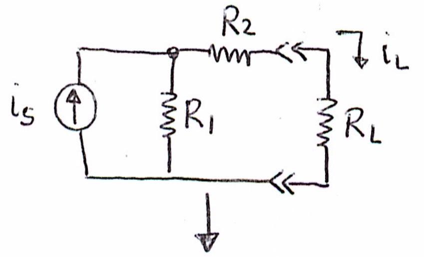
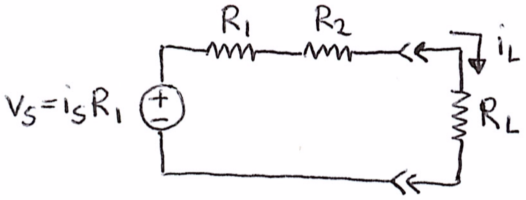
MAE 140 fall 2014
Homework 2 solution

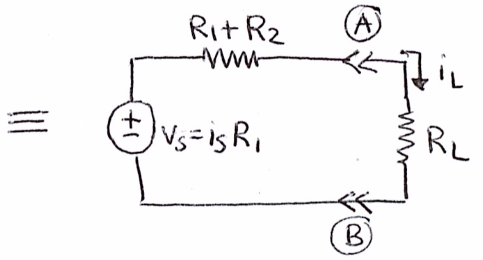
3.50 3.51 3.56 3.3 3.4 3.7 3.11 3.14 3.22 3.24

3.50 (a) Find the Thévenin or Norton equivalent circuit seen by R_L

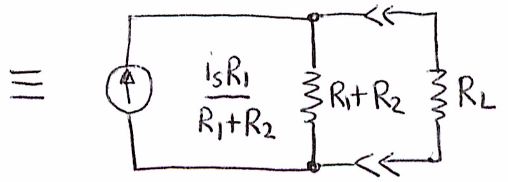


Use source transformation to transform the current source into a voltage source, i.e.





Thevenin equivalent circuit



Norton equivalent circuit

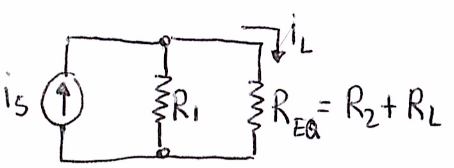
(b) Use (a) to find i_L in terms of R_1, R_2 and R_L
Using the Thévenin equivalent circuit from (a) and KVL we can write

$$-v_S + i_L(R_1+R_2) + i_L R_L = 0$$

$$-i_S R_1 + i_L(R_1+R_2+R_L) = 0$$

$$\boxed{i_L = \frac{R_1}{R_1+R_2+R_L}\, i_S}$$

(c) Check your answer in (b) using current division.



$$i_L = \frac{G_{eq}}{G_1+G_{eq}}\, i_S = \frac{\frac{1}{R_{eq}}}{\frac{1}{R_1}+\frac{1}{R_{eq}}}\, i_S = \frac{R_1}{R_1+R_{eq}}\, i_S$$

