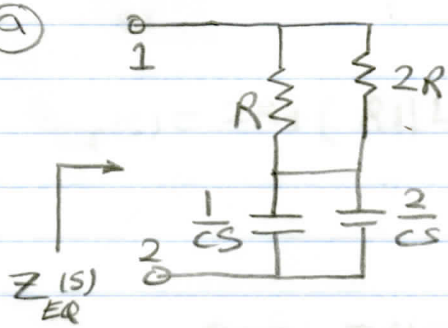
HW#7
MAE140
Fall 2013

10-4

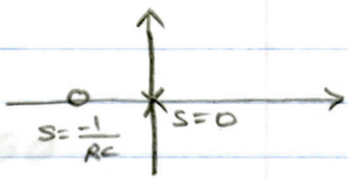
ⓐ



$$Z_{EQ}(S) = (R \| 2R) + \left(\frac{1}{CS} \| \frac{2}{CS}\right) = \frac{2}{3}R + \frac{2}{3}\frac{1}{CS}$$

$$\Rightarrow Z_{EQ}(S) = \frac{2}{3}\left(\frac{RCS+1}{CS}\right)$$

Zero: $RCS+1=0 \Rightarrow S = \frac{-1}{RC}$
pole: $CS=0 \Rightarrow S=0$



ⓑ Zero: $S = -33$ krad/s $= -33\times10^3$ rad/s

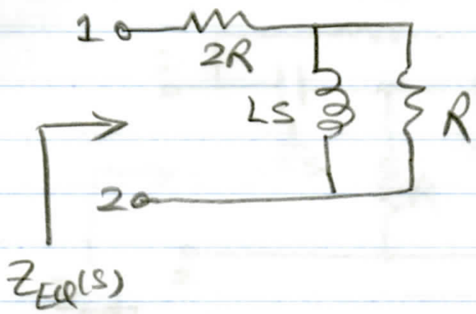
$$S = \frac{-1}{RC} = -33\times10^3 \Rightarrow \frac{1}{RC} = 33\times10^3$$

we can choose $R = 10\ k\Omega$

$\Rightarrow C = 0.003\ \mu F$

10-6

(a)

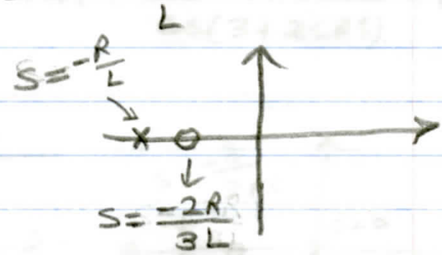


$$Z_{EQ}(s) = 2R + (R \| Ls) = 2R + \frac{RLs}{R+Ls}$$

$$= \frac{3RLs + 2R^2}{R + Ls}$$

zero: $3RLs + 2R^2 = 0 \Rightarrow s = -\frac{2R}{3L}$

pole: $R + Ls = 0 \Rightarrow s = -\frac{R}{L}$



(b) pole: $s = -150$ rad/s

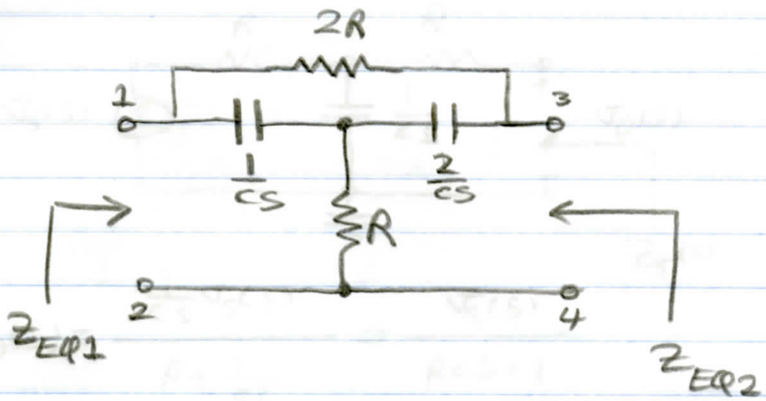
$$s = -\frac{R}{L} = -150 \Rightarrow \frac{R}{L} = 150$$

