**MAE140 - Linear Circuits - Winter 16**
**Midterm, February 5**

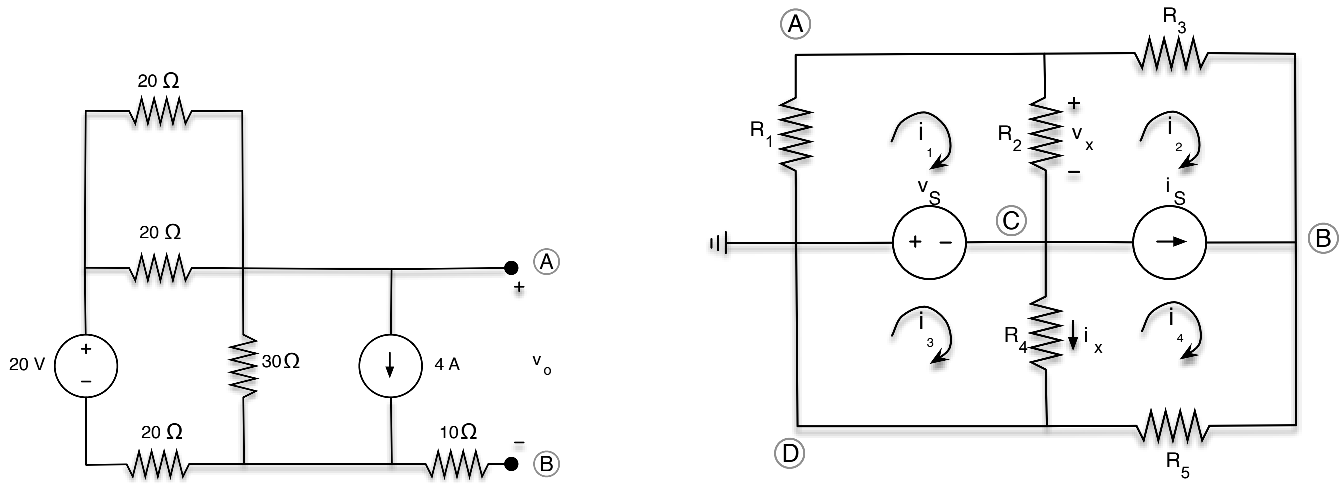
**Instructions**

(i) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities

(ii) You have 50 minutes

(iii) Do not forget to write your **name** and **student number**

Good luck!



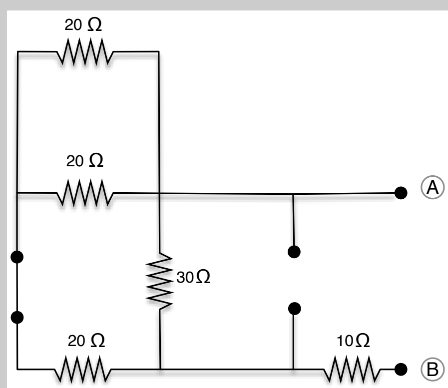

(a) Question 1 (b) Question 2

Figure 1: Circuits for all questions.

1. **Equivalent circuits**

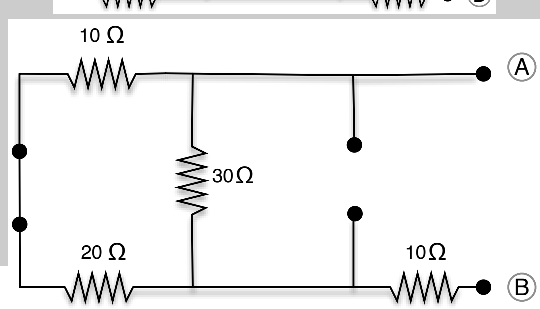
**Part I:** [3 points] Turn off all the sources in the circuit of Figure 1(a) and find the equivalent resistance as seen from terminals Ⓐ and Ⓑ.

