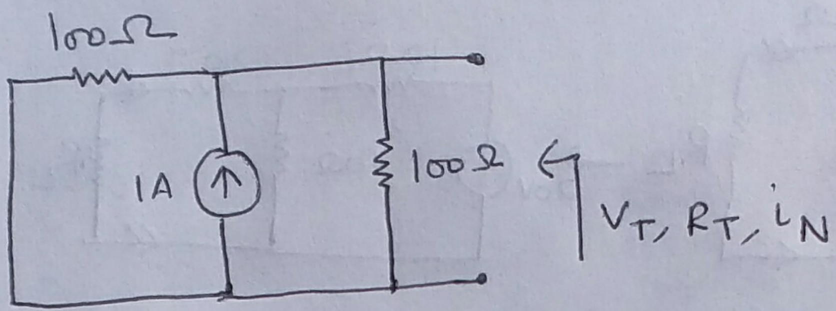
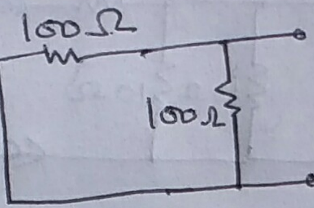
3.48)

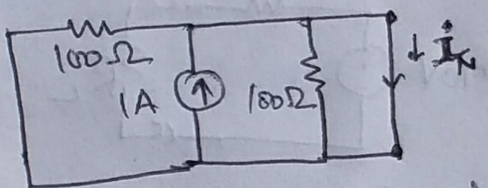



For $R_T$, switch off current source.




$$R_T = 100 \parallel 100 = 50\Omega$$

For $i_N$, short circuit the load side.

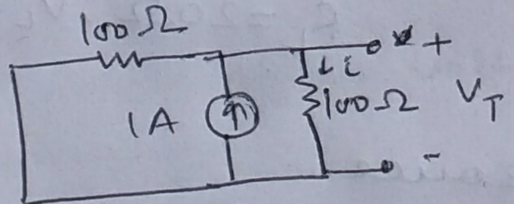



Since the resistors are short circuited, no current will flow in resistors.

$$i_N = 1A$$

$$V_T = i_N \times R_T = 1 \times 50 = 50\,V.$$

Verify:

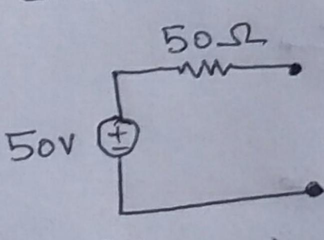



By current division, current $i$ in 100Ω resistor

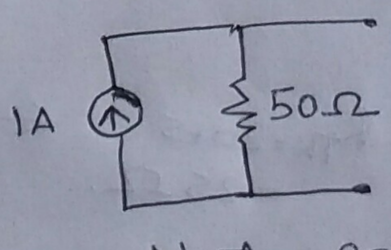
$$= \frac{1/100\Omega \times 1A}{\frac{1}{100\Omega} + \frac{1}{100\Omega}} = \frac{100}{100+100} \times 1 = 0.5A.$$

$$\therefore V_T = 100 \times 0.5 = 50V.$$

Any two pairs can be found & the third can be calculated.