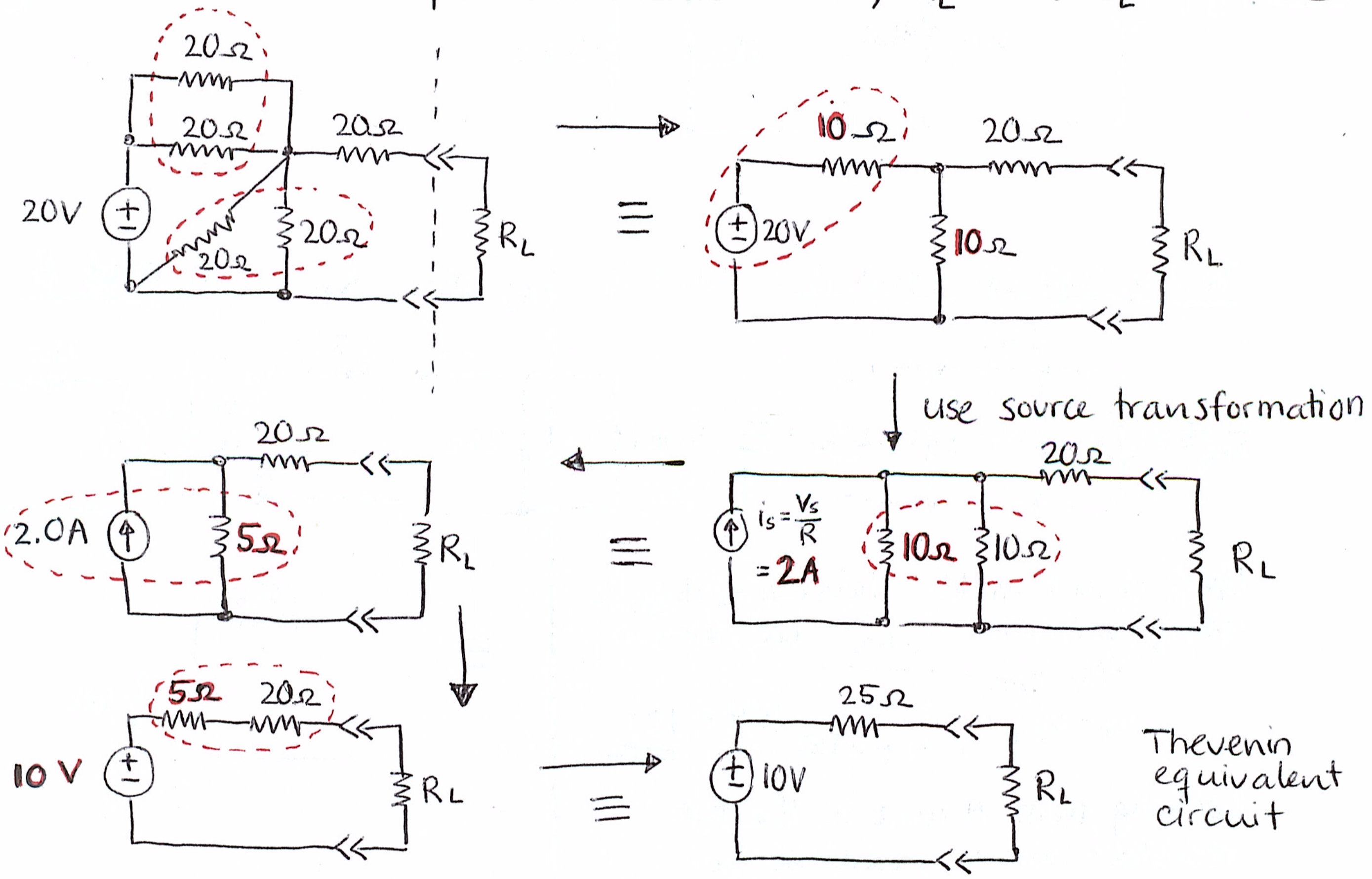
$$\boxed{i_L = \frac{R_1}{R_1+R_2+R_L}\, i_S}$$

Note: You can solve for i_L directly using the following rule

$$i_L = i_S \frac{R_1}{R_1+R_2+R_3}$$

← resistance of the 'other' path
← total resistance of the loop

$\boxed{3.51}$ Find Thevenin equivalent circuit seen by R_L and V_L

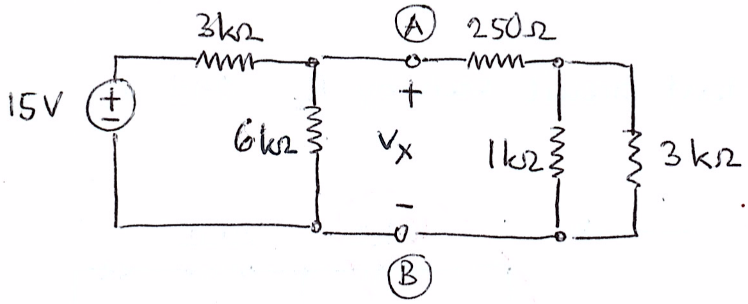


Use voltage divider to find voltage across R_L

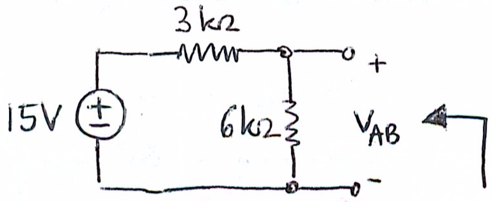
$$V_L = \frac{R_L}{R_L + 25\Omega} 10V$$

$$V_L\ (R_L = 5\Omega) = \boxed{1.67\ V}$$
$$V_L\ (R_L = 10\Omega) = \boxed{2,86\ V}$$
$$V_L\ (R_L = 20\Omega) = \boxed{4.44\ V}$$

3.56



To the left of A and B



R_T ?
V_T ?