we can select $R = 10\,k\Omega$

$$\Rightarrow L = 0.06 \times 10^3 \text{ H}$$

zero: $s = -\frac{2}{3}\frac{R}{L} = \frac{-2}{3}(150)$ rad/s $= -100$ rad/s

10-10



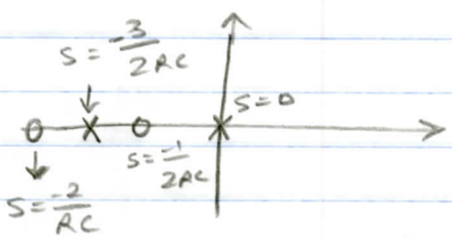
$$Z_{EQ1} = \left[\left(\frac{2}{CS}+2R\right) \| \frac{1}{CS}\right] + R$$

$$= \frac{\left(\frac{2}{CS}+2R\right)\left(\frac{1}{CS}\right)}{\frac{2}{CS}+2R+\frac{1}{CS}} + R = \frac{2(1+RCS)}{CS(3+2CRS)} + R$$

$$= \frac{2(CR)^2S^2+5CRS+2}{CS(3+2CRS)}$$

zeros: $S = \frac{-1}{2RC}$ and $S = \frac{-2}{RC}$

poles: $S=0$ and $S = \frac{-3}{2RC}$



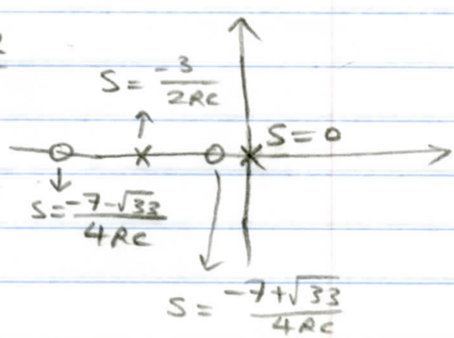
$$Z_{EQ2} = \left[\left(\frac{1}{CS}+2R\right) \| \frac{2}{CS}\right] + R$$

$$= \frac{\left(\frac{1}{CS}+2R\right)\left(\frac{2}{CS}\right)}{\frac{1}{CS}+2R+\frac{2}{CS}} + R = \frac{2(1+2RCS)}{CS(3+2RCS)} + R$$

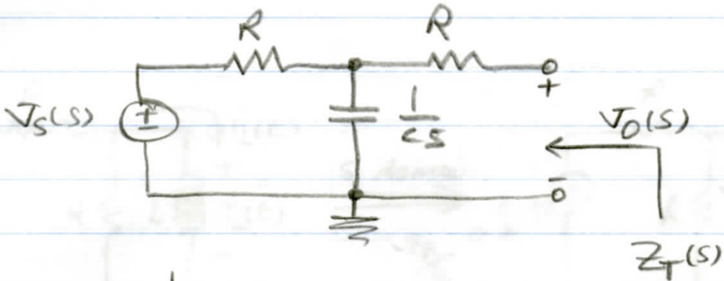
$$= \frac{2(CR)^2S^2+7CRS+2}{CS(3+2RCS)}$$

zeros: $S = \frac{-7 \pm \sqrt{33}}{4RC}$

poles: $S=0$ and $S = \frac{-3}{2RC}$



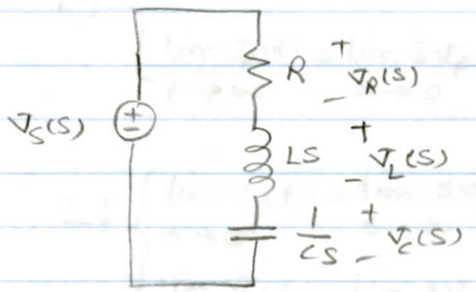
10-14



(a) $$V_O(S) = \frac{\frac{1}{CS} V_S(S)}{R + \frac{1}{CS}} = \frac{V_S(S)}{RCS+1}$$

(b) $$Z_T(S) = \left(R \,||\, \frac{1}{CS}\right) + R = \frac{R\frac{1}{CS}}{R+\frac{1}{CS}} + R = \frac{R(RCS+2)}{RCS+1}$$