**Solution: Part I:** We start by switching off the sources.




We substitute the voltage source by a short circuit, and the current source by an open circuit. Then, we get the circuit on the right

**(+ 1 point)**

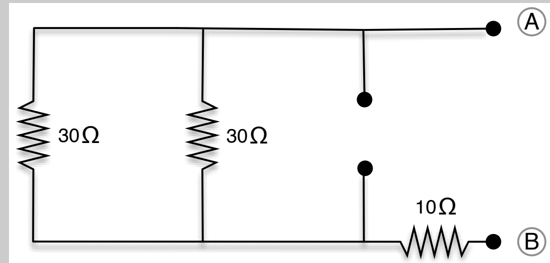



Next, we combine the two resistances in parallel in the upper left corner to get the circuit

**(+ .5 point)**

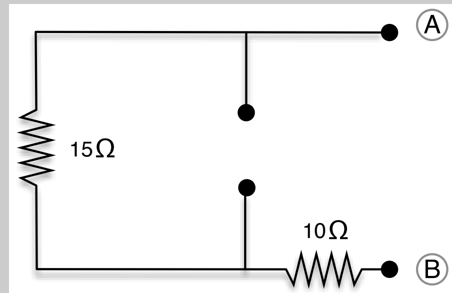
Next, we combine the two resistances in series on the left to get the circuit

**(+ .5 point)**



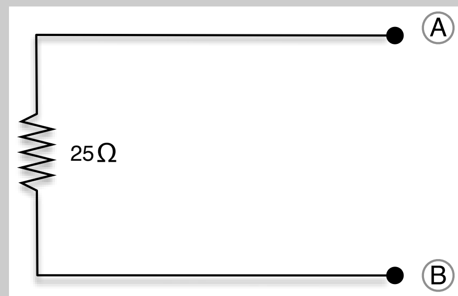

Next, we combine the two resistances in parallel on the left to get the circuit

**(+ .5 point)**




Finally, we combine the two resistances in series to get the equivalent resistance as seen from terminals Ⓐ and Ⓑ.

**(+ .5 point)**

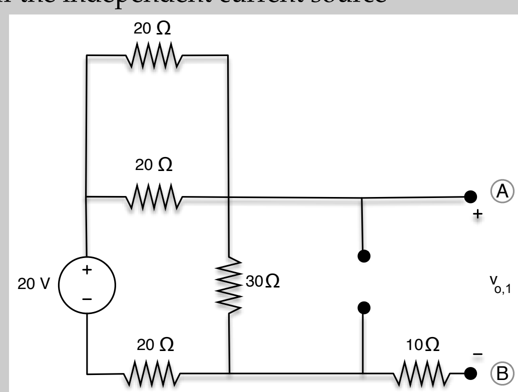



**Part II:** [5 points] Find the voltage $v_0$ using only superposition, association of resistors, voltage division, and current division.

**Solution: Part II:** To use superposition, we first turn off the independent current source

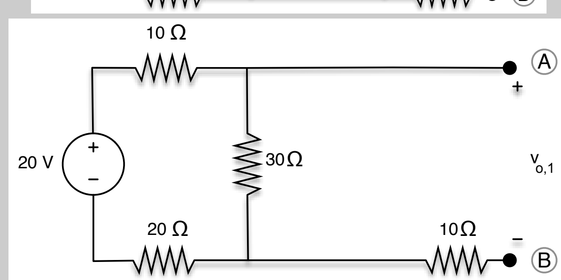
We substitute the current source by an open circuit. Then, we get the circuit on the right

**(+ .5 point)**




Combining the two resistors in parallel at the top left corner, we obtain the circuit on the right

**(+ .5 point)**



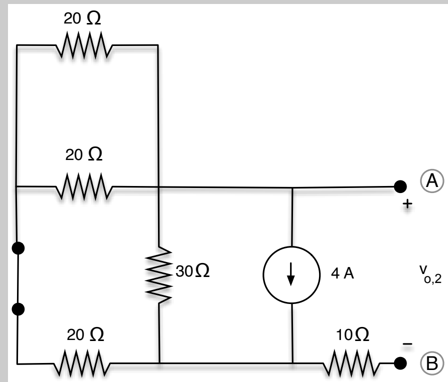

The 10Ohms resistor on the bottom right does not have any current going through it, therefore $v_{0,1}$ is simply the voltage drop that the 30Ohms resistor sees.