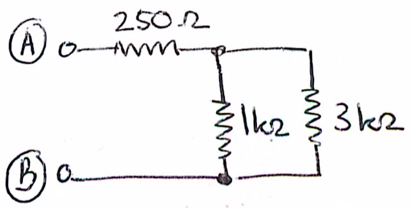
Open circuit yields Thévenin voltage, so using voltage division we obtain

$$V_T = V_{AB} = \frac{6\,k\Omega}{3k\Omega + 6\,k\Omega} 15V = \underline{\underline{10\,V}}$$

Looking from A and B, the two resistors are in parallel so

$$R_T = 3k\Omega \| 6\,k\Omega = \frac{3k\Omega \cdot 6\,k\Omega}{(3+6)\,k\Omega} = \underline{\underline{2\,k\Omega}}$$

To the right of A and B



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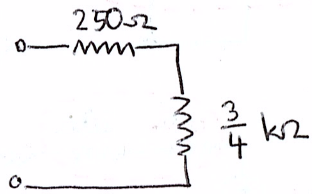

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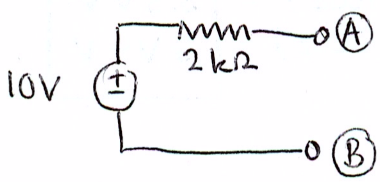
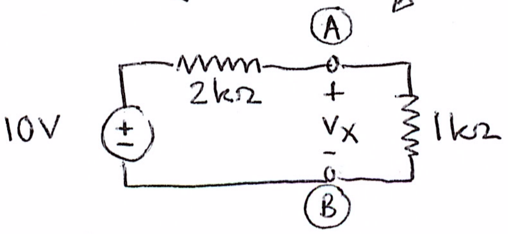
$\underline{\underline{1k\Omega}} = R_T$

$\underline{\underline{V_T = 0\,V}}$ (no sources

Thus, we have for the Norton equ. circuit



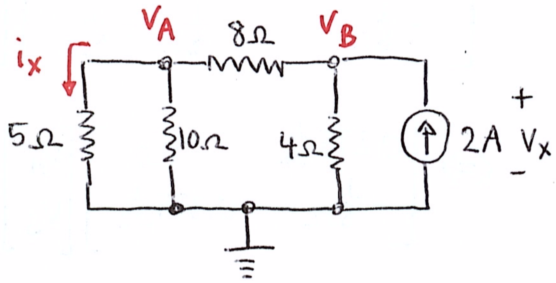
connecting both



Using voltage division we have $V_X = \dfrac{1k\Omega}{1k\Omega + 2k\Omega} 10V$

$$\boxed{V_X = 3.33\,V}$$

3.3 a) Node voltage equation



By inspection we can write

$$\begin{bmatrix} \frac{1}{5\Omega}+\frac{1}{10\Omega}+\frac{1}{8\Omega} & -\frac{1}{8\Omega} \\ -\frac{1}{8\Omega} & \frac{1}{8\Omega}+\frac{1}{4\Omega} \end{bmatrix}\begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 0 \\ 2A \end{bmatrix} \iff \underbrace{\begin{bmatrix} 0.425\frac{1}{\Omega} & -\frac{1}{8\Omega} \\ -\frac{1}{8\Omega} & 0.375\frac{1}{\Omega} \end{bmatrix}}_{A}\underbrace{\begin{bmatrix} V_A \\ V_B \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 0 \\ 2A \end{bmatrix}}_{b}$$

b) Solve for V_A and V_B. Using Cramer's rule we have

$$V_A = \frac{\begin{vmatrix} 0 & -\frac{1}{8\Omega} \\ 2A & \frac{3}{8\Omega} \end{vmatrix}}{|A|} = \frac{0.25 \ \frac{A^2}{V}}{0.14375 \ \frac{A^2}{V^2}} = \boxed{1.74\,V}$$

