

MAE 140 - HWK 1

1.18 1.5-V AA battery delivers 40 kJ of energy
application draws 10 mA continuously.
time the battery will last = ?

$$P = V \cdot I \quad P = \frac{dW}{dt}$$

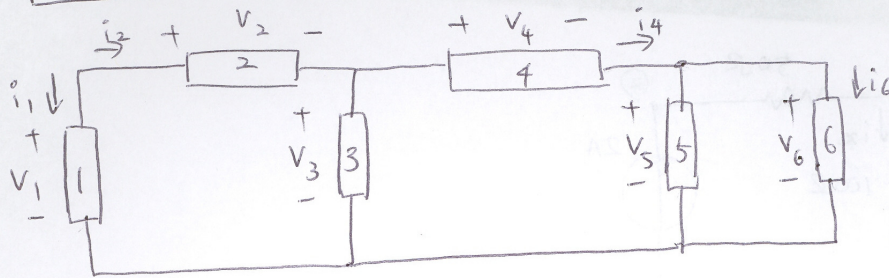
$$P = 1.5 (10 \text{ mA})$$
$$= 15 \text{ mW}$$

$$E = 40 \text{ kJ}$$

$$P = \frac{dW}{dt} \approx \frac{\Delta W}{\Delta t} \Rightarrow t = \frac{W}{P} = \frac{40 \text{ kJ}}{15 \text{ mW}} = 2.67 \times 10^6 \text{ s}$$

$$\Rightarrow \boxed{t = 2.67 \times 10^6 \text{ s}}$$

1.23



Analysis

	Device 1	Device 2	Device 3	Device 4	Device 5	Device 6
v	20V	?	?	?	?	?
i	-2A	?	?	1A	?	?
p	?	20W	10W	?	2.5W	2.5W

Using $P = V_i$,

$$P_1 = V_1 i_1 = (20V)(-2A) = -40W \rightarrow \text{Device 1 delivers power}$$

$$\text{Power balance: } P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = 0$$

$$-40 + 20 + 10 + P_4 + 2.5 + 2.5 = 0$$

$$\therefore P_4 = 5W \rightarrow \boxed{\text{Device 4 absorbs power}}$$

$$V_4 = P_4 / i_4 = (5W) / (1A) = \boxed{5V}$$

$$1.28) \quad W = \int_0^Q v \, dq$$

\downarrow
 Energy
 stored
 in capacitor

$$q(t) = 10^{-7} v(t) \quad \text{and} \quad v(t) = 10(1 - e^{-5000t})$$

$$\Rightarrow q(t) = 10^{-7} (10(1 - e^{-5000t}))$$

$$= 10^{-6} (1 - e^{-5000t})$$

$$\text{Then } \frac{dq}{dt} = +5 \cdot 10^{-3} e^{-5000t} \Rightarrow dq = 5 \cdot 10^{-3} e^{-5000t} dt$$

Substitute $v(t)$ and dq into energy equation

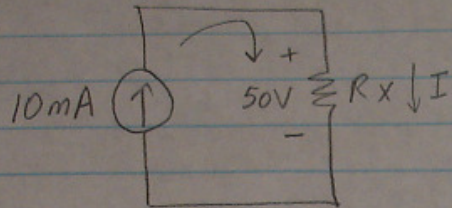
$$W = \int_0^{200 \mu s} \underbrace{[10(1 - e^{-5000t})]}_{v(t)} \underbrace{[5 \cdot 10^{-3} e^{-5000t}]}_{dq} dt$$

Take the integral and consider that $200 \mu s = 2 \cdot 10^{-4} s$

$$W = -10^{-5} e^{-5000t} + 0.5 \cdot 10^{-5} e^{-10000t} \Big|_0^{2 \cdot 10^{-4}}$$

$$W = -3 \cdot 10^{-6} \text{ J} + 5 \cdot 10^{-6} \text{ J} = 2 \cdot 10^{-6} \text{ J}$$

2.5) In figure, find R_x and the power delivered to the resistor



$$V = I R_x$$

$$R_x = \frac{V}{I} = \frac{50V}{10mA} = 5k\Omega$$

$$P = V \cdot I = 50V (10mA)$$

$$P = 500mW$$

2.8

$$R: 10\Omega \sim 100M\Omega$$

maximum rating for voltage : $V_{\max} = 500V$

" " " power : $P_{\max} = \frac{1}{4}W$

Show that the voltage rating is the controlling limit for $R > 1M\Omega$ and the power rating is the controlling limit when $R < 1M\Omega$.

