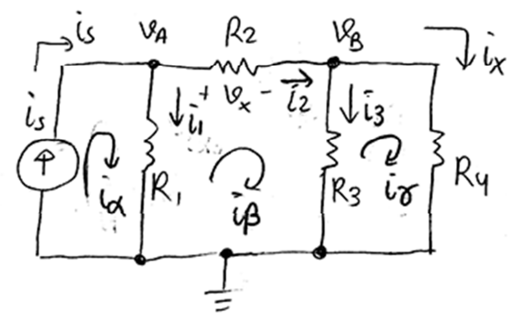
MAE 140 Hw 4 Solutions

3-2



a) Formulate mesh current to get $A\vec{x} = \vec{b}$ form!

Using mesh current analysis, we can get :

$$\boxed{\begin{aligned} i_s &= i_\alpha \\ i_1 &= i_\alpha - i_\beta \\ i_2 &= i_\beta \\ i_3 &= i_\beta - i_\gamma \\ i_x &= i_\gamma \end{aligned}}$$

Using KVL , we can get :

$$\boxed{\begin{aligned} KVL\, i_\beta &\rightarrow -i_1 R_1 + i_2 R_2 + i_3 R_3 = 0 \\ KVL\, i_\gamma &\rightarrow -i_3 R_3 + i_x R_4 = 0 \end{aligned}}$$

Combining both, we can get :

$$\left.\begin{aligned} i_\alpha &= i_s \\ -(i_\alpha - i_\beta)R_1 + i_\beta R_2 + (i_\beta - i_\gamma) R_3 &= 0 \\ -(i_\beta - i_\gamma) R_3 + i_\gamma R_4 &= 0 \end{aligned}\right\} \rightarrow \begin{bmatrix} 1 & 0 & 0 \\ -R_1 & R_1+R_2+R_3 & -R_3 \\ 0 & -R_3 & R_3+R_4 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_\gamma \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \\ 0 \end{bmatrix}$$

$$\hookrightarrow A\vec{x} = \vec{b}$$

b) Solve for v_A and v_B!

Using $i_\alpha = i_s$, we can reduce the matrix into =

$$\begin{bmatrix} R_1+R_2+R_3 & -R_3 \\ -R_3 & R_3+R_4 \end{bmatrix} \begin{bmatrix} i_\beta \\ i_\gamma \end{bmatrix} = \begin{bmatrix} R_1 i_s \\ 0 \end{bmatrix} \rightarrow A_2 \vec{x_2} = \vec{b_2} \rightarrow$$

Solving for $\vec{x_2} = A_2^{-1} \vec{b_2}$, we can get :

$$\begin{bmatrix} i_\beta \\ i_\gamma \end{bmatrix} = \frac{1}{(R_1+R_2+R_3)(R_3+R_4) - R_3^2} \begin{bmatrix} R_3+R_4 & R_3 \\ R_3 & R_1+R_2+R_3 \end{bmatrix} \begin{bmatrix} R_1 i_s \\ 0 \end{bmatrix}$$

$$\boxed{\hookrightarrow \begin{bmatrix} i_\beta \\ i_\gamma \end{bmatrix} = \frac{i_s}{(R_1+R_2)(R_3+R_4)+R_3R_4} \begin{bmatrix} R_1(R_3+R_4) \\ R_1R_3 \end{bmatrix}}$$

Then $v_A = i_1 R_1 = (i_\alpha - i_\beta) R_1$

$$\boxed{v_A = i_s \left(1 - \frac{R_1(R_3+R_4)}{(R_1+R_2)(R_3+R_4)+R_3R_4}\right) R_1 = \frac{R_2(R_3+R_4)+R_3R_4}{(R_1+R_2)(R_3+R_4)+R_3R_4} R_1 i_s}$$

$$v_B = i_x R_4 = i_\gamma R_4 \rightarrow \boxed{v_B = \frac{i_s R_1 R_3 R_4}{(R_1+R_2)(R_3+R_4)+R_3R_4}}$$

c) Solve for v_x and i_x!

$$\boxed{v_x = v_A - v_B = i_s \left[\frac{R_1R_2(R_3+R_4)}{(R_1+R_2)(R_3+R_4)+R_3R_4}\right]}$$

$$\boxed{i_x = i_\gamma = \frac{i_s R_1 R_3}{(R_1+R_2)(R_3+R_4)+R_3R_4}}$$

3-4

a) Formulate mesh current!

