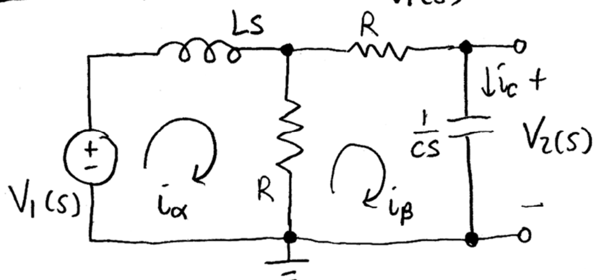
MAE 140 HW 8 Solution

11-9

Find $T_V(s) = \frac{V_2(s)}{V_1(s)}$!



Circuit is already in the impedance form.
Treat each non-source components like resistors.

Mesh Current:

$$\begin{bmatrix} Ls+R & -R \\ -R & R+R+\frac{1}{Cs} \end{bmatrix} \begin{bmatrix} i_\alpha \\ i_\beta \end{bmatrix} = \begin{bmatrix} V_1(s) \\ 0 \end{bmatrix}$$

Solve the mesh current equation (find $i_\beta = i_C$ to calculate $V_2(s)$)

↳ $-Ri_\alpha + (2R+\frac{1}{Cs})i_\beta = 0 \rightarrow i_\alpha = (2+\frac{1}{RCs})i_\beta$

↳ $(Ls+R)i_\alpha - Ri_\beta = 0 \rightarrow (Ls+R)(2+\frac{1}{RCs})i_\beta - Ri_\beta = V_1(s)$

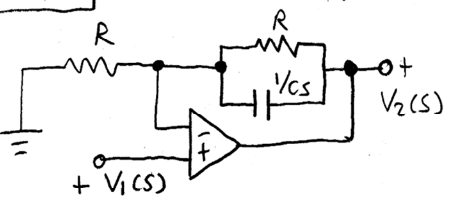
$$\boxed{i_\beta = \frac{V_1(s)}{(Ls+R)(2+\frac{1}{RCs})-R} = \frac{RCs}{(Ls+R)(1+2RCs)-R^2Cs} \cdot V_1(s)}$$

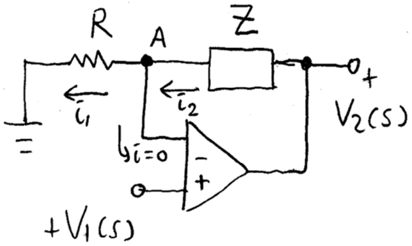
Then $V_2(s) = \frac{1}{Cs} \cdot i_\beta = \frac{1}{\cancel{Cs}} \cdot \frac{R\cancel{Cs}}{(Ls+R)(1+2RCs)-R^2Cs} \cdot V_1(s)$

$$\boxed{T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{R}{2LCRs^2 + (L+CR^2)s + R}}$$

11-11

Find $T_V(s) = \frac{V_2(s)}{V_1(s)}$! Select R and C s.t. $T_V(s)$ has a pole at $s = -100$ krad/s !





↳ Non-inverting OP Amp Circuit

$Z = R \parallel \frac{1}{Cs} = (\frac{1}{R}+Cs)^{-1} = \frac{R}{1+RCs}$

Using nodal analysis at node A ($V_A(s) = V_1(s)$)

$i_1 + 0 = i_2 \rightarrow \frac{V_A(s)-0}{R} = \frac{V_2(s)-V_A(s)}{Z}$

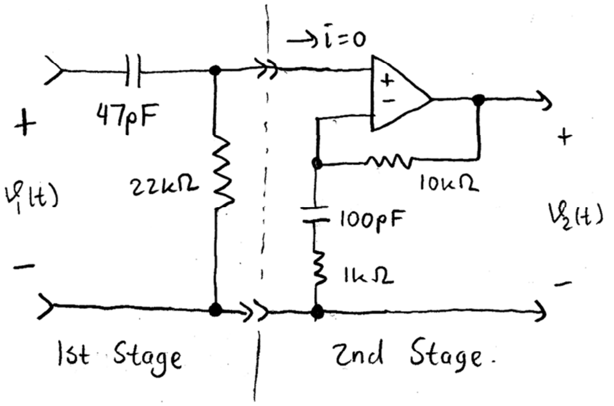
$\frac{\frac{R}{1+RCs}}{R} V_1(s) + V_1(s) = V_2(s)$

$$\boxed{T_V(s) = \frac{V_2(s)}{V_1(s)} = 1 + \frac{1}{1+RCs} = \frac{2+RCs}{1+RCs}}$$

Pole at $\boxed{s = -\frac{1}{RC}}$ $\rightarrow$ Want $\frac{1}{RC} = 100\ \text{krad/s} = 10^5\ \text{rad/s}$

C is usually in the μF (10^{-6}) or pF (10^{-9}) range, so $\boxed{C = 10pF \text{ and } R = 1k\Omega}$ works.

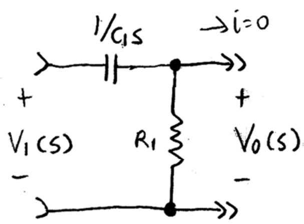
11-15 Find $T_V(s) = \frac{V_2(s)}{V_1(s)}$! Locate the poles and zeros of $T_V(s)$!



Note that this is a cascade of 2 circuits: Voltage divider and non-inverting OP Amp.

We can use chain rule because the 2nd stage is not loading the first one because of the infinite input impedance of the OP Amp.