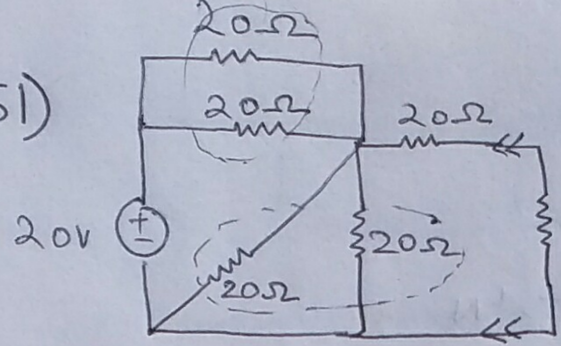


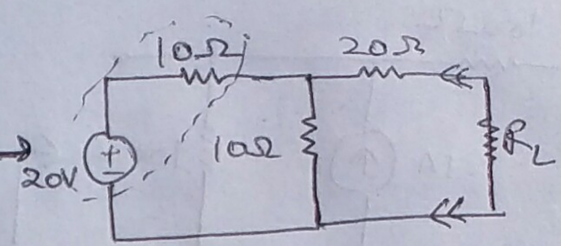

Thevenin equivalent



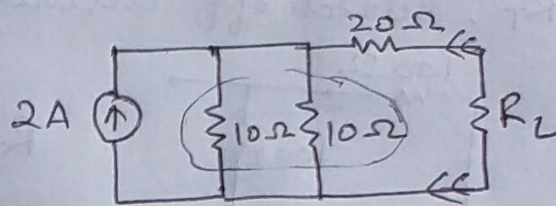

Norton equivalent.

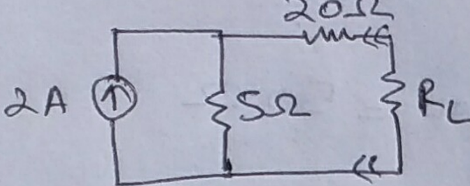
3.51)



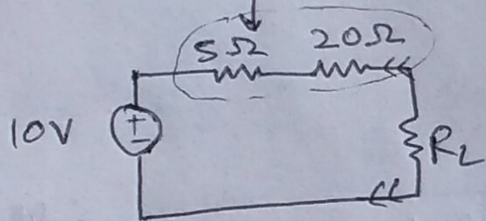



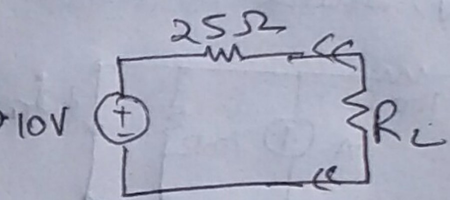

↓ source transformation.




source transformation




Thevenin equivalent circuit.

Using voltage division,

$$\boxed{V_L = \frac{R_L}{R_L+25} \times 10}$$

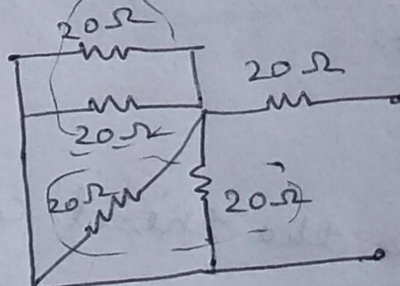
$R_L = 5\Omega$, $V_L = 1.67V$

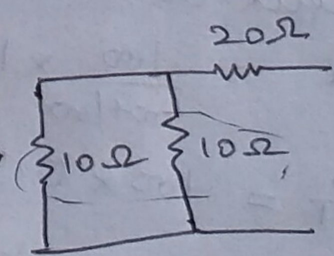
$R_L = 10\Omega$, $V_L = 2.86V$

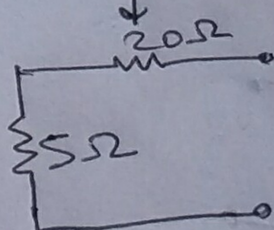
$R_L = 20\Omega$, $V_L = 4.44V$

$R_{Th}$ alternate:

switch off voltage souce.