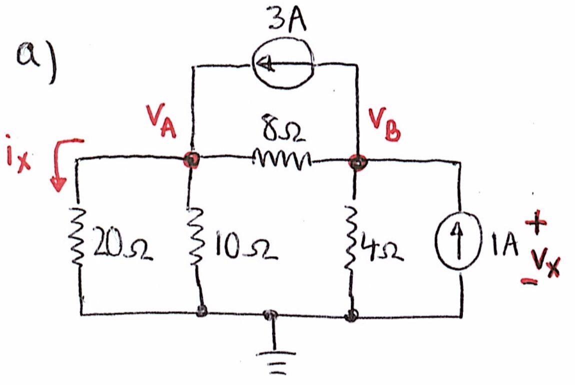
$$V_B = \frac{\begin{vmatrix} 0.425\frac{1}{\Omega} & 0 \\ -\frac{1}{8\Omega} & 2A \end{vmatrix}}{|A|} = \frac{0.85 \ \frac{A^2}{V}}{0.14375 \ \frac{A^2}{V^2}} = \boxed{5.91\,V}$$

c) Find i_x and V_x

$$i_x = \frac{V_A}{5\Omega} = \boxed{347.8\,mA}$$

$$V_x = V_B = \boxed{5.91\,V}$$

3.4 a)



$$\begin{bmatrix} \frac{1}{20\Omega}+\frac{1}{10\Omega}+\frac{1}{8\Omega} & -\frac{1}{8\Omega} \\ -\frac{1}{8\Omega} & \frac{1}{8\Omega}+\frac{1}{4\Omega} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 3A \\ 1A-3A \end{bmatrix}$$

$$\Leftrightarrow \begin{bmatrix} 0.275\frac{1}{\Omega} & -\frac{1}{8\Omega} \\ -\frac{1}{8\Omega} & 0.375\frac{1}{\Omega} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} 3A \\ -2A \end{bmatrix}$$

b)

$$V_A = \frac{\begin{vmatrix} 3A & -\frac{1}{8\Omega} \\ -2A & \frac{3}{8\Omega} \end{vmatrix}}{|A|} = \frac{0.875\ \frac{A^2}{V}}{0.0875\ \frac{A^2}{V^2}} = \boxed{10\ V}$$

$$V_B = \frac{\begin{vmatrix} 0.275\frac{1}{\Omega} & 3A \\ -\frac{1}{8\Omega} & -2A \end{vmatrix}}{|A|} = \frac{-0.175\ \frac{A^2}{V}}{0.0875\ \frac{A^2}{V^2}} = \boxed{-2V}$$

c) Find i_x and V_x.

$$i_x = \frac{V_A}{20\Omega} = \boxed{0.5\ A}$$

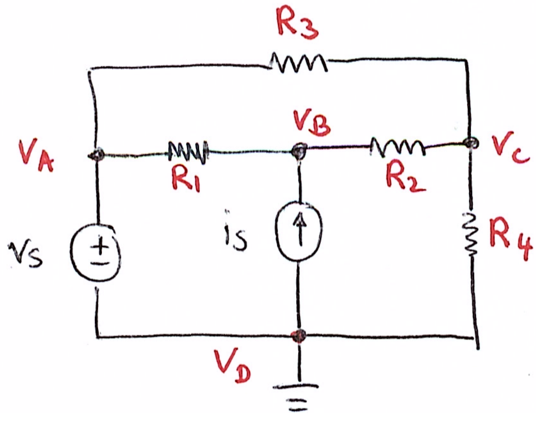
$$V_x = V_B = \boxed{-2V}$$

3.7 Draw the circuit that is represented by the following equations.

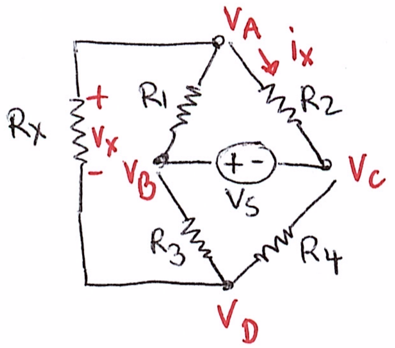
$$\begin{bmatrix} 1 & 0 & 0 \\ \left(-\frac{1}{R_1}\right) & \left(\frac{1}{R_1}+\frac{1}{R_2}\right) & \left(-\frac{1}{R_2}\right) \\ \left(-\frac{1}{R_3}\right) & \left(-\frac{1}{R_2}\right) & \left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right) \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} V_s \\ i_s \\ 0 \end{bmatrix}$$

means that the circuit contains a voltage source between node A and GND

$V_D = 0$ (GND!)