1st Stage

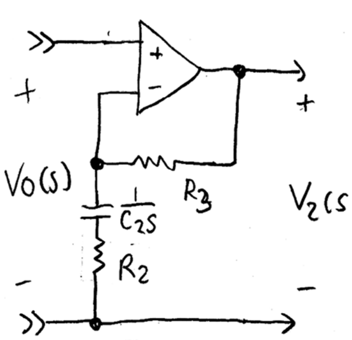


Volt. Divider

$$\frac{V_o(s)}{V_1(s)} = \frac{R_1}{R_1 + \frac{1}{C_1 s}} = \frac{s}{s + \frac{1}{R_1 C_1}} = \frac{s}{s + \frac{1}{(22k\Omega)(47pF)}}$$

$$\boxed{\frac{V_o(s)}{V_1(s)} = \frac{s}{s + 967.1}}$$

2nd Stage



Non-inverting OP-Amp

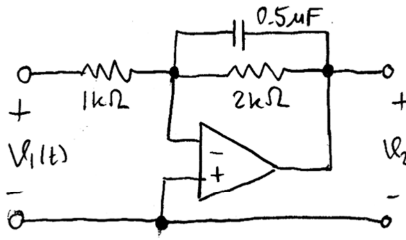
$$\frac{V_2(s)}{V_o(s)} = 1 + \frac{R_3}{\left(\frac{1}{C_2 s} + R_2\right)} = 1 + \frac{R_3/R_2\, s}{s + \frac{1}{C_2 R_2}} = 1 + \frac{\frac{10k\Omega}{1k\Omega} s}{s + \frac{1}{100pF \cdot 1k\Omega}}$$

$$\boxed{\frac{V_2(s)}{V_o(s)} = 1 + \frac{10s}{s + 10000} = \frac{11s + 10000}{s + 10000}}$$

$$\hookrightarrow \boxed{\frac{V_2(s)}{V_1(s)} = \left(\frac{V_o(s)}{V_1(s)}\right)\left(\frac{V_2(s)}{V_o(s)}\right) = \left(\frac{s}{s + 967.1}\right)\left(\frac{11s + 10000}{s + 10000}\right)}$$

$$\boxed{\text{Zeros} = \left\{0, -\frac{10^4}{11}\right\} \text{rad/s}, \quad \text{Poles} = \left\{-967.1, -10^4\right\} \text{rad/s}}$$

11-35 Find $v_{2ss}(t)$ using $v_1(t) = 10 \cdot \cos(\omega t)\,V$, $\omega = \{500, 1k, 10k\}$ rad/s!
Where is the pole?



Inverting OP-Amp Circuit

$$\frac{V_2(s)}{V_1(s)} = \frac{-\left(\frac{1}{0.5\mu F \cdot s} \parallel 2k\Omega\right)}{1k\Omega} = -\frac{\left(\frac{1}{2k\Omega} + 0.5\mu F \cdot s\right)^{-1}}{1k\Omega}$$

$$\boxed{T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{-2}{1+10^{-3}s} = \frac{-2000}{s+1000}} \rightarrow \boxed{\text{Pole} = -1000 \text{ rad/s}}$$

For $v_1(t) = 10 \cdot \cos \omega t \rightarrow$ Sinusoidal input, we can use $T_V(j\omega)$ to find the $v_{2ss}(t)$.

$$T_V(j\omega) = \frac{-2000}{j\omega + 1000} = \frac{-2000(-j\omega + 1000)}{\omega^2 + 10^6} \rightarrow \boxed{T_V(\omega j) = \frac{-2 \cdot 10^6 + 2000\omega j}{\omega^2 + 10^6}}$$

$$\boxed{v_{2ss}(t) = 10 \cdot |T_V(j\omega)| \cdot \cos(\omega t + \angle T_V(j\omega))}$$

↳ $\omega = 500$ rad/s

$$\boxed{\begin{aligned} &|T_V(j\omega)| = 1.789, \quad \angle T_V(j\omega) = 2.678 \text{ rad} \\ &v_{2ss}(t) = 17.89 \cos(500t + 2.678 \text{ rad})\,V = (-16 \cos 500t - 8 \sin 500t)\,V \end{aligned}}$$

↳ $\omega = 1$ krad/s

$$\boxed{\begin{aligned} &|T_V(j\omega)| = 1.414, \quad \angle T_V(j\omega) = 2.356 \text{ rad} \\ &v_{2ss}(t) = 14.14 \cos(1kt + 2.356 \text{ rad})\,V = -10(\cos 1kt + \sin 1kt)\,V \end{aligned}}$$

↳ $\omega = 10$ krad/s

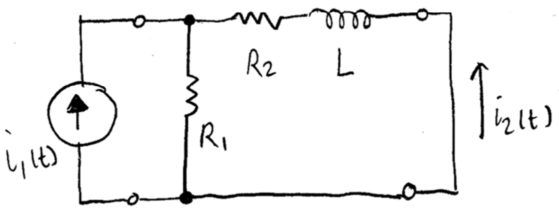
$$\boxed{\begin{aligned} &|T_V(j\omega)| = 0.1990, \quad \angle T_V(j\omega) = 1.670 \text{ rad} \\ &v_{2ss}(t) = 1.99 \cos(10kt + 1.670 \text{ rad})\,V = (-0.198 \cos 10kt - 1.98 \sin 10kt)\,V \end{aligned}}$$

11-37

$R_1 = 100\Omega$, $R_2 = 400\Omega$, $L = 100mH$,

Find $i_{2ss}(t)$ for $i_1(t) = 10\cos(\omega t)\,mA$, $\omega = \{50krad/s,\ 5krad/s\}$!

Find poles!