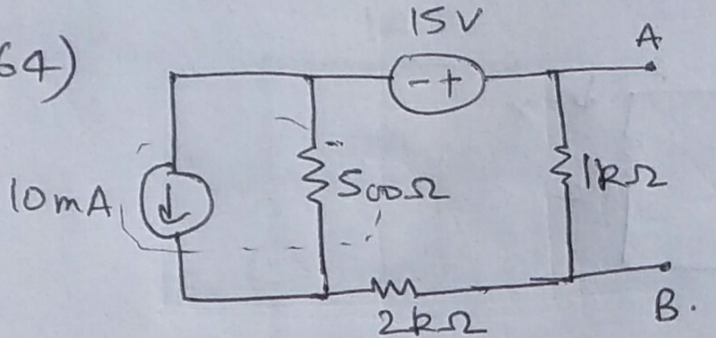


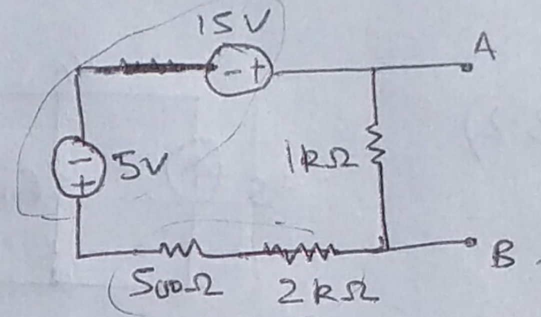




$R_{Th} = 20+5$
$= 25\Omega.$

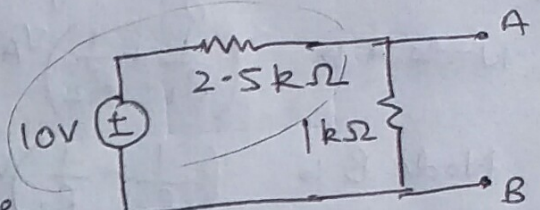
3-64)



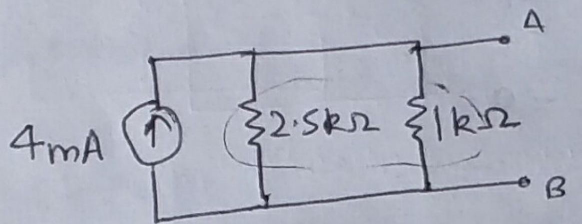

Source transformation




source addition.

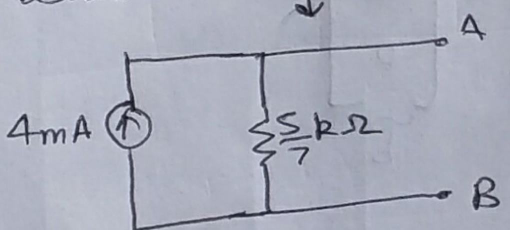



source transformation

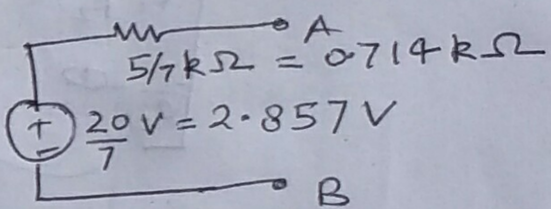



$$\frac{5}{7}k\Omega = \frac{2500\Omega \times 1000\Omega}{2500\Omega + 1000\Omega}$$

$2.5k\Omega \,\|\, 1k\Omega =$



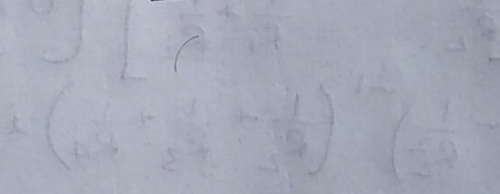

source transformation

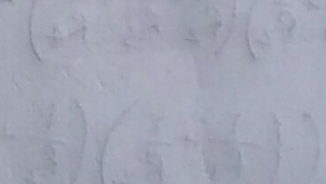



Thevenin eqt.

<u>Alternate</u>

For $R_T$, switch off the voltage & current sources.

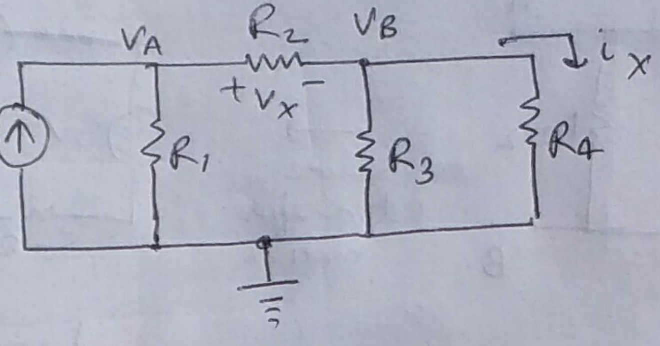



$\rightarrow R_{eq} = 2.5k\Omega \,\|\, 1k\Omega$

$= 0.714 k\Omega$

3.2)




a) Node voltage equations

Node A ↳ $\left(\frac{1}{R_1}+\frac{1}{R_2}\right)V_A - \frac{1}{R_2}V_B - i_{S_1} = 0$.