Analysis

For $R > 1M\Omega$,

$$\therefore P = \frac{V^2}{R}$$

maximum power occurs when V is at maximum and R is at minimum

$$\therefore P = \frac{V_{\max}^2}{10^6\Omega} = \frac{(500V)^2}{(10^6\Omega)} = \frac{1}{4}W = \text{max. rating for power.}$$

\therefore voltage rating is the controlling limit
(i.e. P will always be smaller than $\frac{1}{4}W$ if a smaller value for V and/or a larger value for R is used)

For $R < 1M\Omega$,

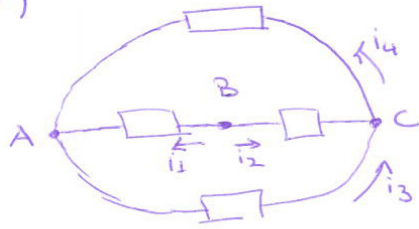
$$\therefore V = \sqrt{RP}$$

maximum voltage occurs maximum values for R and P are used.

$$\therefore V = \sqrt{(10^6\Omega)(\frac{1}{4}W)} = 500V = \text{max. rating for voltage}$$

\therefore Power rating is the controlling limit (i.e. V will always be smaller if a smaller value for R and/or a smaller value for P is used).

2.10)



$$i_2 = 2A$$
$$i_3 = -5A$$

Let's apply KCL

$$\text{Node A: } i_1 + i_4 - i_3 = 0$$

$$\text{Node B: } -i_1 - i_2 = 0$$

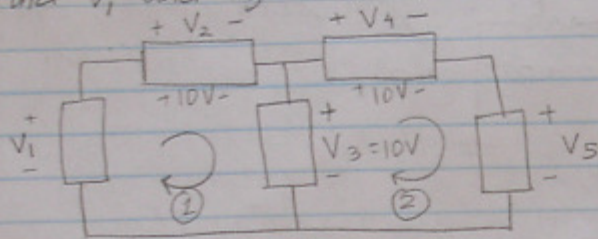
$$\text{Node C: } i_3 + i_2 - i_4 = 0$$

$$\text{From Node B: } \underline{i_1 = -i_2 = -2A}$$

$$\text{From Node C: } \underline{i_4 = i_3 + i_2 = -3A}$$

$$\text{Check Node A: } -2A - 3A - (-5A) = 0 \checkmark$$

2.16 In the figure, $V_2 = 10V$, $V_3 = 10V$, and $V_4 = 10V$.
Find V_1 and V_5



loop ① Applying KVL

$$V_1 - V_2 - V_3 = 0$$

$$V_1 = V_2 + V_3$$

$$V_1 = 10V + 10V$$

$$\boxed{V_1 = 20V}$$

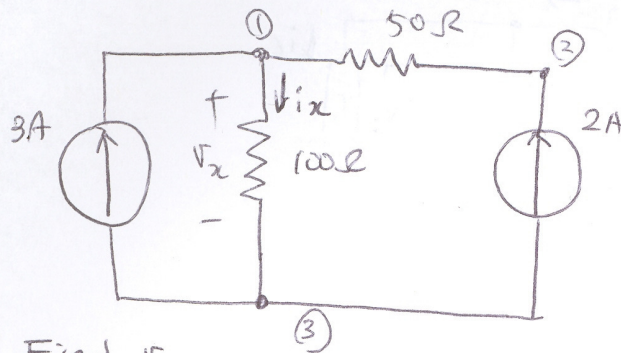
loop ② Applying KVL

$$V_3 - V_4 - V_5 = 0$$

$$10V - 10V = V_5$$

$$\boxed{V_5 = 0V}$$

2.23



Find V_x .