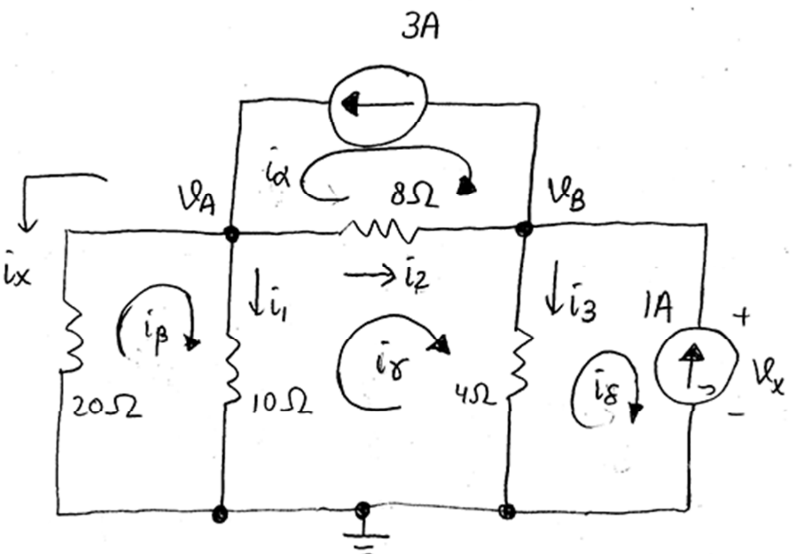
$i_x = -i_\beta \qquad 1A = -i_\delta$

$3A = -i_\alpha$

$i_1 = i_\beta - i_\gamma$

$i_2 = i_\gamma - i_\alpha = i_\gamma + 3A$

$i_3 = i_\gamma - i_\beta = i_\gamma + 1A$



$\rightarrow$ Unknowns: i_β, i_γ

KVL $i_\beta \rightarrow$ $i_1(10\Omega) - i_x(20\Omega) = 0$

$(i_\beta - i_\gamma)(10\Omega) + i_\beta(20\Omega) = 0 \rightarrow (20\Omega + 10\Omega)\, i_\beta - (10\Omega)\, i_\gamma = 0$

KVL $i_\gamma \rightarrow$ $i_3(4\Omega) + i_2(8\Omega) - i_1(10\Omega) = 0$

$(i_\gamma + 1A)(4\Omega) + (i_\gamma + 3A)(8\Omega) - (i_\beta - i_\gamma)(10\Omega) = 0$

$(-10\Omega)\, i_\beta + (4\Omega + 8\Omega + 10\Omega)\, i_\gamma = -4V - 24V$

So, we have :

$$\boxed{\begin{bmatrix} 30\Omega & -10\Omega \\ -10\Omega & 22\Omega \end{bmatrix} \begin{bmatrix} i_\beta \\ i_\gamma \end{bmatrix} = \begin{bmatrix} 0 \\ -28V \end{bmatrix}} \rightarrow \boxed{\begin{bmatrix} i_\beta \\ i_\gamma \end{bmatrix} = \begin{bmatrix} -0.5A \\ -1.5A \end{bmatrix}}$$

b) Solve for v_A and v_B !