3.11 a) Formulate node-voltage equations



Since the circuit does not have a ground designated, we choose node C as our ground. Thus, $V_B = V_S$.
Using method 2 (see book p.85) we can write by inspection

$$\begin{array}{c|c} & \text{node A} \quad\quad \text{node B} \quad\quad \text{node D} \\ \begin{array}{c}\text{node A}\\ \text{node D}\end{array} & \begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_X} & -\frac{1}{R_1} & -\frac{1}{R_X} \\ -\frac{1}{R_X} & -\frac{1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_X} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{array}$$

and $V_B = V_S$

This can be further reduced to

$$\underbrace{\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_X} & -\frac{1}{R_X} \\ -\frac{1}{R_X} & \frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_X} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} V_A \\ V_D \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} \frac{V_S}{R_1} \\ \frac{V_S}{R_3} \end{bmatrix}}_{b}$$

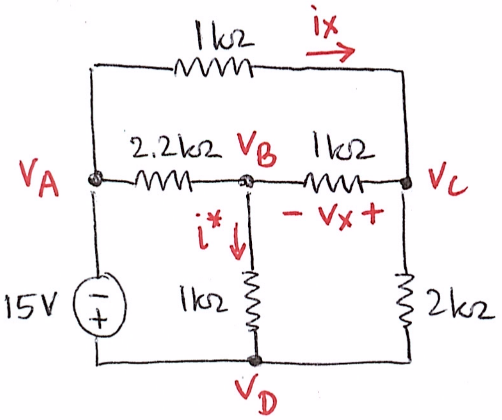
b) Solve for v_x and i_x.
First we solve for V_A and V_D using Cramer's rule or matrix inversion and obtain

$$V_A = 5.3266\ V$$
$$V_B = 9.6774\ V$$

$$i_x = \frac{V_A}{R_2} = \boxed{10.6452\ mA}$$

$$V_x = V_A - V_B = \boxed{-4.3548\ V}$$

3.14 a) Formulate node-voltage equations



We choose node A as our ground and thus $V_D = +15\,V$. Using method 2 (see book page 85) we can write by inspection

$$\begin{array}{cc} & \begin{array}{ccc} \text{node B} & \text{node C} & \text{node D} \end{array} \\ \begin{array}{c} \text{node B} \\ \text{node C} \end{array} & \begin{bmatrix} \frac{1}{2.2k}+\frac{1}{1k}+\frac{1}{1k} & -\frac{1}{1k} & -\frac{1}{1k} \\ -\frac{1}{1k} & \frac{1}{1k}+\frac{1}{1k}+\frac{1}{2k} & -\frac{1}{2k} \end{bmatrix} \end{array} \begin{bmatrix} V_B \\ V_C \\ V_D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and $V_D = 15V$

Rearranging yields

$$\underbrace{\begin{bmatrix} \frac{1}{2.2k}+\frac{1}{1k}+\frac{1}{1k} & -\frac{1}{1k} \\ -\frac{1}{1k} & \frac{1}{1k}+\frac{1}{1k}+\frac{1}{2k} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} V_B \\ V_C \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} \frac{V_D}{1k} \\ \frac{V_D}{2k} \end{bmatrix}}_{b}$$

b) Solve for i_A, i_B and i_C.
First, we solve for V_B and V_C using matrix inversion ($x = A^{-1}b$)

$$V_B = 8.7611\ V$$
$$V_C = 6.5044\ V$$

$$i_C = i_X = \frac{V_A - V_C}{1k\Omega} = \frac{-6.5044V}{1000\ \frac{V}{A}} = \boxed{-6.5044\ mA}$$

$$i_B = \frac{V_C - V_D}{2k\Omega} = \frac{6.5044V - 15V}{2000\ \frac{V}{A}} = \boxed{-4.25\ mA}$$

