MAE140 - Linear Circuits - Winter 16
Final, March 16, 2016

Instructions

(i) The exam is open book. You may use your class notes and textbook. You may use a hand calculator with no communication capabilities.

(ii) You have 180 minutes

(iii) Do not forget to write your **name** and **student number**

(iv) On the questions for which the answers are given, please provide detailed derivations

(v) The exam has 5 questions for a total of 50 points and 2 bonus points

Good luck!

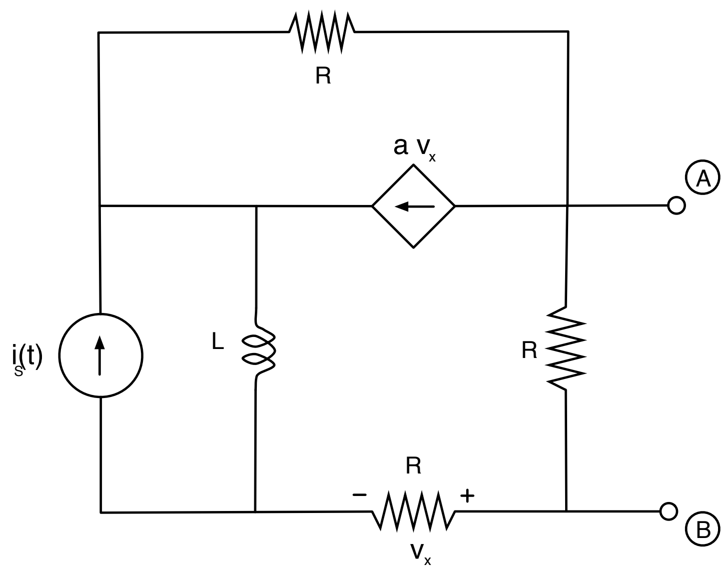

Figure 1: Circuit for Question 1.

1. **Equivalent Circuits**

Part I: [2 points] Assuming zero initial conditions, transform the circuit in Figure 1 into the s-domain.

Part II: [4 points] Find the open-circuit voltage in the circuit obtained in Part I (as seen from terminals A and B). Be mindful of the presence of the dependent source.

Part III: [4 points] Find the s-domain Thévenin equivalent of the circuit obtained in Part I as seen from terminals A and B. Be mindful of the presence of the dependent source.

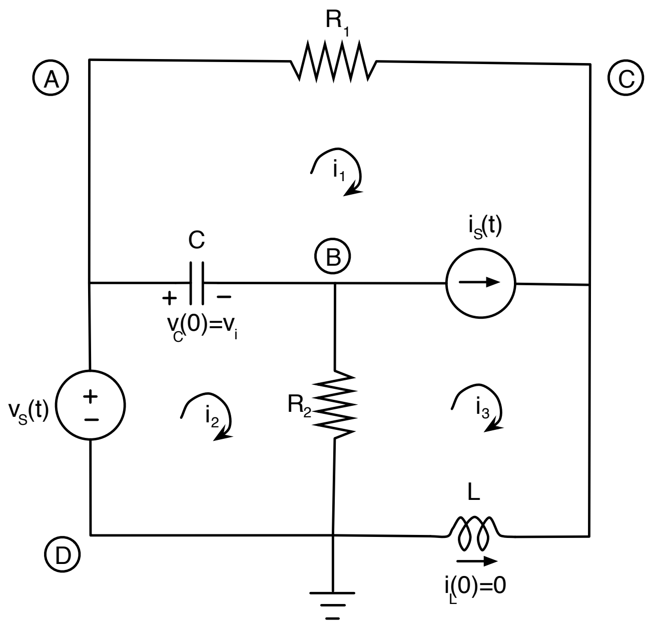

Figure 2: Nodal and Mesh Analysis Circuit for Question 2.

2. **Nodal and Mesh Analysis**

Part I: [5 points] Formulate node-voltage equations in the s-domain for the circuit in Figure 2. Use the reference node and other labels as shown in the figure. Use the initial conditions indicated in the figure and transform them into current sources. Make sure your final answer has the same number of independent equations as unknown variables. No need to solve any equations!