$v_A = (-i_\beta)(20\Omega) = (0.5A)(20\Omega) = \underline{\underline{10V}}$

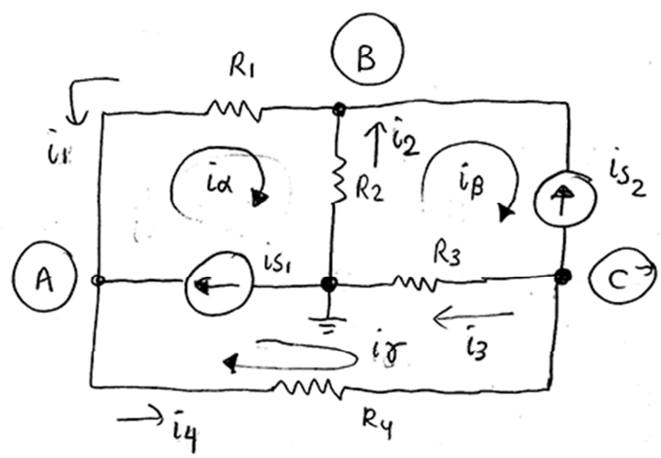
$v_B = i_3(4\Omega) = (1A - 1.5A)\, 4\Omega = \underline{\underline{-2V}}$

c) Solve for v_x and i_x !

$\boxed{v_x = v_B = -2V}$ $\boxed{i_x = -i_\beta = 0.5A}$

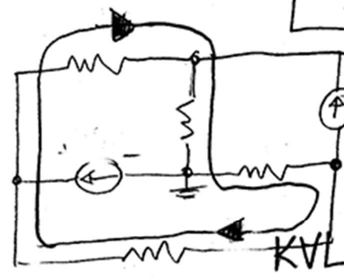
3-9

a) Formulate mesh current !



$$\boxed{\begin{aligned} i_{s1} &= i_\alpha - i_\gamma \qquad i_4 = -i_\gamma \\ i_{s2} &= -i_\beta \\ i_1 &= -i_\alpha \\ i_2 &= i_\beta - i_\alpha \\ i_3 &= i_\beta - i_\gamma \end{aligned}}$$

You might want to avoid the current sources for the KVL



$\hookrightarrow -(i_4 R_4 + i_3 R_3 + i_2 R_2 + i_1 R_1) = 0$

$(i_\alpha) R_1 + (i_\alpha - i_\beta) R_2 + (i_\gamma - i_\beta) R_3 + (i_\gamma) R_4 = 0$

So, we have :

$$\boxed{\begin{bmatrix} +1 & 0 & -1 \\ 0 & -1 & 0 \\ R_1+R_2 & -R_2-R_3 & R_3+R_4 \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_\gamma \end{bmatrix} = \begin{bmatrix} i_{s_1} \\ i_{s_2} \\ 0 \end{bmatrix}}$$

b) Solve symbolically with Matlab for the node voltages!

$$\hookrightarrow \boxed{\begin{bmatrix} i_\alpha \\ i_\beta \\ i_\gamma \end{bmatrix} = \begin{bmatrix} \dfrac{-i_{s_2}(R_2+R_3) + i_{s_1}(R_3+R_4)}{R_1+R_2+R_3+R_4} \\ -i_{s_2} \\ \dfrac{-i_{s_1}(R_1+R_2) + i_{s_2}(R_2+R_3)}{R_1+R_2+R_3+R_4} \end{bmatrix}}$$

KVL $i_\alpha \rightarrow -v_B + i_1 R_1 + v_A = 0 \rightarrow v_A = v_B + i_\alpha R_1$

$v_B = -i_2 R_2 = (i_\alpha - i_\beta) R_2$ $\Big\}$ $v_A = i_\alpha (R_1+R_2) - i_\beta R_2$

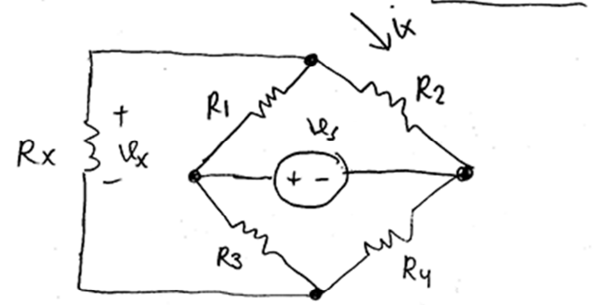
$v_C = i_3 R_3 = (i_\beta - i_\gamma) R_3$