**(+ .5 point)**

This can be easily computed using voltage division as

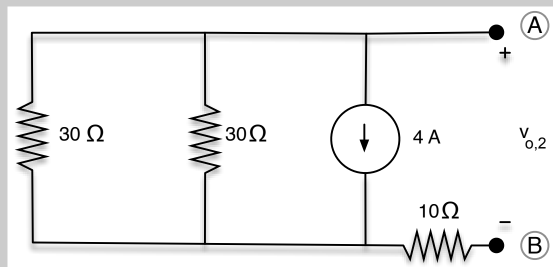
$$v_{0,1} = \frac{30}{30+10+20} 20 = 10V \quad \textbf{(+ .5 point)}$$

Next, we turn off the independent voltage source.




We substitute the voltage source by a closed circuit. Then, we get the circuit on the right

**(+ .5 point)**




Combining the two resistors in parallel in the top left corner, and then the resulting resistor in series with the 20Ohms resistor on the bottom left, we get the circuit on the right

**(+ 1 point)**

The 10Ohms resistor on the bottom right does not have any current going through it, therefore $v_{0,2}$ is simply the voltage drop that any of the 30Ohms resistor sees.

**(+ .5 point)**

This can be easily computed using current division as

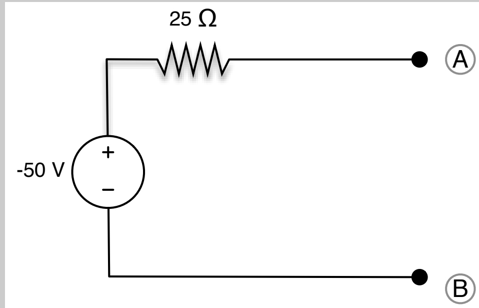
$$v_{0,2} = 30 \frac{1/30}{1/30 + 1/30} (-4) = -60V \quad \textbf{(+ .5 point)}$$

By superposition, we conclude that

$$v_0 = v_{0,1} + v_{0,2} = 10 - 60 = -50V \quad \textbf{(+ .5 point)}$$

**Part III:** [1 point] What is the Thévenin equivalent of the circuit as seen from terminals Ⓐ and Ⓑ?

**Solution: Part III:** We have computed the equivalent resistance from terminals Ⓐ and Ⓑ with all sources turned off in Part I, and the open-circuit voltage in Part II. Therefore, the Thévenin equivalent of the circuit is simply

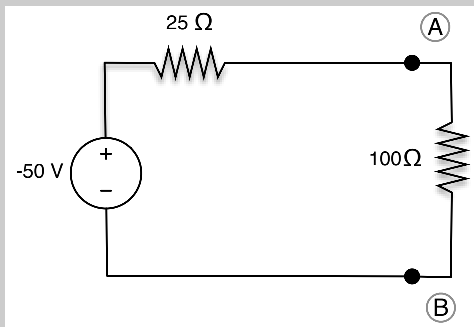



**(+ 1 point)**

**Part IV:** [1 point] Find the power absorbed by a $100\,\Omega$ resistor that is connected to terminals Ⓐ and Ⓑ.

**Solution:**

**Part IV:** We use the Thévenin equivalent of the circuit to obtain the answer in an easy way. Connecting the 100Ohms resistor gives rise to the circuit




**(+ .5 point)**

By voltage division, the voltage drop across the load is

$$v = \frac{100}{100+25}(-50) = -40V$$

Therefore, the power absorbed is

$$P = v^2 G = (-40)^2 \frac{1}{100} = 16W$$

**(+ .5 point)**

2. **Node voltage and mesh current analysis**

**Part I:** [4 points] Formulate node-voltage equations for the circuit in Figure 1(b). Use the node labels Ⓐ through Ⓓ provided in the figure and clearly indicate how you handle the presence of a voltage source. The final equations must depend only on unknown node voltages and the resistor values $R_1$ through $R_5$. **Do not modify the circuit or the labels**. No need to solve any equations!