Node B ↳ $\left(\frac{1}{R_4}+\frac{1}{R_2}+\frac{1}{R_3}\right)V_B - \frac{1}{R_2}V_A = 0$.

in matrix form.

$$\underbrace{\begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} V_A \\ V_B \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} i_s \\ 0 \end{bmatrix}}_{b}$$

$\Rightarrow Ax = b$

$x = A^{-1}b$

b) $\Rightarrow \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4} \end{bmatrix}^{-1} \begin{bmatrix} i_s \\ 0 \end{bmatrix}$

$$= \left(\frac{1}{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right) - \frac{1}{R_2^2}}\right) \begin{bmatrix} \frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4} & \frac{1}{R_2} \\ \frac{1}{R_2} & \frac{1}{R_1}+\frac{1}{R_2} \end{bmatrix} \begin{bmatrix} i_s \\ 0 \end{bmatrix}$$

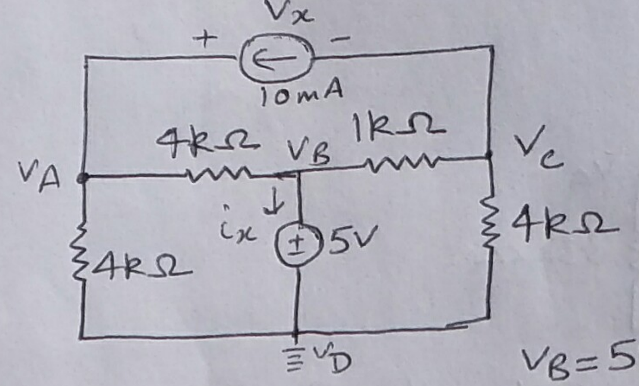
$$\Rightarrow V_A = \left(\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right) - \frac{1}{R_2^2}\right)^{-1} \left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right) i_s$$

$$V_B = \left(\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right) - \frac{1}{R_2^2}\right)^{-1} \frac{i_s}{R_2}$$

c) $v_x = V_A - V_B$

$i_x = \frac{V_B}{R_4}$

3.5)




$V_B = 5V$ $\quad V_D = 0.$

a)

Node A: $\left(\frac{1}{4\times10^3} + \frac{1}{4\times10^3}\right)V_A - \frac{V_B}{4} - 10\times10^{-3} = 0$

Node C: $\left(\frac{1}{4\times10^3} + \frac{1}{1\times10^3}\right)V_C + 10\times10^{-3} - \frac{V_B}{1\times10^3} = 0$

$$\underbrace{\begin{bmatrix} \frac{1}{4\times10^3}+\frac{1}{4\times10^3} & 0 \\ 0 & \frac{1}{4\times10^3}+\frac{1}{1\times10^3} \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} V_A \\ V_C \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 10\times10^{-3} + 5/4 \\ 5\times10^{-3} - 10\times10^{-3} \end{bmatrix}}_{b}$$

(substituting value of B.)

b) Solving the above equation

$Ax = b.$

as $x = A^{-1}b.$

$V_A = 22.5V$

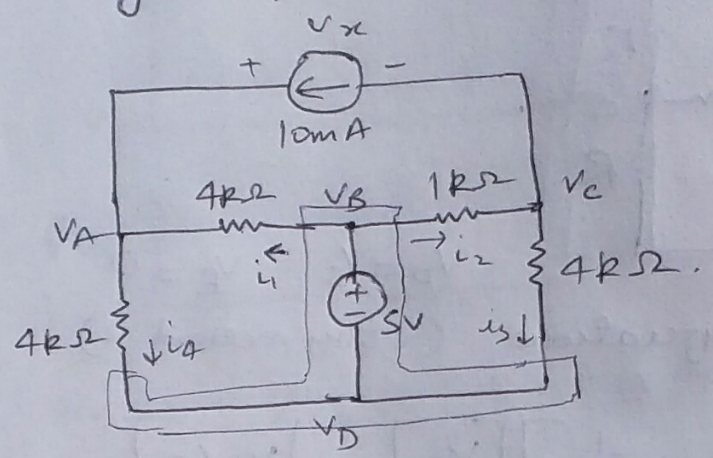
$V_C = -4V$

c) $V_x = V_A - V_C = 22.5 - (-4) = 26.5V$

at Node B: $\frac{V_A - V_B}{4\times10^3} + \frac{V_C - V_B}{1\times10^3} = i_x$

$\Rightarrow i_x = -4.625mA$

3(b) Using supernode.