10-18



$V_S(t) = u(t) \Longrightarrow V_S(S) = \frac{1}{S}$

(a) using voltage division:

$$V_R(S) = \frac{R V_S(S)}{R + LS + \frac{1}{CS}} = \frac{R}{LS^2 + RS + \frac{1}{C}}$$

$$V_L(S) = \frac{LS\, V_S(S)}{R + LS + \frac{1}{CS}} = \frac{LCS}{LCS^2 + RCS + 1} = \frac{LS}{LS^2 + RS + \frac{1}{C}}$$

$$V_C(S) = \frac{\frac{1}{CS} V_S(S)}{R + LS + \frac{1}{CS}} = \frac{1}{CLS^3 + CRS^2 + S} = \frac{\frac{1}{CS}}{LS^2 + RS + \frac{1}{C}}$$

(b) $V_R(S)$: two poles

$V_L(S)$: one zero & two poles

$V_C(S)$: three poles

Note that the denominator is the same for all V_R, V_L, and V_C

$\Rightarrow$ the two poles are the same for all V_R, V_L, V_C. And the zero for V_L is due to impedance of L and pole for V_C is due to impedance of C.

(c)

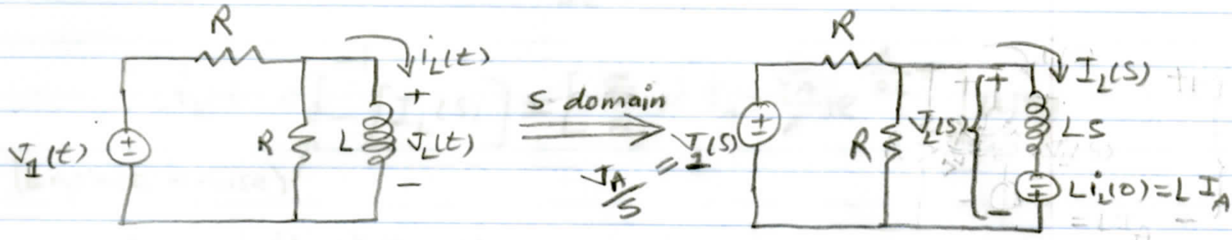
$$R: \begin{cases} \lim_{t\to 0^+} v_R(t) = \lim_{s\to\infty} s V_R(s) = 0 \text{ V, initial value} \\ \lim_{t\to\infty} v_R(t) = \lim_{s\to 0} s V_R(s) = 0 \text{ V, final value} \end{cases}$$

$$L: \begin{cases} \lim_{t\to 0^+} V_L(t) = \lim_{s\to\infty} s V_L(s) = 1 \text{ V, initial value} \\ \lim_{t\to\infty} V_L(t) = \lim_{s\to 0} s V_L(s) = 0 \text{ V, final value} \end{cases}$$

$$C: \begin{cases} \lim_{t\to 0^+} v_C(t) = \lim_{s\to\infty} s V_C(s) = 0 \text{ V, initial value} \\ \lim_{t\to\infty} v_C(t) = \lim_{s\to 0} s V_C(s) = 1 \text{ V, final value} \end{cases}$$

Because the input voltage is $v_S(t) = u(t)$, therefore at $t=0$, the whole input voltage drops across L. But at $t=\infty$, the whole input voltage drops across C. At $t=0$ and $t=\infty$, there is no voltage drop across R.

10-23

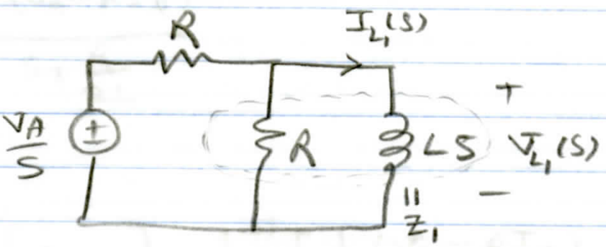


$v_1(t) = V_A u(t)$

$i_L(0) = I_A$

Using superposition as:

(I) turn off "LI_A"