This is a current divider circuit.

$$\frac{-I_2(s)}{I_1(s)} = \frac{(R_2+Ls)^{-1}}{R_1^{-1} + (R_2+Ls)^{-1}} = \frac{R_1}{R_1+(R_2+Ls)}$$

$$= \frac{100\Omega}{100\Omega + 400\Omega + 100mH\cdot s} = \underline{\underline{\frac{100}{0.1s+500}}}$$

$$\boxed{T_I(s) = \frac{I_2(s)}{I_1(s)} = \frac{-1000}{s+5000}}$$

$\rightarrow$ $\boxed{\text{Pole} = -5000\ rad/s}$

$i_1(t) = 10\cdot\cos(\omega t)\,mA$ $\rightarrow$ $\boxed{i_{2ss}(t) = 10\cdot|T_I(j\omega)|\cos(\omega t + \measuredangle T_I(j\omega))\,mA}$

$\omega = 50krad/s$ $\rightarrow$ $|T_I(j\omega)| = 0.0199$, $\measuredangle T_I(j\omega) = 1.670\ rad$

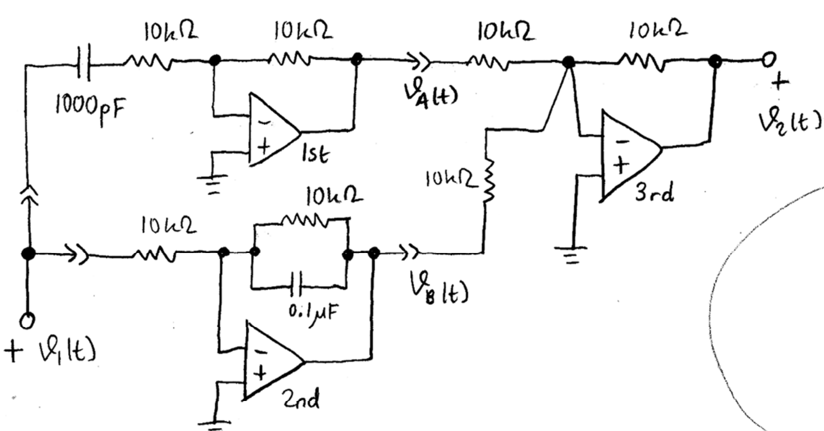
$i_{2ss}(t) = 0.199\cos(50kt + 1.670)\,mA = (-0.0198\cos 50kt - 0.198\sin 50kt)\,mA$

$\omega = 5krad/s$ $\rightarrow$ $|T_I(j\omega)| = 0.1414$, $\measuredangle T_I(j\omega) = 2.356\ rad$

$i_{2ss}(t) = 1.414\cos(5kt + 2.356)\,mA = -0.1(\cos 5kt + \sin 5kt)\,mA$

11-80

Interconnection of 3 basic OP Amp modules (all are inverting OP Amp).



All $R = 10k\Omega$,

$C_1 = 1000pF$

$C_2 = 0.1\mu F$

a) Does this interconnection involve loading?

No → OP Amp circuits do not involve loading.

b) Find the overall TF and locate its poles and zeros!

1st OP Amp → Inverting OP Amp

$$T_1(s) = \frac{V_A(s)}{V_1(s)} = \frac{-R}{R+\frac{1}{C_1 s}} = \frac{-s}{s+\frac{1}{RC_1}} = \frac{-s}{s+\frac{1}{10k\Omega \cdot 1000pF}} \rightarrow \boxed{T_1(s) = \frac{-s}{s+100}}$$

2nd OP Amp → Inverting OP Amp

$$T_2(s) = \frac{V_B(s)}{V_1(s)} = \frac{-(R \parallel \frac{1}{C_2 s})}{R} = \frac{-(\frac{1}{R}+C_2 s)^{-1}}{R} = \frac{-1}{1+RC_2 s} = \frac{-1}{1+10k\Omega \cdot 0.1\mu F \cdot s} = \frac{-1}{1+10^{-3}s}$$

$$\hookrightarrow \boxed{T_2(s) = \frac{-1000}{s+1000}}$$

3rd OP Amp → Inverting OP Amp Summing junction.

$$V_2(s) = -\frac{R}{R} \cdot V_A(s) - \frac{R}{R} V_B(s) = -(V_A(s)+V_B(s)) = -(T_1(s)V_1(s)+T_2(s)V_1(s))$$

$$= -\left(\frac{-s}{s+100} - \frac{1000}{s+1000}\right)V_1(s) = \frac{s(s+1000)+1000(s+100)}{(s+100)(s+1000)} V_1(s)$$

$$\boxed{T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{s^2+2000s+10^5}{(s+100)(s+1000)} = \frac{(s+1948.7)(s+51.3167)}{(s+100)(s+1000)}}$$

$\hookrightarrow$ $\boxed{\text{Zeros} = \{-1948.7, -51.3167\} \text{ rad/s}, \quad \text{Poles} = \{-100, -1000\} \text{ rad/s}}$

c) Find $v_{2SS}(t)$ when $v_1(t) = \cos\omega t$ V, $\omega = \{500, 10k, 200k\}$ rad/s!