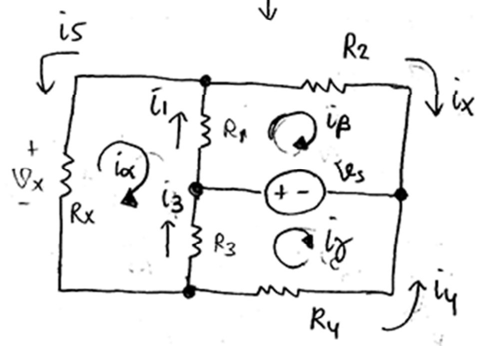
$$\hookrightarrow \boxed{\begin{aligned} v_A &= (R_1+R_2)\, i_\alpha - R_2\, i_\beta \\ v_B &= R_2 (i_\alpha - i_\beta) \\ v_C &= R_3 (i_\beta - i_\gamma) \end{aligned}}$$

c) Solve for v_A, v_B, v_C using $R_1 = 1.5k\Omega$, $R_2 = 2.2k\Omega$, $R_3 = 3.3k\Omega$, $R_4 = 4.7k\Omega$, $i_{s_1} = i_{s_2} = 2mA$!

$$\boxed{\begin{aligned} i_\alpha &= 0.427\ mA \\ i_\beta &= -2\ mA \\ i_\gamma &= -1.6\ mA \end{aligned}} \longrightarrow \boxed{\begin{aligned} v_A &= 5.9812\ V \\ v_B &= 5.3402\ V \\ v_C &= -1.4103\ V \end{aligned}}$$

3-11





a) Formulate mesh-current !

$i_1 = i_\beta - i_\alpha \qquad i_4 = -i_\gamma$

$i_x = i_\beta \qquad i_5 = -i_\alpha$

$i_3 = i_\gamma - i_\alpha$

KVL $i_\alpha \rightarrow -i_1R_1 - i_3R_3 - i_5R_x = 0$

$(i_\alpha - i_\beta)R_1 + (i_\alpha - i_\gamma)R_3 + i_\alpha R_x = 0$

KVL $i_\beta \rightarrow +i_xR_2 + i_1R_1 - v_s = 0$

$i_\beta R_2 + (i_\beta - i_\alpha)R_1 = +v_s$

KVL $i_\gamma \rightarrow +i_3R_3 + i_4R_4 + v_s = 0$

$(i_\gamma - i_\alpha)R_3 + i_\gamma R_4 = -v_s$

$$\boxed{\begin{bmatrix} R_1+R_3+R_x & -R_1 & -R_3 \\ -R_1 & R_2+R_1 & 0 \\ -R_3 & 0 & R_3+R_4 \end{bmatrix}\begin{bmatrix} i_\alpha \\ i_\beta \\ i_\gamma \end{bmatrix} = \begin{bmatrix} 0 \\ +v_s \\ -v_s \end{bmatrix}}$$

b) Solve for v_x, i_x using $R_1 = R_4 = 2k\Omega$, $R_2 = R_3 = 500\Omega$, $R_x = 750\Omega$, $v_s = 15V$!

Matrix becomes :

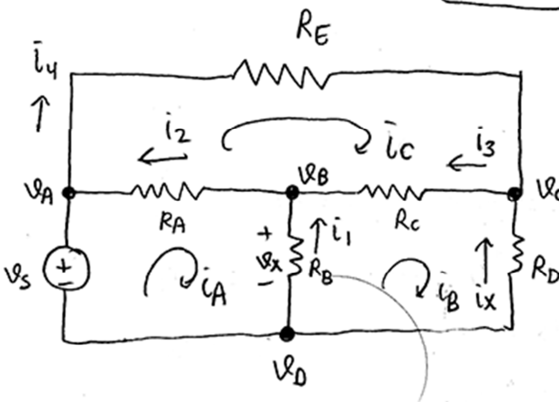
(with Matlab)