$$-i_1 - i_2 + i_3 + i_4 = 0$$

$$\Rightarrow -\frac{V_B - V_A}{4\times10^3} - \frac{V_B - V_C}{1\times10^3} + \frac{V_A - V_D}{4\times10^3} + \frac{V_C - V_D}{4\times10^3} = 0.$$

$$V_B - V_D = 5\,V.$$

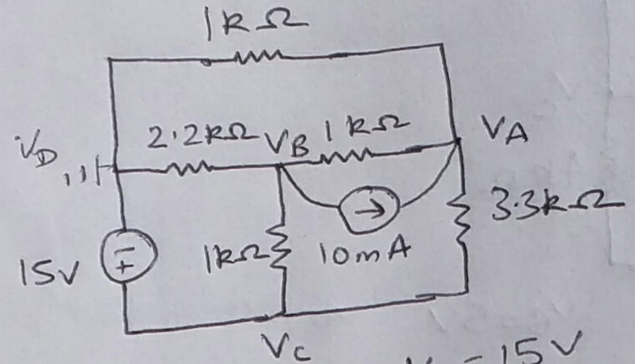
KCL at A :- $\dfrac{V_B - V_A}{4\times10^3} + 10\times10^{-3} - \dfrac{V_A - V_D}{4\times10^3} = 0$

KCL at C : $\dfrac{V_B - V_C}{1\times10^3} - \dfrac{V_C - V_D}{4\times10^3} - 10\times10^{-3} = 0.$

Solving this gives.

$V_A = 22.5\,V \qquad V_C = -4\,V.$

3.8)




$V_C = 15V$ $\quad V_D = 0$

Using method 2.

a) Node A: $\left(\frac{1}{1\times10^3} + \frac{1}{3.3\times10^3} + \frac{1}{1\times10^3}\right)V_A - \frac{1}{1\times10^3}V_B - \frac{1}{3.3\times10^3}V_C - 10\times10^{-3} = 0$

Node B: $\left(\frac{1}{1\times10^3} + \frac{1}{2.2\times10^3} + \frac{1}{1\times10^3}\right)V_B - \frac{1}{1\times10^3}V_A - \frac{1}{1\times10^3}V_C + 10\times10^{-3} = 0$

Multiplying both equations by $10^3$ & substituting for $V_C$, we get.

$$\begin{bmatrix} 1+\frac{1}{3.3}+1 & -1 \\ -1 & 1+\frac{1}{2.2}+1 \end{bmatrix}\begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} \frac{15}{3.3}+10 \\ 15-10 \end{bmatrix} = \begin{bmatrix} 14.546 \\ 5 \end{bmatrix}$$

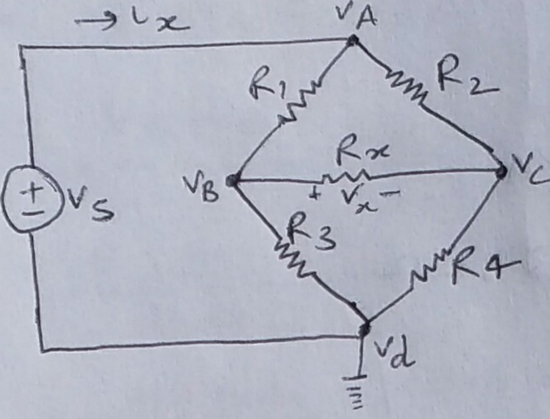
b) Solving the above equation, we get.