Same method as 11-35 and 11-37

$\hookrightarrow$ $\boxed{v_{2SS}(t) = |T_V(j\omega)| \cdot \cos(\omega t + \angle T_V(j\omega)) \text{ V}}$

$\omega = 500$ rad/s → $|T_V(j\omega)| = 1.774$, $\angle T_V(j\omega) = -0.1174$ rad

$$v_{2SS}(t) = 1.774\cos(500t - 0.1174) \text{ V} = (1.7615\cos 500t + 0.2077\sin 500t) \text{ V}$$

$$\boxed{\begin{array}{l} \omega = 10k\,rad/s \rightarrow |T_V(j\omega)| = 1.014 \quad , \quad \measuredangle T_V(j\omega) = -0.0879\ rad \\ V_{2ss}(t) = 1.014\cos(10kt - 0.0879)V = (1.010\cos 10kt + 0.0890\sin 10kt)\ V \end{array}}$$

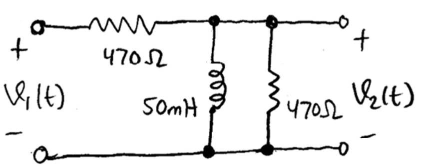
$$\boxed{\begin{array}{l} \omega = 200k\,rad/s \rightarrow |T_V(j\omega)| = 1. \quad \measuredangle T_V(j\omega) = -0.0045\ rad \\ V_{2ss}(t) = \cos(200kt - 0.0045)V = (\cos 200kt + 0.0045\sin 200kt)\ V \end{array}}$$

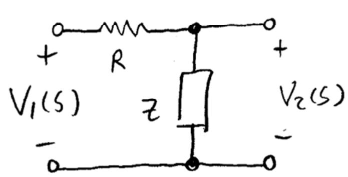
d) Can you think of a use for this circuit?

We have the input signal passed to 2 different OP Amps and then the output from each of these OP Amps are summed by the 3rd OP Amp. The 1st OP Amp is a differentiator which attenuates at high frequency while the 2nd OP Amp is a low pass filter.

↳ Differentiator + Low pass → $\boxed{\text{Proportional Differentiator (PD) Circuit which rejects high frequency noise}}$

$\boxed{\text{12-5}}$ Find $T_V(s) = \dfrac{V_2(s)}{V_1(s)}$! $\qquad R = 470\Omega,\ L = 50mH$





This is a voltage divider.

$$Z = (Ls \| R) = \left(\frac{1}{Ls} + \frac{1}{R}\right)^{-1} = \frac{RLs}{R + Ls}$$

$$T_V(s) = \frac{Z}{Z + R} = \frac{\frac{RLs}{R+Ls}}{\frac{RLs}{R+Ls} + R} = \frac{RLs}{RLs + R^2 + RLs} = \frac{s}{2\left(s + \frac{R}{2L}\right)}$$

$$\boxed{T_V(s) = \frac{s}{2\left(s + \frac{470\Omega}{2 \cdot 0.05H}\right)} = \frac{s}{2(s + 4700)}}$$

a) Find DC Gain, ∞ freq. gain, cutoff freq.! Identify the type of gain response!

DC Gain → $G_{DC} = \lim_{s \to 0} T_V(s) = \dfrac{0}{2(0 + 4700)} \rightarrow \boxed{G_{DC} = 0}$

∞ Gain → $G_\infty = \lim_{s \to \infty} T_V(s) = \lim_{s \to \infty} \dfrac{1}{2\left(1 + \frac{4700}{s}\right)} \rightarrow \boxed{G_\infty = \dfrac{1}{2}}$

Cutoff frequency ω_c where $|T_v(j\omega_c)| = \frac{1}{\sqrt{2}} T_{v\,max}$, $T_{v\,max} = 1/2$ in this case

$$T_v(j\omega) = \frac{j\omega}{2(j\omega+4700)} = \frac{j\omega(-j\omega+4700)}{2(\omega^2+4700^2)} = \frac{\omega^2+4700\omega j}{2(\omega^2+4700^2)}$$

$$\hookrightarrow |T_v(j\omega)| = \frac{\sqrt{\omega^4+\omega^2\cdot 4700^2}}{2(\omega^2+4700^2)} = \frac{1}{2}\frac{\omega}{\sqrt{\omega^2+4700^2}}$$

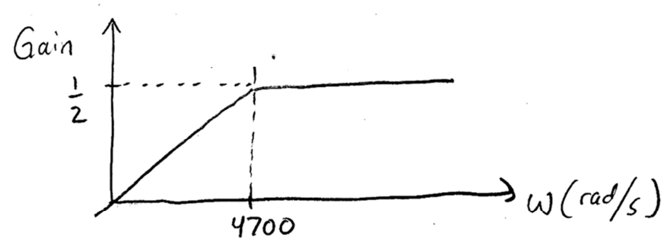
Same as the magnitude of the pole!

$$\hookrightarrow |T_v(j\omega_c)| = \frac{1}{\sqrt{2}}\cdot\frac{1}{2} = \frac{1}{2}\frac{\omega_c}{\sqrt{\omega_c^2+4700^2}} \rightarrow \frac{1}{8} = \frac{\omega_c^2}{4}\cdot\frac{1}{(\omega_c^2+4700^2)}$$

$$\hookrightarrow \omega_c^2+4700^2 = 2\omega_c^2 \rightarrow \omega_c^2 = 4700^2 \rightarrow \boxed{\omega_c = 4700 \text{ rad/s}}$$

$$T_v(s) = \frac{s}{s+4700} \rightarrow \boxed{\text{High pass filter}}$$

b) Sketch the straight line approximate of the gain response!



c) Calculate gain at $\omega = \{0.5\omega_c, \omega_c, 2\omega_c\}$!

Using gain $= |T_v(j\omega)|$ and $\omega_c = 4700$ rad/s $\rightarrow$

$$\boxed{\begin{aligned} |T_v(0.5\omega_c j)| &= 0.2236 \\ |T_v(\omega_c j)| &= 0.3536 = \frac{1}{\sqrt{2}\cdot 2} \\ |T_v(2\omega_c j)| &= 0.4472 \end{aligned}}$$

d) Plot the bode plot! $\rightarrow$ (Matlab Plot at the end)

e) How many dB down the passband is the filter one octave below the ω_c?