$$\begin{bmatrix} 3250\Omega & -2000\Omega & -500\Omega \\ -2000\Omega & 2500\Omega & 0 \\ -500\Omega & 0 & 2500\Omega \end{bmatrix}\begin{bmatrix} i_A \\ i_B \\ i_c \end{bmatrix} = \begin{bmatrix} 0 \\ 15V \\ -15V \end{bmatrix} \rightarrow \boxed{\begin{bmatrix} i_A \\ i_B \\ i_c \end{bmatrix} = \begin{bmatrix} 5.8\ mA \\ 10.6\ mA \\ -4.8\ mA \end{bmatrix}}$$

$v_x = i_5R_x = (-i_\alpha)R_x = (-5.8mA)(750\Omega) \rightarrow \boxed{v_x = -4.35V}$

$i_x = i_\beta \rightarrow \boxed{i_x = 10.6\ mA}$

3-17



a) Formulate mesh-current!

$i_1 = i_B - i_A$ $\quad$ $i_4 = i_C$

$i_2 = i_C - i_A$ $\quad$ $i_x = -i_B$

$i_3 = i_C - i_B$

mesh KVL =

KVL $i_A \rightarrow$ $-v_s - i_2 R_A - i_1 R_B = 0 \rightarrow (i_A - i_C)R_A + (i_A - i_B)R_B = v_s$

KVL $i_B \rightarrow$ $i_1 R_B - i_3 R_C - i_x R_D = 0 \rightarrow (i_B - i_A)R_B + (i_B - i_C)R_C + i_B R_D = 0$

KVL $i_C \rightarrow$ $i_4 R_E + i_2 R_A + i_3 R_C = 0 \rightarrow i_C R_E + (i_C - i_A)R_A + (i_C - i_B)R_C = 0$

Matrix form
↳

$$\boxed{\begin{bmatrix} R_A + R_B & -R_B & -R_A \\ -R_B & R_B + R_C + R_D & -R_C \\ -R_A & -R_C & R_A + R_C + R_E \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} v_s \\ 0 \\ 0 \end{bmatrix}}$$

c) Which set of equations would be easier and why?

For nodal vs mesh analysis, the nodal analysis can be easier to solve if you choose v_D to be your ground. Then you have a known v_A and v_D, leading to only 2 node equations. mesh still needs 3 equations, so nodal is easier.

d) Using Matlab, find v_x and i_x in terms of mesh current variables!

Using Matlab,

$$\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \frac{1}{\det(A)} \begin{bmatrix} v_s\left(R_A(R_B + R_C + R_D) + R_C(R_B + R_D + R_E) + R_E(R_D + R_B)\right) \\ v_s\left(R_A(R_B + R_C) + R_B(R_C + R_E)\right) \\ v_s\left(R_A(R_B + R_C + R_D) + R_B R_C\right) \end{bmatrix}$$

$$\det(A) = R_A(R_B R_D + R_B R_E + R_C R_D + R_C R_E + R_D R_E) + R_B(R_C R_D + R_C R_E + R_D R_E)$$

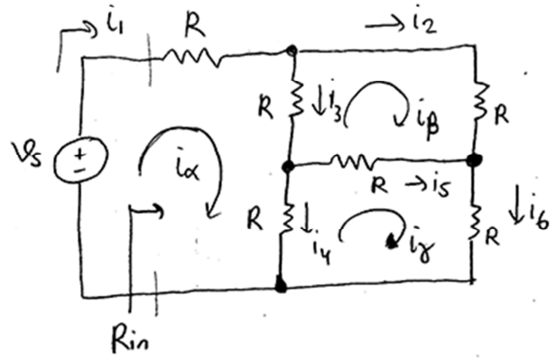
Now, we have:

$$i_X = -i_\beta \rightarrow \boxed{i_X = -\frac{R_A(R_B+R_C) + R_B(R_C+R_E)}{\det(A)} \cdot v_s}$$

$$v_X = (i_\alpha - i_\beta) R_B \rightarrow \boxed{v_X = \frac{R_A R_D + R_C(R_D+R_E) + R_E R_D}{\det(A)} \cdot R_B v_s}$$

3-23

Use mesh current to find Rin!