Part II: [5 points] Formulate mesh-current equations in the s-domain for the circuit in Figure 2. Use the currents shown in the figure. Use the initial conditions indicated in the figure and transform them into voltage sources. Make sure your final answer has the same number of independent equations as unknown variables. No need to solve any equations!

Part III: [1 bonus point] Express the transform of the capacitor voltage in terms of your unknown variables of Part I and also in terms of your unknown variables of Part II.

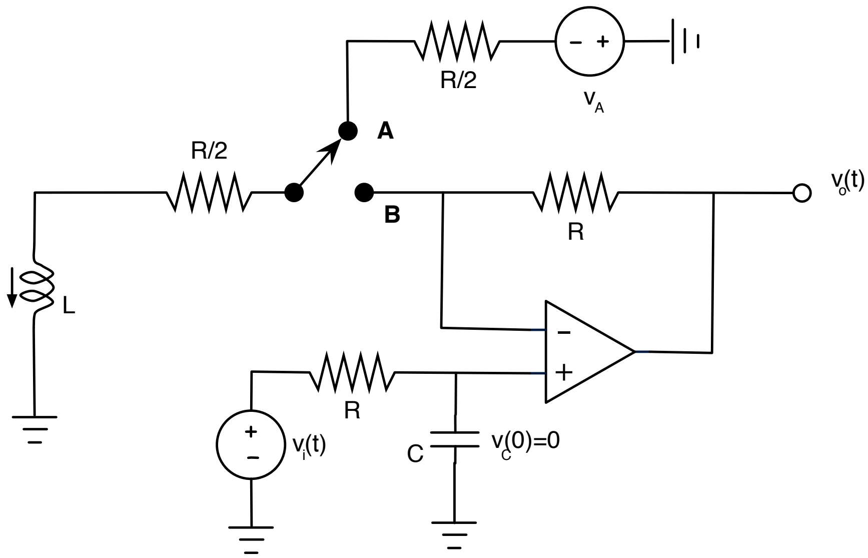

Figure 3: RCL circuit for Laplace Analysis for Question 3.

3. **Laplace Domain Circuit Analysis**

Part I: [2 points] Consider the circuit depicted in Figure 3. The value v_A of the voltage source at the top is constant. The switch is kept in position **A** for a very long time. At $t = 0$ it is moved to position **B**. Show that the initial condition for the inductor is given by

$$i_L(0^-) = -\frac{v_A}{R}.$$

[Show your work]

Part II: [4 points] Use this initial condition to transform the circuit into the s-domain for $t \geq 0$. Use an equivalent model for the inductor in which the initial condition appears as a voltage source. Do you recognize the resulting circuit as one of the basic op-amp building blocks? Express the output response transform $V_o(s)$ as a function of $V_i(s)$ and v_A.

Part III: [4 points] Use partial fractions and inverse Laplace transforms to show that the output voltage $v_o(t)$ when $v_A = 1\,V$, $v_i(t) = e^{-1500t}u(t)\,V$, $C = 1$ mF, $L = 1$ mH, and $R = 1\,\Omega$ is

$$v_o(t) = (e^{-500t} - 2e^{-1000t})u(t).$$

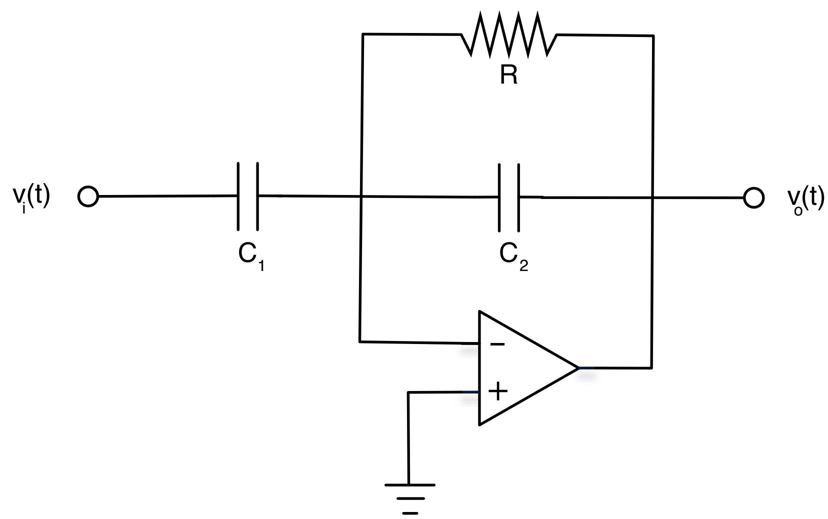

Figure 4: Frequency Response Analysis for Question 4.