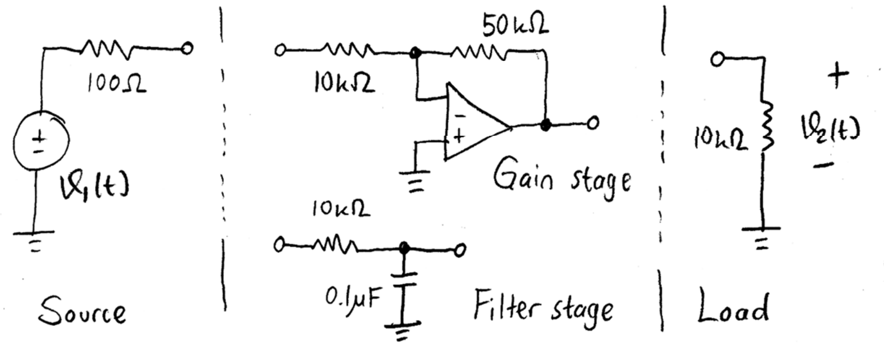
One octave lower $\rightarrow$ $\omega = 0{,}5\,\omega_c$.

Gain at ω_c $\rightarrow$ $G_{\omega_c} = 20\cdot\log_{10}\left(\frac{1}{\sqrt{2}\cdot 2}\right) = -9.03$ dB

Gain at $\omega = 0.5\omega_c$ $\rightarrow$ $G_\omega = 20\cdot\log_{10}(0.2236) = -13.01$ dB

$\hookrightarrow$ $\boxed{3.98 \text{ dB lower}}$

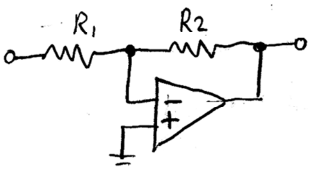
12-9 We need a low-pass filter with $\omega_c = 1\,krad/s$ and gain of -5.
The filter must fit in between the source and load circuit. The following circuits can never get the desired result no matter how you connect them. Explain why and suggest a way to get the desired result!



We have 2 configurations:
a) Gain then filter
b) Filter then gain

Individually, the gain and filter circuits seem to do what they are supposed to do:

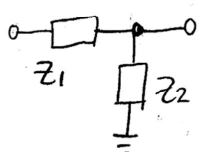
Gain stage → Inverting OP Amp.



$R_2 = 50k\Omega$
$R_1 = 10k\Omega$

$$T_G(s) = -\frac{R_2}{R_1} = -\frac{50k\Omega}{10k\Omega} = \underline{\underline{-5}}$$

Filter Stage → Voltage divider



$Z_1 = 10k\Omega$
$Z_2 = (0.1\mu F \cdot s)^{-1}$

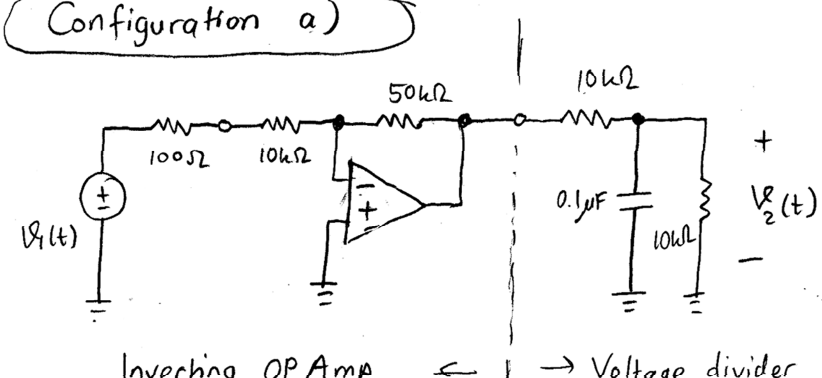
$$T_F(s) = \frac{Z_2}{Z_1 + Z_2} = \frac{\frac{1}{0.1\mu F \cdot s}}{10k\Omega + \frac{1}{0.1\mu F \cdot s}}$$

$$= \frac{1000}{s + 1000} \rightarrow \text{DC Gain} = \underline{\underline{1}}$$

↳ $\underline{\underline{\omega_c = 1000\ rad/s}}$

However, when plugged into the circuit, the source internal resistance and the load resistance will affect the transfer functions in a bad way.

Configuration a)



This configuration will have

$R_1 = 100\Omega + 10k\Omega$
$R_2 = 50k\Omega$
$Z_1 = 10k\Omega$
$Z_2 = \left(10k\Omega \parallel \frac{1}{0.1\mu F \cdot s}\right)$

$$T_{G_a}(s) = -\frac{R_2}{R_1} = \frac{-50k\Omega}{10.1k\Omega} = \underline{\underline{-4.95}}$$

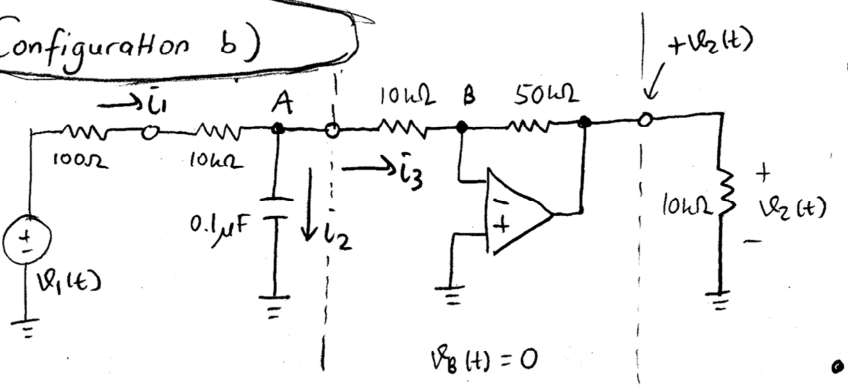
$$T_{F_a}(s) = \frac{Z_2}{Z_1+Z_2} = \frac{\left(\frac{1}{10k\Omega} + 0.1\mu F \cdot s\right)^{-1}}{10k\Omega + \left(0.1\mu F \cdot s + \frac{1}{10k\Omega}\right)^{-1}} = \frac{\frac{1}{1+10^{-3}s}}{1+\frac{1}{1+10^{-3}s}} = \frac{1}{2+10^{-3}s} = \underline{\underline{\frac{1000}{s+2000}}}$$

We can do the chain rule because the OP Amp circuit is not loading to the volt. divider.