$i_1 = i_\alpha$ $\qquad$ $i_4 = i_\alpha - i_\gamma$

$i_2 = i_\beta$ $\qquad$ $i_5 = i_\gamma - i_\beta$

$i_3 = i_\alpha - i_\beta$ $\qquad$ $i_6 = i_\gamma$

$$KVL_{i_\alpha} \rightarrow -v_s + i_1 R + i_3 R + i_4 R = 0$$
$$R(i_\alpha + i_\alpha - i_\beta + i_\alpha - i_\gamma) = v_s$$

$$KVL_{i_\beta} \rightarrow -i_3 R + i_2 R - i_5 R = 0$$
$$R(i_\beta - i_\alpha + i_\beta + i_\beta - i_\gamma) = 0$$

$$KVL_{i_\gamma} \rightarrow -i_4 R + i_5 R + i_6 R$$
$$R(i_\gamma - i_\alpha + i_\gamma - i_\beta + i_\gamma) = 0$$

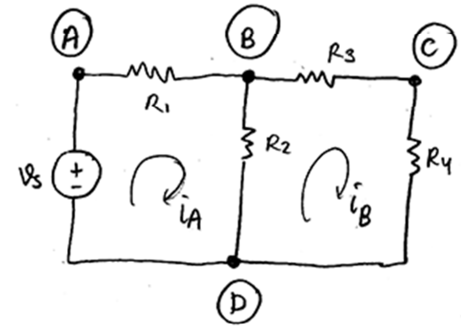
Matrix form:

$$\boxed{\begin{bmatrix} 3R & -R & -R \\ -R & 3R & -R \\ -R & -R & 3R \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \\ i_\gamma \end{bmatrix} = \begin{bmatrix} v_s \\ 0 \\ 0 \end{bmatrix}}$$

By definition, $R_{in} = \dfrac{v_s}{i_1} = \dfrac{v_s}{i_\alpha}$

Solving i_α using Matlab $\rightarrow$ $\boxed{i_\alpha = \dfrac{v_s}{2R}}$

Therefore, $\boxed{R_{in} = 2R}$



3-24 $R_i = 1k\Omega$, $v_s = 10V$, $v_c = -2V$ when $v_B = 0$

Find v_A, v_D, i_A, i_B!

mesh A → $-v_s + i_A R_1 + (i_A - i_B) R_2 = 0$

mesh B → $(i_B - i_A) R_2 + (R_3 + R_4) i_B = 0$

Matrix form → $$\boxed{\begin{bmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 + R_4 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} v_s \\ 0 \end{bmatrix}}$$

Plugging in the values → $$\begin{bmatrix} 2k\Omega & -1k\Omega \\ -1k\Omega & 3k\Omega \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} 10V \\ 0 \end{bmatrix}$$

$\boxed{\begin{array}{l} i_A = 6mA \\ i_B = 2mA \end{array}}$ → $\boxed{\begin{array}{l} v_A = i_A R_1 = 6mA(1k\Omega) = \underline{\underline{6V}} \\ v_D = (i_B - i_A) R_2 = (2mA - 6mA)(1k\Omega) = \underline{\underline{-4V}} \end{array}}$

(Note that we can solve i_B directly because v_B and v_C are known)

↳ $\boxed{i_B = -\dfrac{v_C - v_B}{R_3} \rightarrow (R_1 + R_2) i_A = v_s + R_2 i_B}$ → alternative sol.