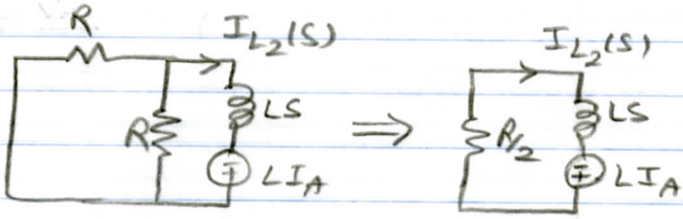
$$I_{L_1}(S) = \frac{V_{L_1}(S)}{LS}$$

voltage division: $V_{L_1}(S) = \dfrac{Z_1\left(\frac{V_A}{S}\right)}{R+Z_1}$

$$Z_1 = \frac{LRS}{R+LS}$$

$$\Rightarrow I_{L_1}(S) = \frac{V_A}{S(R+2LS)} = \frac{V_A}{2L}\left(\frac{1}{S(S+\frac{R}{2L})}\right)$$

(II) turn off "$\frac{V_A}{S}$"



$$\Rightarrow I_{L_2}(S) = \frac{LI_A}{\frac{R}{2}+LS} = \frac{I_A}{S+\frac{R}{2L}}$$

$$I_L(S) = I_{L_1}(S) + I_{L_2}(S) = \frac{V_A}{2L}\left(\frac{1}{S(S+\frac{R}{2L})}\right) + \frac{I_A}{S+\frac{R}{2L}}$$

$$\Rightarrow I_L(s) = \frac{\frac{V_A}{R}}{s} + \frac{I_A - \frac{V_A}{R}}{s + \frac{R}{2L}}$$

$$i_L(t) = \mathcal{L}^{-1}\left[I_L(s)\right] = \left[\frac{V_A}{R} + \left(I_A - \frac{V_A}{R}\right)e^{-\frac{R}{2L}t}\right]u(t)$$

(Laplace inverse)

$$V_L(s) = (Ls)I_L(s) - LI_A$$

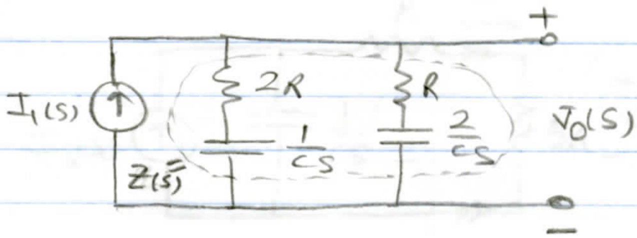
$$= Ls\left[\frac{\frac{V_A}{R}}{s} + \frac{I_A - \frac{V_A}{R}}{s + \frac{R}{2L}}\right] - LI_A$$

$$= \frac{\frac{1}{2}(V_A - RI_A)}{s + \frac{R}{2L}}$$

$$V_L(t) = \mathcal{L}^{-1}\left[V_L(s)\right] = \mathcal{L}^{-1}\left[\frac{\frac{1}{2}(V_A - RI_A)}{s + \frac{R}{2L}}\right]$$

$$\Rightarrow V_L(t) = \frac{1}{2}(V_A - RI_A)e^{-\frac{R}{2L}t}\,u(t)$$

10 - 28



$$V_0(s) = Z(s) I_1(s) \Rightarrow \frac{V_0(s)}{I_1(s)} = Z(s)$$

$$Z(s) = \left(2R + \frac{1}{cs}\right) \| \left(R + \frac{2}{cs}\right)$$

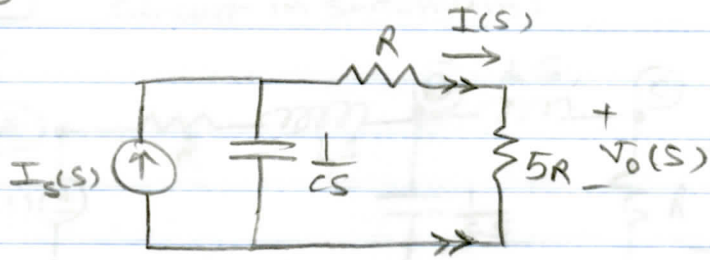
$$= \frac{\left(2R + \frac{1}{cs}\right)\left(R + \frac{2}{cs}\right)}{2R + \frac{1}{cs} + R + \frac{2}{cs}}$$