↳ $\boxed{T_{V_a}(s) = T_{G_a}(s) \cdot T_{F_a}(s) = -4.95 \cdot \frac{1000}{(s+2000)}}$ → $\boxed{\begin{aligned} \omega_c &= 2000 \ \text{rad/s} \\ \text{DC gain} &= -4.95 \cdot \frac{1000}{2000} \\ &= -2.475 \end{aligned}}$

→ Wrong gain and wrong ω_c !

Configuration b)



- Note that we lose the Voltage divider because $i_3 \neq 0$.
- Need to do a nodal analysis at node A to find v_A, then use v_A as the input to the OP Amp.
- Note that $v_2(t)$ = the OP Amp's output.

Find v_A!

Node A → $i_1 = i_2 + i_3$

$$\frac{V_1(s) - V_A(s)}{10k\Omega + 100\Omega} = \frac{V_A(s) - 0}{(0.1\mu F \cdot s)^{-1}} + \frac{V_A(s) - 0}{10k\Omega} = \left(\frac{1}{10k\Omega} + 0.1\mu F \cdot s\right) V_A(s)$$

$$\frac{V_1(s)}{10.1k\Omega} = \left(\frac{1}{10k\Omega} + \frac{1}{10.1k\Omega} + 0.1\mu F s\right) V_A(s) \rightarrow \frac{V_A(s)}{V_1(s)} = \frac{1/10.1k\Omega}{\left(\frac{1}{10k\Omega} + \frac{1}{10.1k\Omega} + 0.1\mu F \cdot s\right)}$$

$$T_{F_b}(s) = \frac{V_A(s)}{V_1(s)} = \frac{1}{\left(\frac{10.1}{10} + 1 + 0.00101s\right)} = \underline{\underline{\frac{1}{(2.01 + 0.00101s)}}}$$

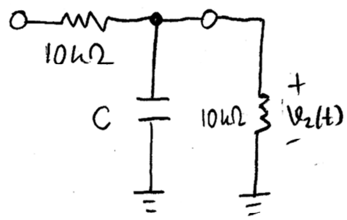
$$T_{G_b}(s) = \frac{V_2(s)}{V_A(s)} = \frac{-50k\Omega}{10k\Omega} = \underline{\underline{-5}}$$

So, we have:

$$\boxed{T_{V_b}(s) = \frac{V_2(s)}{V_1(s)} = \frac{V_A(s)}{V_1(s)}\,\frac{V_2(s)}{V_A(s)} = \left(\frac{1}{2.01+0.001010s}\right)(-5)} \rightarrow \boxed{\begin{matrix}\omega_c = 1990 \text{ rad/s} \\ \text{DC Gain} = -2.488\end{matrix}}$$

So, the loading and source resistances significantly altered the gain and ω_c. We need to take these resistances into account when designing the circuits.

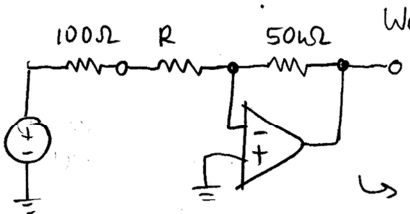
I think configuration a) is easier to fix.



$$T_{F_a}(s) = \frac{(\frac{1}{cs} \parallel 10k\Omega)}{10k\Omega + (\frac{1}{cs} \parallel 10k\Omega)} = \frac{(\frac{1}{10k\Omega} + Cs)^{-1}}{10k\Omega + (\frac{1}{10k\Omega}+Cs)^{-1}} = \frac{\frac{10k\Omega}{1+10k\Omega\cdot Cs}}{10k\Omega + \frac{10k\Omega}{1+10k\Omega\cdot Cs}}$$

$$= \frac{1}{1+10k\Omega\cdot Cs+1} = \frac{1}{2+10k\Omega\cdot Cs}$$

Want: $\omega_c = \dfrac{2}{10k\Omega\cdot C} = 1000 \text{ rad/s} \rightarrow \boxed{C = 0.2\mu F}$



$\hookrightarrow T_{G_a}(s) = -\dfrac{50k\Omega}{R+100\Omega} \rightarrow$ Find R where the gain is -5

$$T_{V_a}(s) = T_{G_a}(s)\cdot T_{F_a}(s) = \left(-\frac{50k\Omega}{R+100\Omega}\right)\left(\frac{1}{2+2\cdot 10^{-3}s}\right)$$

$\hookrightarrow$ DC Gain $= \lim_{s\to 0} T_{V_a}(s) = -\dfrac{1}{2}\cdot\dfrac{50k\Omega}{R+100\Omega} = -5 \rightarrow 25k\Omega = 5R + 500\Omega$

$$\boxed{R = 4900\Omega}$$

Then, we have:

$$\boxed{T_{V_a}(s) = (-10)\frac{500}{(s+1000)} = \frac{-5000}{s+1000}}$$

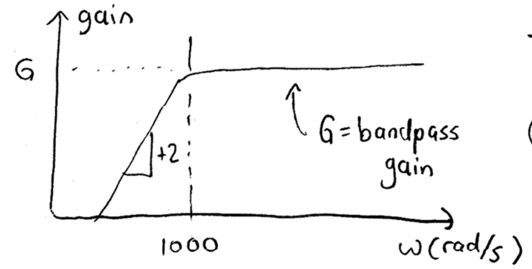
$\hookrightarrow$ DC Gain $= -5$

$\omega_c = 1000$ rad/s

Correct now

$\boxed{12\text{-}47}$ $T(s) = \dfrac{10^6 s^2}{(s+1000)^2}$

a) Construct bode plot for the gain! Is this a low/high/band-pass/band-stop filter?
Estimate cutoff freq. and bandpass gain!