**Solution: Part I:** There are four nodes in this circuit and the ground node Ⓓ (hence $v_D = 0$), which has already been chosen for us, is directly connected to the voltage source. Therefore, we take care of the voltage source using method 2 and set $v_C = -v_S$.

**(+ 1 point)**

We need to derive equations for the other two unknown node voltages $v_A$ and $v_B$. We do this using KCL and write equations by inspection. The matrix is $2 \times 3$ and the independent vector has 2 components.

**(+ .5 point)**

We write,

$$\begin{pmatrix} G_1+G_2+G_3 & -G_3 & -G_2 \\ -G_3 & G_3+G_5 & 0 \end{pmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \end{pmatrix} = \begin{pmatrix} 0 \\ i_S \end{pmatrix}$$ **(+ 2 points)**

where, as we do usually, $G_i = 1/R_i$.
Since we know that $v_C = -v_S$, we can rewrite the equations above as

$$\begin{pmatrix} G_1+G_2+G_3 & -G_3 \\ -G_3 & G_3+G_5 \end{pmatrix} \begin{pmatrix} v_A \\ v_B \end{pmatrix} = \begin{pmatrix} -G_2 v_S \\ i_S \end{pmatrix}$$ **(+ .5 point)**

**Part II:** [4 points] Formulate mesh-current equations for the circuit in Figure 1(b). Use the mesh currents shown in the figure and clearly indicate how you handle the presence of the current source. The final equations should only depend on the unknown mesh currents and the resistor values $R_1$ through $R_5$. **Do not modify the circuit or the labels**. No need to solve any equations!

**Solution: Part II:** There are four meshes in this circuit. The current source, $i_S$, belongs to two meshes and is not in parallel with any resistor, so we need to use a supermesh (combining meshes 2 and 4) to deal with it.

**(+ 1 point)**

The current source imposes the constraint

$$i_4 - i_2 = i_S.$$ **(+ .5 point)**

KVL for the supermesh reads like

$$R_3 i_2 + R_5 i_4 + R_4(i_4 - i_3) + R_2(i_2 - i_1) = 0$$ **(+ .5 point)**

The remaining equations come from KVL for mesh 1

$$R_1 i_1 + R_2(i_1 - i_2) - v_S = 0$$ **(+ 1 point)**

and KVL for mesh 3

$$R_4(i_3 - i_4) + v_S = 0$$ **(+ 1 point)**

**Part III:** [2 points] Provide two expressions for the voltage $v_x$ and the current $i_x$, one in terms of node voltages and the other one in terms of mesh currents.

**Solution: Part III:** In terms of the node voltages, $v_x$ and $i_x$ can be expressed as

$$v_x = v_A - v_C$$ **(+ .5 point)**

$$i_x = \frac{1}{R_4} v_C$$ **(+ .5 point)**

In terms of the mesh currents, $v_x$ and $i_x$ can be expressed as

$$v_x = R_2(i_1 - i_2)$$ **(+ .5 point)**

$$i_x = i_3 - i_4$$ **(+ .5 point)**

**Part IV:** [2 bonus points] Is the independent voltage source in parallel with the resistor $R_4$? Removing the resistor $R_4$ and substituting it by an open circuit would eliminate mesh 3. Would this removal have any effect on the values of the node voltages or the other mesh currents? Would it have any effect on the current that flows through the independent voltage source? Justify your answers.

**Solution:**

**Part IV:** The independent voltage source is in parallel with the resistor (both elements form a loop, mesh 3, that contains no other element).

**(+ .5 extra point)**

From our discussion in class of equivalent sources, we know that a voltage source connected in parallel with a resistor can be transformed into just a voltage source, with the rest of the circuit being oblivious to this transformation.

**(+ .5 extra point)**

Therefore, removing the resistor $R_4$ does not have any effect on the values of the node voltages or the other mesh currents (this in fact can be verified by checking the equations in Parts I and II above).

**(+ .5 extra point)**

Finally, the removal of $R_4$ would have an effect on the current that flows through the independent voltage source. With the resistor, the current is $i_3 - i_1$, and without the resistor it would be $i_4 - i_1$.

**(+ .5 extra point)**