$V_A \simeq 8.75\ V \qquad V_B \simeq 5.6\ V$

$V_x = V_A - 0 = 8.75V.$

$i_x = \frac{0 - V_B}{2.2\times10^3} \simeq -2.55\ mA.$

3.10)




a) $V_D = 0\,V$

$V_A = V_S$

~~node B:~~

~~node C:~~

node B: $\left(\frac{1}{R_3} + \frac{1}{R_x} + \frac{1}{R_1}\right) V_B - \frac{1}{R_1} V_A - \frac{1}{R_3} V_D - \frac{1}{R_x} V_C = 0$

$\Rightarrow \left(\frac{1}{R_3} + \frac{1}{R_x} + \frac{1}{R_1}\right) V_B - \frac{V_S}{R_1} - \frac{V_C}{R_x} = 0.$

node C: $\left(\frac{1}{R_x} + \frac{1}{R_4} + \frac{1}{R_2}\right) V_C - \frac{1}{R_x} V_B - \frac{1}{R_2} V_A - \frac{1}{R_4} V_D = 0$

$\Rightarrow \left(\frac{1}{R_x} + \frac{1}{R_4} + \frac{1}{R_2}\right) V_C - \frac{V_B}{R_x} - \frac{V_S}{R_2} = 0.$

$$\begin{bmatrix} \frac{1}{R_3} + \frac{1}{R_x} + \frac{1}{R_1} & -\frac{1}{R_x} \\ -\frac{1}{R_x} & \frac{1}{R_x} + \frac{1}{R_4} + \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} \frac{V_S}{R_1} \\ \frac{V_S}{R_2} \end{bmatrix}$$

for $R_1 = R_4 = 2k\Omega$, $R_2 = R_3 = 500\Omega$, $R_x = 750\Omega$, $V_S = 15V$.

b)

$$\Rightarrow \begin{bmatrix} \frac{1}{500} + \frac{1}{750} + \frac{1}{2000} & -\frac{1}{750} \\ -\frac{1}{750} & \frac{1}{750} + \frac{1}{2000} + \frac{1}{500} \end{bmatrix} \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 15/2000 \\ 15/500 \end{bmatrix}$$

$\Rightarrow V_B = 5.32V$

$V_C = 9.68V.$

$\Rightarrow \quad V_x = V_B - V_C = 5.32 - 9.68 = -4.36\,V$

$$i_x = -\frac{V_B - V_A}{R_1} + -\frac{V_C - V_A}{R_2}$$

$$= -\frac{5.32 - 15}{2000} + -\frac{9.68 - 15}{500}$$

$$= 4.8\,mA + 10.6\,mA$$

$$= 15.4\,mA$$

c) $R_4$ is variable;

$V_x = 0$

$\Rightarrow V_B = V_C$

$$\begin{bmatrix} \frac{1}{500} + \frac{1}{750} + \frac{1}{2000} & -\frac{1}{750} \\ -\frac{1}{750} & \frac{1}{750} + \frac{1}{500} + \frac{1}{R_4} \end{bmatrix} \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 15/2000 \\ 15/500 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 0.0038 & -0.0013 \\ -0.0013 & \frac{1}{R_4} + 0.0033 \end{bmatrix} \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 0.0075 \\ 0.03 \end{bmatrix}$$

$$0.0038\,V_B - 0.0013\,V_B = 0.0075$$

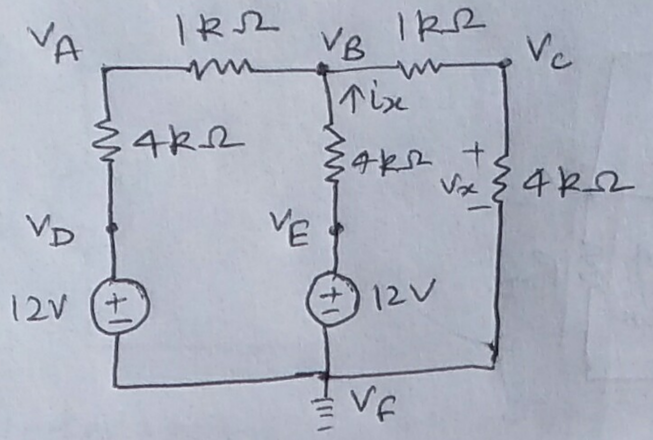
$$\Rightarrow V_B = 3V.$$

$$-0.0013 \times 3 + \left(\frac{1}{R_4} + 0.0033\right) \times 3 = 0.03.$$

$$\Rightarrow \frac{1}{R_4} + 0.0033 = 0.0113$$

$$\Rightarrow R_4 = \frac{1}{0.0080} = 125\,\Omega.$$

3.15)