→ $\boxed{\text{High Pass Filter}}$

$$G = \lim_{s\to\infty} T(s) = \lim_{s\to\infty}\left(\frac{10^6}{1+\frac{2000}{s}+\frac{10^6}{s^2}}\right) \rightarrow \boxed{G = 10^6}$$

Cutoff freq: ω_c where $|T(j\omega_c)| = \frac{1}{\sqrt{2}}\cdot G$

$$|T(j\omega_c)| = \left|\frac{10^6(-\omega_c^2)}{-\omega_c^2 + j2000\omega_c + 10^6}\right| = \left|\frac{-10^6\omega_c^2(10^6-\omega_c^2+j2000\omega_c)}{(10^6-\omega_c^2)^2+(2000\omega_c)^2}\right|$$

$$= \frac{\cancel{10^6}\omega_c^2}{((10^6-\omega_c^2)^2+(2000\omega_c)^2)}\left((10^6-\omega_c^2)^2+(2000\omega_c)^2\right)^{1/2} = \frac{\cancel{10^6}}{\sqrt{2}}$$

$$\rightarrow \sqrt{2}\omega_c^2 = \sqrt{(10^6-\omega_c^2)^2+(2000\omega_c)^2} \rightarrow 2\omega_c^4 = \omega_c^4 - 2\cdot10^6\omega_c^2 + 2000^2\omega_c^2 + 10^{12}$$

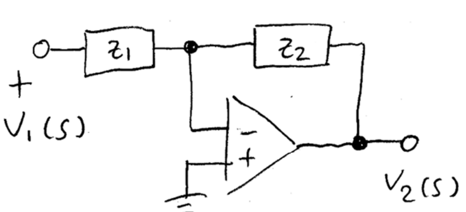
$$\rightarrow \omega_c^4 - 2\cdot10^6\omega_c^2 - 10^{12} = 0 \rightarrow \omega_c^2 = \{2.414\cdot10^6, -0.414\cdot10^6\}$$

Frequency is a positive real number $\longrightarrow$ $\omega_c^2 = 2.414\cdot10^6$ → $\boxed{\omega_c = 1554 \text{ rad/s}}$

b) Use MatLab to plot the bodeplot! → (at the end)

c) Design a circuit with practical components with the same transfer function!

We have $T(s) = \left(\dfrac{10^3 s}{s+1000}\right)^2$, so try cascading 2 OP Amps with $TF = \dfrac{10^3 s}{s+1000}$



$$T_1(s) = \frac{V_2(s)}{V_1(s)} = \frac{-Z_2}{Z_1} = \frac{10^3 s}{s+1000}$$

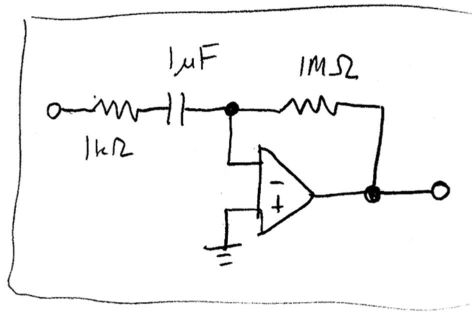
$$= \frac{10^3}{1+\frac{1000}{s}}$$ → R circuit → R, C in series

$Z_1 = R_1 + \frac{1}{Cs}$

$Z_2 = R_2$

$$T_1(s) = \frac{-R_2}{R_1+\frac{1}{Cs}} = \frac{-R_2/R_1}{1+\left(\frac{1}{R_1C}\right)\frac{1}{s}} \rightarrow \boxed{\frac{R_2}{R_1} = 10^3 \quad \frac{1}{R_1C} = 10^3}$$

We can use $\boxed{C = 1\mu F,\ R_1 = 1k\Omega,\ R_2 = 1M\Omega}$ →



Cascade 2 of this circuit to form $T(s) = T_1(s)^2$

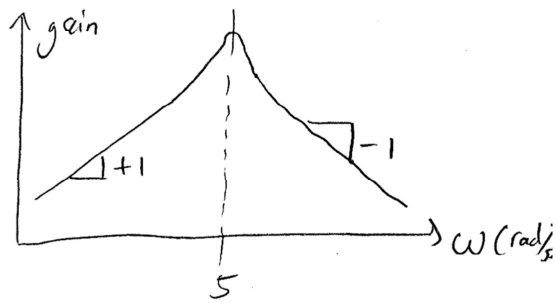
d) Use orcAD to compare this with the Matlab's plot! → (not needed)

$\boxed{12-51}$ $T_v(s) = \dfrac{4s}{0.04s^2 + 0.2s + 1} = \dfrac{100s}{s^2+5s+25}$

a) Construct the body-plot of the gain! Is this low/high/band-pass/band-stop?

$T_v(s) = \dfrac{100s}{s^2+5s+25}$ → Complex poles,

$\omega_c = \sqrt{25} = 5$ rad/s



$\boxed{\text{Band-pass filter}}$

b) Estimate the max gain!