3-25

a) Use Matlab to find symbollic expression for i_A for Prob 3-24!

$$\boxed{\begin{array}{l} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \dfrac{v_s}{\det(A)} \begin{bmatrix} R_2 + R_3 + R_4 \\ R_2 \end{bmatrix} \\ \det(A) = R_1 (R_2 + R_3 + R_4) + R_2 (R_3 + R_4) \end{array}}$$

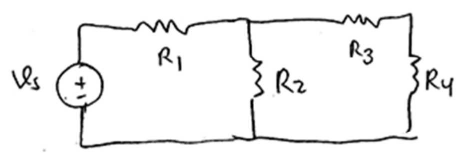
→ $$\boxed{i_A = \frac{R_2 + R_3 + R_4}{(R_1 + R_2)(R_3 + R_4) + R_1 R_2} v_s}$$

or use $i_B = -\dfrac{v_C - v_B}{R_3}$ and solve for $\boxed{i_A = \dfrac{v_s + R_2 i_B}{R_1 + R_2}}$ mentioned above

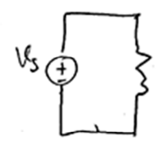
b) Compute ratio of i_A / v_s !

$$\boxed{\frac{i_A}{v_s} = \frac{R_2+R_3+R_4}{(R_1+R_2)(R_3+R_4)+R_1R_2}}$$

c) Find symbollic expression for equivalent circuit using series/parallel resistance!



$\rightarrow$



$R_{eq} = R_1 + (R_2 \| (R_3+R_4))$

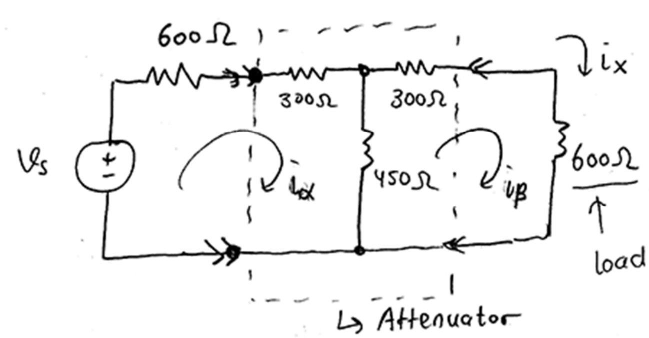
$$\hookrightarrow \quad R_{eq} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3+R_4}\right)^{-1} = R_1 + \frac{R_2(R_3+R_4)}{R_2+R_3+R_4}$$

$$= \frac{R_1(R_2+R_3+R_4)+R_2(R_3+R_4)}{R_1+R_2+R_3} = \boxed{\frac{(R_1+R_2)(R_3+R_4)+R_1R_2}{R_1+R_2+R_3}}$$

$$\boxed{\frac{i_A}{v_s} = \frac{1}{R_{eq}} = \frac{R_1+R_2+R_3}{(R_1+R_2)(R_3+R_4)+R_1R_2}} \rightarrow \underline{\text{Same as b)}}$$

3-92

Find the power delivered to the load with & without the attenuator!



Using mesh, we have:

$i_x = i_\beta$, mesh $\alpha \rightarrow -v_s + i_\alpha(600\Omega + 300\Omega + 450\Omega) - i_\beta(450\Omega) = 0$

mesh $\beta \rightarrow i_\beta(450\Omega + 300\Omega + 600\Omega) - i_\alpha(450\Omega) = 0$

$$\hookrightarrow \begin{bmatrix} 1350\,\Omega & -450\,\Omega \\ -450\,\Omega & 1350\,\Omega \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} v_s \\ 0 \end{bmatrix} \rightarrow \boxed{\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} v_s / 1200\,\Omega \\ v_s / 3600\,\Omega \end{bmatrix}}$$

So, we have $P_W = i_x^2 \cdot R_{Load} = (v_s / 3600\,\Omega)^2 (600\,\Omega) = \boxed{\frac{1}{21600\,\Omega} v_s^2}$