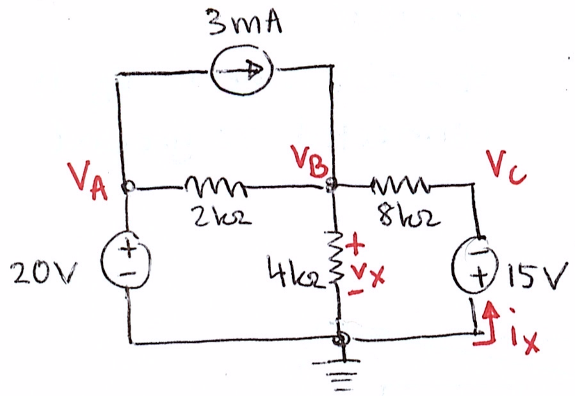
$$i^* = \frac{V_B - V_D}{1k\Omega} = i_A - i_B \rightarrow i_A = i_B + \frac{8.7611V - 15V}{1000\ \frac{V}{A}} = \boxed{-10.4889\ mA}$$

c) Find V_X and i_X

$$i_X = i_C = \boxed{-6.5044\ mA}$$

$$V_X = V_C - V_B = \boxed{-2.2567\ V}$$

$\boxed{3.22}$ b) formulate node-voltage equations



Because of the two voltage sources in the circuit we know that

$\boxed{V_A = 20V}$ and $\boxed{V_C = -15V}$

Thus, we can write by inspection

$$\begin{array}{cc} & \begin{array}{ccc} \text{node A} & \text{node B} & \text{node C} \end{array} \\ \text{node B} & \left[\begin{array}{ccc} -\frac{1}{2k\Omega} & \frac{1}{2k\Omega}+\frac{1}{4k\Omega}+\frac{1}{8k\Omega} & -\frac{1}{8k\Omega} \end{array} \right] \end{array} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = 3mA$$

with $V_A = 20V$
$V_C = -15V$

Rearranging yields

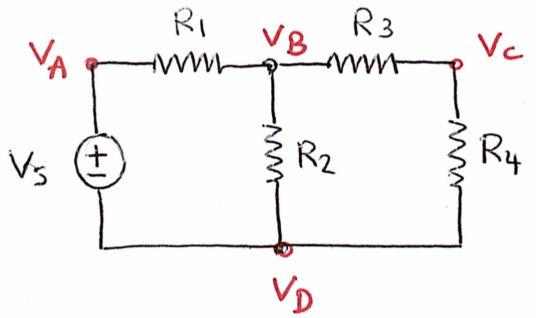
$$\left[\frac{1}{2k\Omega} + \frac{1}{4k\Omega} + \frac{1}{8k\Omega} \right] V_B = 3mA + \frac{V_A}{2k\Omega} + \frac{V_C}{8k\Omega}$$

d) Find v_x and i_x.
First we have to solve for V_B: $\boxed{V_B = 12.7143\ V}$

$$V_x = V_B = \boxed{12.7143\ V}$$

$$i_x = \frac{V_C - V_B}{8k\Omega} = \frac{-15V - 12.7143V}{8000\ \frac{V}{A}} = \boxed{-3.5\ mA}$$

3.24



$R_1 = R_2 = R_3 = R_4 = 1\,k\Omega$

$V_S = 10\,V$

$V_C = -2V$ when V_B is connected to ground

Note that we already know the voltage at node C [$V_C = -2V$] and that $V_A - V_D = V_S = 10\,V$.

Thus, we can write by inspection (using method 3) ↳ 'supernode'

$$\begin{array}{cc} & \begin{array}{ccc} \text{node A} & \text{node C} & \text{node D} \end{array} \\ \text{node C} & \left[\begin{array}{ccc} 0 & \frac{1}{R_3}+\frac{1}{R_4} & -\frac{1}{R_4} \end{array}\right] \end{array} \begin{bmatrix} V_A \\ V_C \\ V_D \end{bmatrix} = 0$$

$$V_A - V_D = V_S$$

$$V_C = -2\,V$$

Rearranging yields

$$\left[-\frac{1}{R_4}\right] V_D = -\left(\frac{1}{R_3} + \frac{1}{R_4}\right) V_C$$

$$V_D = \frac{0.002}{0.001}(-2V)$$

$$\boxed{V_D = -4V}$$

$$V_A = V_D + V_S = -4V + 10\,V$$

$$\boxed{V_A = 6\,V}$$