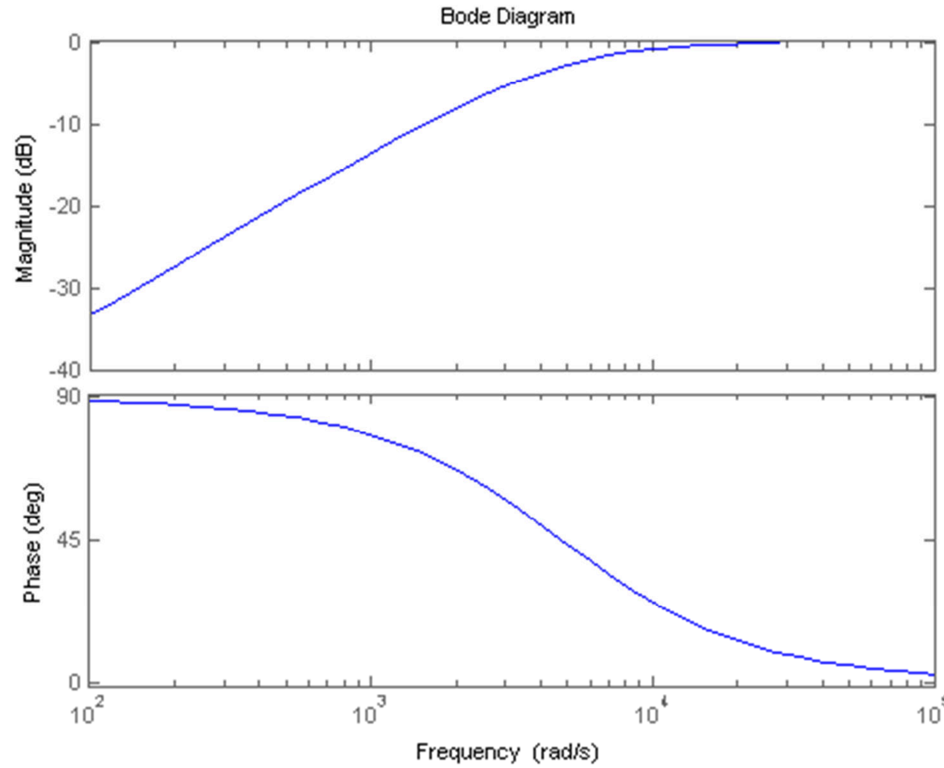
Max gain is at the resonant frequency $\omega_c = 5$ rad/s

↳ $|T_v(j\omega_c)| = |T_v(5j)| = \left|\dfrac{4(5j)}{(5j)^2 \cdot 0.04 + 0.2(j5) + 1}\right| = \left|\dfrac{20j}{-1+j+1}\right|$

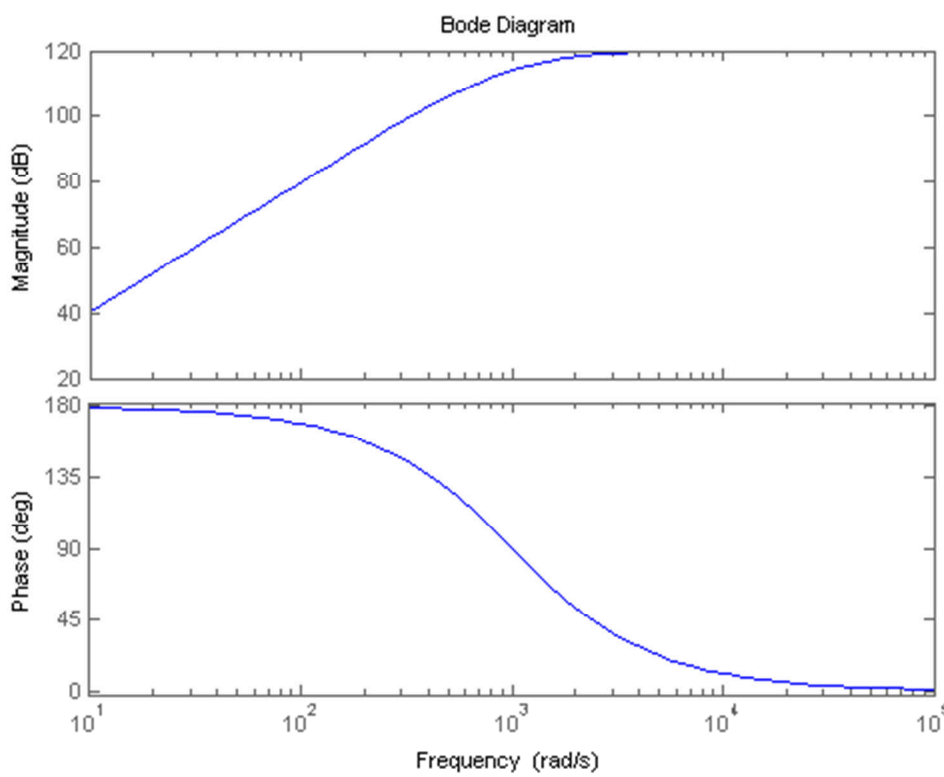
$\boxed{|T_v(j\omega_c)| = 20}$

c) Use Matlab to draw the bode plot! → (at the end)

Matlab's bode plot for Problem 12-5, for $T(s) = s/(s+4700)$



Matlab's bode plot for Problem 12-47, for $T(s) = 10^6 s^2/(s+1000)^2$



Matlab's bode plot for Problem 12-51, for $T(s) = 4\,s/(0.04\,s^2 + 0.2\,s + 1)$