Without attenuator, the circuit is simply $\longrightarrow$

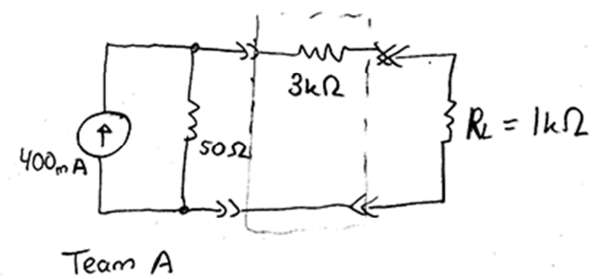
$i_x = \frac{v_s}{1200\,\Omega} \rightarrow P_{W/O} = i_x^2 R_{Load} = \left(\frac{v_s}{1200\,\Omega}\right)^2 (600\,\Omega)$

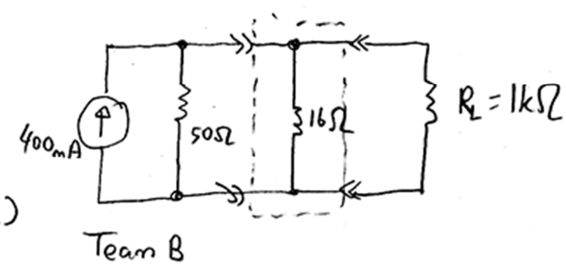
$$\boxed{P_{W/O} = \frac{1}{2400\,\Omega} v_s^2}$$

$\frac{P_W}{P_{W/O}} = \frac{1/21600}{1/2400} = \boxed{\frac{1}{9}} \rightarrow$ Attenuator reduces power by $\log^{10}(1/9) \cdot 20\,dB = \boxed{-19.08\,dB}$

3-96

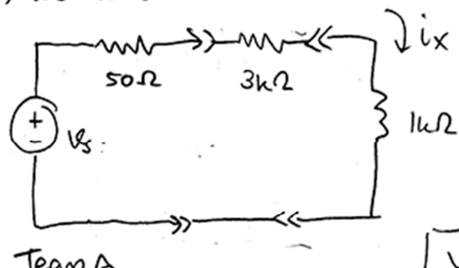
Want $P = 25mW \pm 10\%$ to the R_L. Which one is better? What if $P = 25mW \pm 5\%$?





Note that the source can be simplified.

So, we have



$v_s = 400mA\,(50\,\Omega)$

$$\boxed{v_s = 20\,V}$$

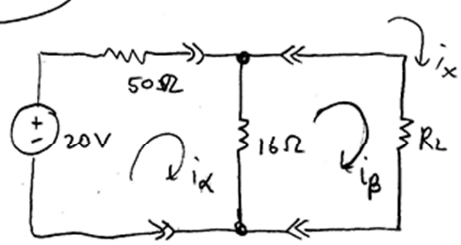
So, we have:

Team A

$$i_x = \frac{V_s}{50\Omega + 3k\Omega + 1k\Omega} = 4.9\ mA \rightarrow P_A = i_x^2 R_L = (4.9mA)^2(1k\Omega)$$

$$\boxed{P_A = 24.4\ mW}$$

Team B



Using mesh, we have:

$i_A \rightarrow -20V + (50\Omega + 16\Omega)\, i_\alpha - (16\Omega)\, i_\beta = 0$

$i_B \rightarrow (16\Omega + 1k\Omega)\, i_\beta - (16\Omega)\, i_\alpha = 0$

$$\hookrightarrow \begin{bmatrix} 66\Omega & -16\Omega \\ -16\Omega & 1016\Omega \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} 20V \\ 0 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} 0.3042\ A \\ 4.8\ mA \end{bmatrix}}$$

$\rightarrow i_x = 4.8mA$

$P_B = i_x^2 R_L = (4.8mA)^2(1k\Omega)$

$$\boxed{P_B = 23\ mW}$$

For $P = 25mW \pm 10\%$, we have a range of $P_1 \in [22.5mW, 27.5mW]$

For $P = 25mW \pm 5\%$, we have a range of $P_2 \in [23.75mW, 26.25mW]$

Both team uses the same amount of components. Both team fulfilled the ±10% power requirement, but team B's design used less power. The ±5% power requirement is only fulfilled by team A.

$\hookrightarrow$ **Choose Team B for ±10%, Team A for ±5%**