$V_D = 12V \quad V_E = 12V \quad V_F = 0.$

Using method 2.

a) Node A: $\left(\frac{1}{4\times10^3} + \frac{1}{1\times10^3}\right)V_A - \frac{1}{1\times10^3}V_B - \frac{V_D}{4\times10^3} = 0$

Node B: $\left(\frac{1}{4\times10^3} + \frac{1}{1\times10^3} + \frac{1}{1\times10^3}\right)V_B - \frac{1}{1\times10^3}V_A - \frac{1}{1\times10^3}V_C - \frac{V_E}{4\times10^3} = 0$

Node C: $\left(\frac{1}{4\times10^3} + \frac{1}{1\times10^3}\right)V_C - \frac{1}{1\times10^3}V_B - \frac{1}{4\times10^3}V_F = 0.$

Substituting for $V_D, V_E$ & $V_F$ & multiplying by $10^3$ bothsides

$$\begin{bmatrix} 1.25 & -1 & 0 \\ -1 & 2.25 & -1 \\ 0 & -1 & 1.25 \end{bmatrix}\begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$$

$V_A = 9.05V \quad V_B = 8.31V \quad V_C = 6.65V.$

b) $i_A = \frac{V_D - V_A}{4\times10^3} = \frac{12-9.05}{4\times10^3} \approx 0.74mA$

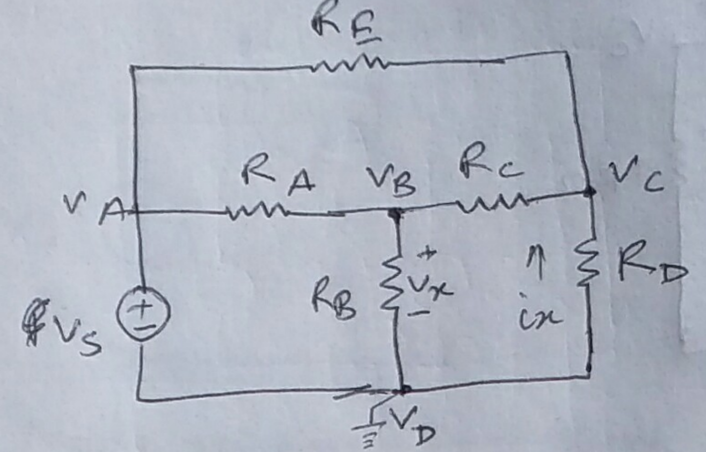
$i_B = \frac{V_C - V_F}{4\times10^3} = \frac{6.65-0}{4\times10^3} \approx 1.67mA.$

c) $V_x = V_C - V_F = V_C - 0 = 6.65V.$

$i_x = \frac{V_E - V_B}{4\times10^3} = \frac{12-8.31}{4\times10^3} = 0.93mA.$

another way: $i_x = i_B - i_A = 0.93mA.$

3.17)




$V_D = 0 \quad V_A = V_S$. Using method 2

b) at Node B: $\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)V_B - \frac{1}{R_A}V_A - \frac{1}{R_B}V_D - \frac{1}{R_C}V_C = 0$.

$$\Rightarrow \left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)V_B - \frac{V_C}{R_C} = \frac{V_S}{R_A}$$

node C: $\left(\frac{1}{R_C}+\frac{1}{R_D}+\frac{1}{R_E}\right)V_C - \frac{1}{R_C}V_B = \frac{V_S}{R_E}$