$$\Rightarrow Z(s) = \frac{(2Rcs + 1)(Rcs + 2)}{3cs(Rcs + 1)}$$

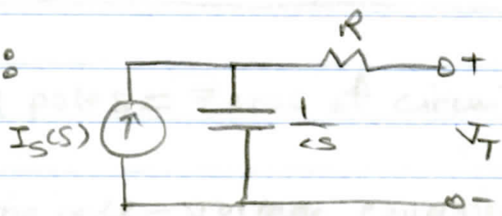
zeros: $\begin{cases} 2Rcs + 1 = 0 \Rightarrow s = \frac{-1}{2Rc} \\ Rcs + 2 = 0 \Rightarrow s = \frac{-2}{Rc} \end{cases}$

poles: $\begin{cases} cs = 0 \Rightarrow s = 0 \\ Rcs + 1 = 0 \Rightarrow s = -\frac{1}{Rc} \end{cases}$

$\boxed{10-34}$



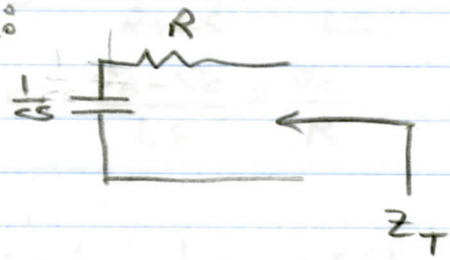
V_T:



$V_T = V_{OC}$

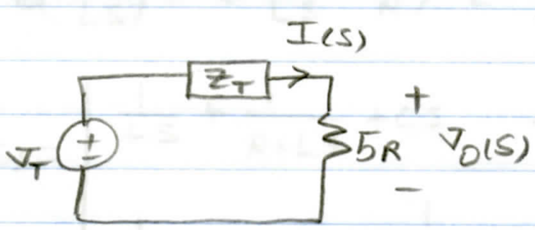
$$V_T = V_{OC} = \frac{I_s(S)}{CS}$$

Z_T:



$$Z_T = R + \frac{1}{CS} = \frac{RCS+1}{CS}$$

$\Rightarrow$

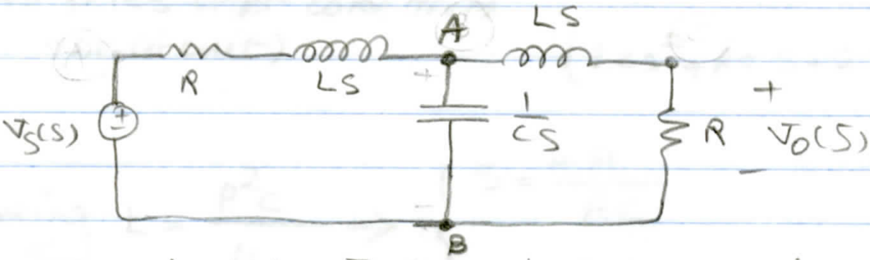


$$I(S) = \frac{V_T}{Z_T + 5R} \Rightarrow I(S) = \frac{I_s(S)}{cs\left(\frac{RCS+1}{cs} + 5R\right)}$$

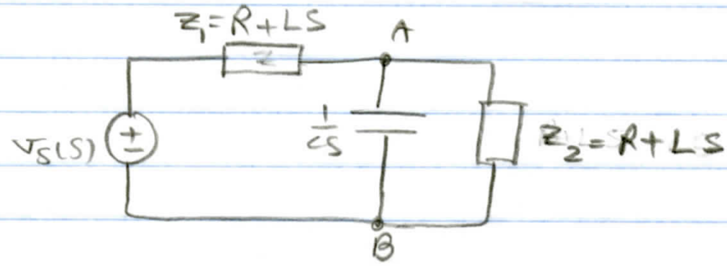
$$\Rightarrow \frac{I(S)}{I_s(S)} = \frac{1}{6RCS+1}$$

10 – 62

Circuit in S-domain:



First, we calculate $V_o(S)$ using circuit reduction and voltage division as:

⇒


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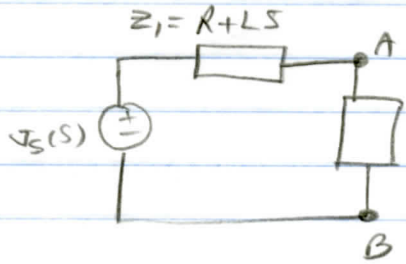


$$Z_3 = \underbrace{(R+LS)}_{Z_2} \,||\, \frac{1}{CS} = \frac{(LS+R)\frac{1}{CS}}{\frac{1}{CS}+LS+R}$$

$$= \frac{LS+R}{1+RCS+LCS^2}$$

Using voltage division: $V_{AB} = \dfrac{Z_3 V_s(S)}{Z_3 + Z_1}$

inserting Z_1 and Z_3 $\Rightarrow V_{AB} = \dfrac{V_S(S)}{2+RCS+LCS^2}$

using another voltage division:

$$V_o(S) = \frac{R}{LS+R} V_{AB} \xrightarrow[V_{AB}]{\text{inserting}} V_o(S) = \frac{R V_S(S)}{(LS+R)(LCS^2+RCS+2)}$$

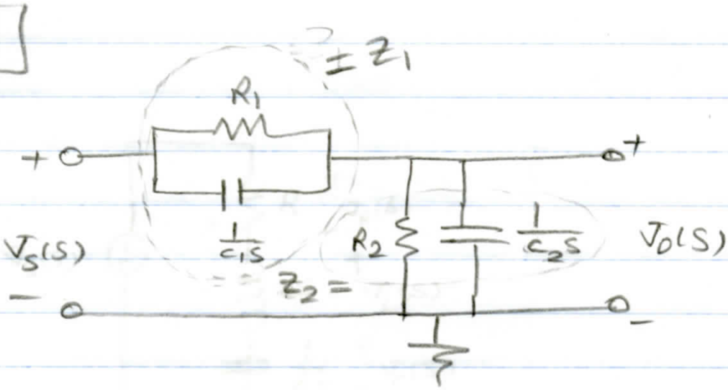
Natural poles for $V_o(s)$:
(the poles that come from the circuit)

$$\begin{cases} Ls + R = 0 \\ LCs^2 + RCs + 2 = 0 \end{cases}$$

inserting $L = \frac{R^2 C}{4} \Rightarrow \begin{cases} s = \frac{-4}{RC} \\ s = \frac{2}{RC}(-1 \pm j) \end{cases}$

10-67

a



$$Z_1 = R_1 \| \frac{1}{C_1S} = \frac{R_1 \frac{1}{C_1S}}{R_1 + \frac{1}{C_1S}} = \frac{R_1}{R_1C_1S+1}$$

$$Z_2 = R_2 \| \frac{1}{C_2S} = \frac{R_2 \frac{1}{C_2S}}{R_2 + \frac{1}{C_2S}} = \frac{R_2}{R_2C_2S+1}$$

voltage division:

$$V_o(S) = \frac{Z_2 V_s(S)}{Z_1 + Z_2} = \frac{R_2(R_1C_1S+1)V_s(S)}{R_1(C_2R_2S+1) + R_2(C_1R_1S+1)}$$

$$\Rightarrow \frac{V_o(S)}{V_s(S)} = \frac{R_1R_2C_1S + R_2}{(R_1R_2C_2 + R_1R_2C_1)S + R_1 + R_2}$$

b $R_2 = 15\,M\Omega$

$C_2 = 3\,pF$

$$\frac{V_o(S)}{V_s(S)} = \frac{1}{2} \Rightarrow \frac{R_1R_2C_1S + R_2}{(R_1R_2C_2 + R_1R_2C_1)S + R_1 + R_2} = \frac{1}{2}$$

Text

$$\Rightarrow R_1R_2C_2S + R_1 = R_1R_2C_1S + R_2$$

$$\Rightarrow \left.\begin{array}{l} R_1 = R_2 \\ C_1 = C_2 \end{array}\right\} \Rightarrow \begin{array}{l} R_1 = 15\,M\Omega \\ C_1 = 3\,pF \end{array}$$