4. **Frequency Response Analysis**

Part I: [1 point] Assuming zero initial conditions, transform the circuit in Figure 4 into the s-domain.

Part II: [2 points] Show that the transfer function from $V_i(s)$ to $V_o(s)$ is given by

$$T(s) = \frac{V_o(s)}{V_i(s)} = -\frac{RC_1 s}{1 + RC_2 s}.$$

[Show your work]

Part III [5.5 points] Let $R = 1\ \text{k}\Omega$, $C_1 = 20\,\mu\text{F}$ and $C_2 = 1\,\mu\text{F}$. Compute the gain and phase functions of $T(s)$. What are the DC gain and the ∞-freq gain? What are the corresponding values of the phase function? What is the cut-off frequency ω_c and its phase? Sketch plots for the gain and phase functions. What type of filter is this one?
[Explain your answer]

Part IV [1.5 points] Using what you know about frequency response, compute the steady state response $v_o^{SS}(t)$ of this circuit when $v_i(t) = \frac{1}{4}\cos(500t + \frac{\pi}{4})$ using the same values of R, C_1, and C_2 as in Part III.

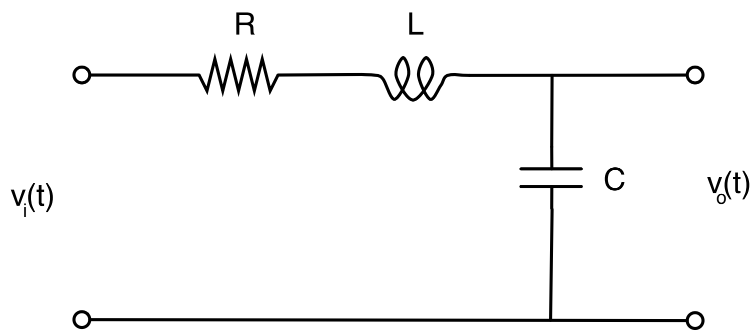

Figure 5: Circuit for Question 5.

5. **Loading and the Chain Rule**

Consider the circuit in Figure 5.

Part I: [2 points] Assuming zero initial conditions, plot the circuit in the s-domain. Find the transfer function $T(s)$ and determine its poles and zeros assuming $R > 2\sqrt{L/C}$.

Part II: [3 points] A student with a rusty recollection of MAE140 connected two identical copies of this circuit in series and was surprised to observe that the transfer function of the resulting circuit is not $T(s) \cdot T(s) = T(s)^2$. Could you explain to him why this is so and suggest an easy way to solve the problem so that he obtains $T(s)^2$ as the transfer function of the resulting circuit? Properly justify your answer.

Part III: [2 points] We could help the student of Part II even further by designing a different circuit with the same transfer function that does not have the same problem. To make the computations concrete, let $R = 100\ \Omega$, $C = 10\ \mu F$ and $L = 5$ mH, and compute the corresponding numerical values for the poles of the transfer function $T(s)$. Next, write $T(s)$ as a product of the form

$$T(s) = \frac{-A}{s+\alpha} \cdot \frac{-B}{s+\beta}$$

and determine appropriate values for $A > 0$, $B > 0$, $\alpha > 0$, and $\beta > 0$.

Part IV: [3 points] Design an inverting op-amp circuit whose transfer function is $\frac{-A}{s+\alpha}$ and another inverting op-amp circuit whose transfer function is $\frac{-B}{s+\beta}$, with the numerical values for A, B, α, β you chose in Part III. What is the transfer function of the series connection of these two inverting op-amps?

Part V: [1 bonus point] Let us call circuit "B" the series connection of the two inverting op-amps that you obtained in Part IV. Would the student with the rusty recollection of MAE140 have the same problem he had in Part II if he were to connect two identical copies of circuit "B" in series? Why?