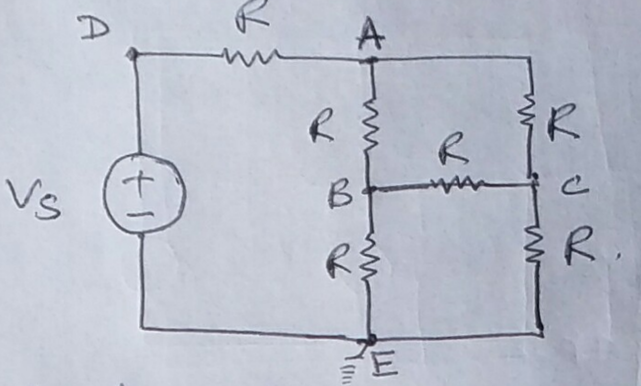
$$\begin{bmatrix} \frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C} & -\frac{1}{R_C} \\ -\frac{1}{R_C} & \frac{1}{R_C}+\frac{1}{R_D}+\frac{1}{R_E} \end{bmatrix}\begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} V_S/R_A \\ V_S/R_E \end{bmatrix}$$

c) $$\begin{bmatrix} V_B \\ V_C \end{bmatrix} = \frac{1}{\left(\left(\frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C}\right)\left(\frac{1}{R_C}+\frac{1}{R_D}+\frac{1}{R_E}\right)-\frac{1}{R_C^2}\right)}\begin{bmatrix} \frac{1}{R_C}+\frac{1}{R_D}+\frac{1}{R_E} & \frac{1}{R_C} \\ \frac{1}{R_C} & \frac{1}{R_A}+\frac{1}{R_B}+\frac{1}{R_C} \end{bmatrix}\begin{bmatrix} V_S/R_A \\ V_S/R_E \end{bmatrix}$$

$$V_x = V_B$$

$$i_x = \frac{-V_C}{R_D}$$

3.23)




$V_D = V_s \quad V_E = 0$

Node voltage equations (Using method 2)

$$\begin{bmatrix} \frac{1}{R}+\frac{1}{R}+\frac{1}{R} & -\frac{1}{R} & -\frac{1}{R} \\ -\frac{1}{R} & \frac{1}{R}+\frac{1}{R}+\frac{1}{R} & -\frac{1}{R} \\ -1/R & -\frac{1}{R} & \frac{1}{R}+\frac{1}{R}+\frac{1}{R} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \end{bmatrix} = \begin{bmatrix} V_s/R \\ 0 \\ 0 \end{bmatrix}$$

$\Rightarrow \quad 3V_A - V_B - V_C = V_s \quad -\text{(i)}$

$-V_A + 3V_B - V_C = 0 \quad -\text{(ii)}$

$-V_A - V_B + 3V_C = 0 \quad -\text{(iii)}$

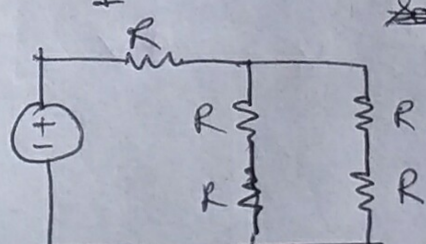
Solving this,

$V_A = 2V_B$

$V_B = V_C$

As $V_B = V_C$, no current flows through resistor between B & C.
~~So it is an open circuit~~.
So it acts as an ~~open~~ short circuit.

putting in (i) $\Rightarrow$




$$\boxed{R_{Eq} = R_{IN} = R + (2R \parallel 2R) = 2R}$$

Alternatively,

current through D to A = current from A to B + current from A to C.

(KCL at node A).

$$\Rightarrow \quad \frac{V_D - V_A}{R} = \frac{V_A - V_B}{R} + \frac{V_A - V_C}{R}$$

$$\Rightarrow \quad V_S - V_A = 2V_A - (V_B + V_C) = V_A.$$

$$\Rightarrow \quad V_A = \frac{V_S}{2}, \quad V_B = \frac{V_S}{4}, \quad V_C = \frac{V_S}{4}$$

current through resistor between D & A is

$$i = \frac{V_S - \frac{V_S}{2}}{R} = \frac{V_S}{2R}.$$

equivalent resistance = ?

$$\frac{i}{V_S} = \left(\frac{V_S}{2R}\right)\Big/V_S = \frac{1}{2R}.$$

$\therefore$ eq$^{nt}$ resistance $= \dfrac{V_S}{i} = 2R.$