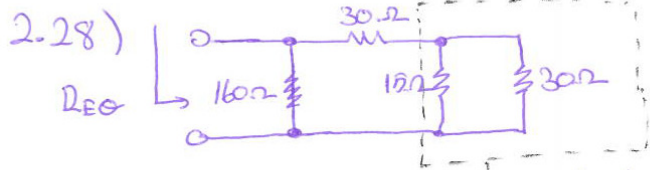
Analysis

KCL at node ①:

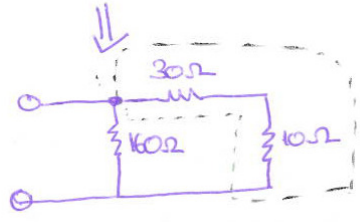
$$3 + 2 = i_x$$

$$i_x = 5A$$

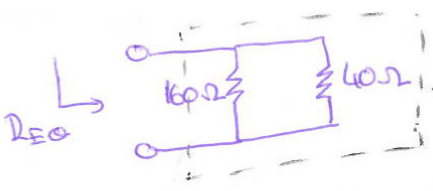
$$V_x = 100\Omega \cdot i_x = (100\Omega)(5A) = \underline{\underline{500V}}$$



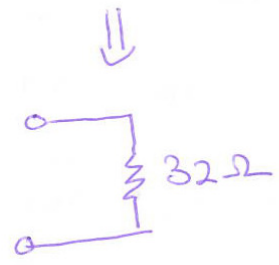
$$R_{EQ1} = \left(\frac{1}{15} + \frac{1}{30} \right)^{-1} = 10\Omega \text{ (parallel)}$$



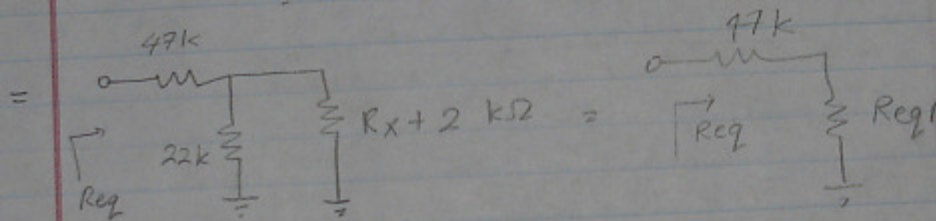
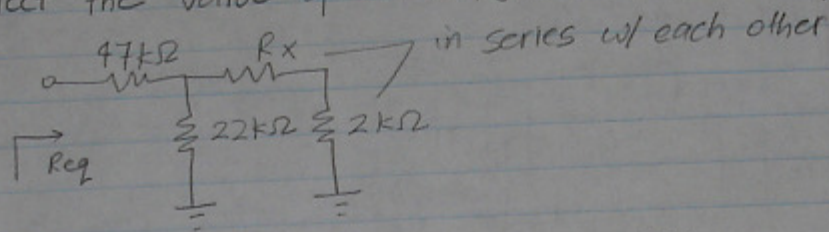
$$R_{EQ2} = (30 + 10)\Omega = 40\Omega \text{ (series)}$$



$$R_{EQ} = \left(\frac{1}{16} + \frac{1}{40} \right)^{-1} = \underline{\underline{32\Omega}} \text{ (Parallel)}$$



2.36 Select the value of R_x in figure so that $R_{eq} = 60k\Omega$



$$R_{eq1} = \frac{(22k)(R_x + 2k\Omega)}{22k + R_x + 2k}$$

($R_x + 2k\Omega$ and $22k$ are parallel to each other)

$$R_{eq} = 60k = R_{eq1} + 47k$$

$$13k = \frac{22k R_x + 44 \times 10^6 \Omega}{R_x + 24k}$$

$$13k(R_x + 24k) = 22k R_x + 44 \times 10^6 \Omega$$

$$13k R_x + 312 \times 10^6 = 22k R_x + 44 \times 10^6 \Omega$$

$$268 \times 10^6 \Omega = 9k R_x$$

$$R_x = 29.78 k\Omega$$