 + \dfrac{C_1}{s+100} + \dfrac{C_2}{s+500}$

Solve partial fraction expansion we get : $C_1 = 125$, $C_2 = -3125$

$f_2(t) = 5\delta(t) + \left[125 \cdot e^{-100t} - 3125 e^{-500t} \right] u(t)$ ※

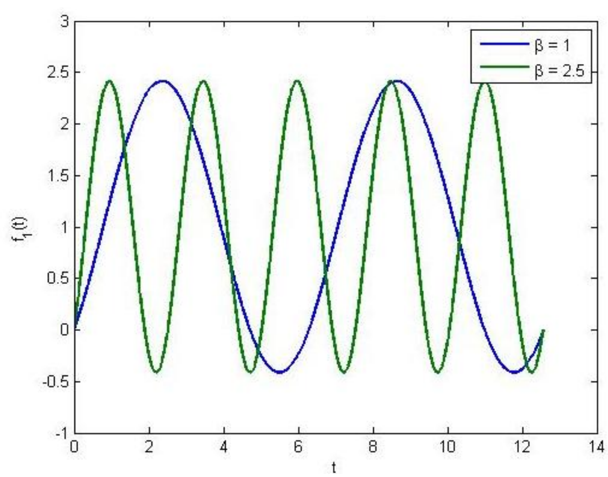
9-24 (a) $F_1(s) = \dfrac{\beta(s+\beta)}{s(s^2+\beta^2)} = \dfrac{C_1}{s} + \dfrac{C_2}{s+\beta j} + \dfrac{C_2^*}{s-\beta j}$

$C_1 = \lim\limits_{s \to 0} F_1(s) \cdot s = 1$. $C_2 = \lim\limits_{s \to -\beta j} F_1(s)(s+\beta j) = -0.5 + 0.5j$

$= \dfrac{\sqrt{2}}{2} \cdot e^{j\frac{3}{4}\pi}$

$f(t) = \left[1 + \dfrac{\sqrt{2}}{2} e^{-\beta j t + j\frac{3}{4}\pi} + \dfrac{\sqrt{2}}{2} e^{\beta j t - j\frac{3}{4}\pi} \right] u(t)$

$= \left[1 + \sqrt{2} \cdot \cos\left(\beta t - \frac{3}{4}\pi\right) \right] u(t)$ $\quad \left[\because e^{\theta j} + e^{-\theta j} = 2 \cdot \cos\theta \right]$



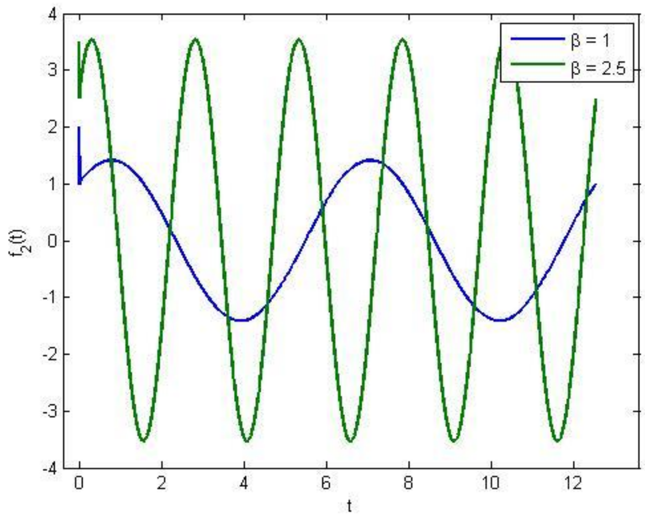
9-24 (b)

$$F_2(s) = \frac{s(s+\beta)}{s^2+\beta^2} = 1 + \frac{s\beta - \beta^2}{s^2+\beta^2}$$

$$f_2(t) = \delta(t) + \beta \cdot \cos\beta t - \beta \cdot \sin\beta t$$

$$= \delta(t) + \sqrt{2}\,\beta \cdot \cos\left(\beta t - \frac{\pi}{4}\right)$$

$$f_2(0) = 1 + \sqrt{2}\beta \cdot \left(\frac{1}{\sqrt{2}}\right) = 1 + \beta$$



9-26 $$F_1(s) = \frac{600}{(s+10)(s+20)(s+30)} = \frac{C_1}{s+10} + \frac{C_2}{s+20} + \frac{C_3}{s+30}$$

$C_1 = \lim\limits_{s\to -10} F_1(s)\cdot(s+10) = 3$ $\quad C_2 = \lim\limits_{s\to -20} F_1(s)\cdot(s+20) = -6,$ $\quad C_3 = \lim\limits_{s\to -30} F_1(s)(s+30) = 3$

$$f_1(t) = \left[3\cdot e^{-10t} - 6\cdot e^{-20t} + 3\cdot e^{-30t}\right] u(t) \quad \#$$

(b) $$F_2(s) = \frac{2(s+10)}{(s+15)(s+20)} = \frac{C_1}{s+15} + \frac{C_2}{s+20}$$

$C_1 = \lim\limits_{s\to -15} F_2(s)\cdot(s+15) = -2$ $\quad C_2 = \lim\limits_{s\to -20} F_2(s)\cdot(s+20) = 4$

$$f_2(t) = \left[-2e^{-15t} + 4e^{-20t}\right] u(t) \quad \#$$

9-34

a) $F_1(s) = \dfrac{s^2}{(s+5)} = s - 5 + \dfrac{25}{s+5}$

$f_1(t) = \dfrac{d\delta(t)}{dt} - 5\delta(t) + \left[25 \cdot e^{-5t}\right] * u(t)$ ※

b) $F_2(s) = \dfrac{(s+1000)^2}{(s+2000)^2} = 1 - \dfrac{2000s + 3000000}{(s+2000)^2} = 1 - \dfrac{2000}{s+2000} + \dfrac{1000000}{(s+2000)^2}$

$f_2(t) = \delta(t) + \left[-2000 \cdot e^{-2000t} + 1000000 \cdot t \cdot e^{-2000t}\right] * u(t)$ ※