MAE140 - Linear Circuits - Winter 17
Midterm, February 9

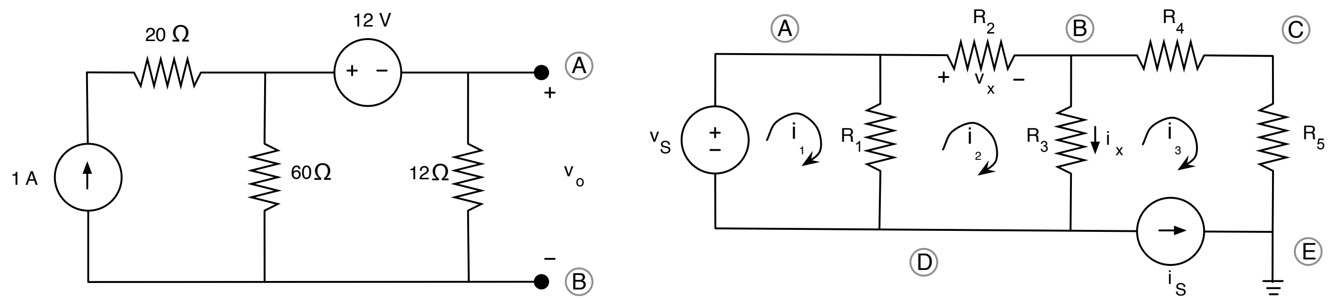
Instructions

(i) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities

(ii) You have 75 minutes

(iii) Do not forget to write your **name** and **student number**

Good luck!



(a) Question 1 (b) Questions 2&3

Figure 1: Circuits for all questions.

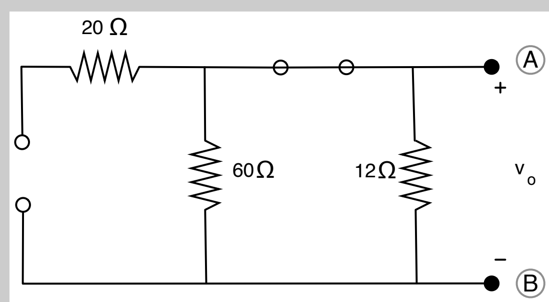
1. **Equivalent circuits**

Part I: [2 points] Turn off all the sources in the circuit of Figure 1(a) and find the equivalent resistance as seen from terminals A and B.

Solution: Part I: We start by switching off the sources.

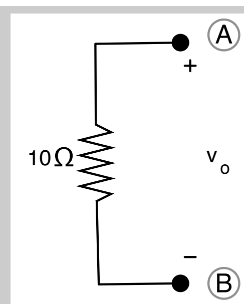
We substitute the voltage source by a short circuit, and the current source by an open circuit. Then, we get the circuit on the right

(+ 1 point)



No current flows through the 20Ohms resistor, so we can discard it. Therefore, we combine the two resistances in parallel to get the circuit

(+ 1 point)

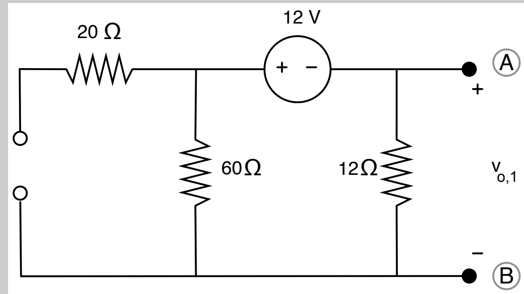


Part II: [4 points] Find the voltage v_0 using only superposition, source transformations, voltage division, and current division.

Solution: **Part II:** To use superposition, we first turn off the independent current source

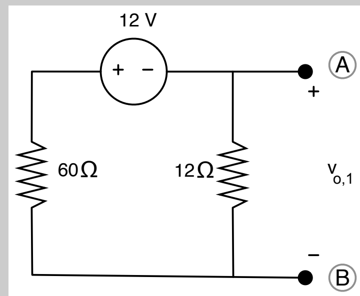
We substitute the current source by an open circuit. Then, we get the circuit on the right

(+ .5 point)



The 20Ohms resistors does not have any current going through it, so we can just redraw the circuit as follows

(+ .5 point)



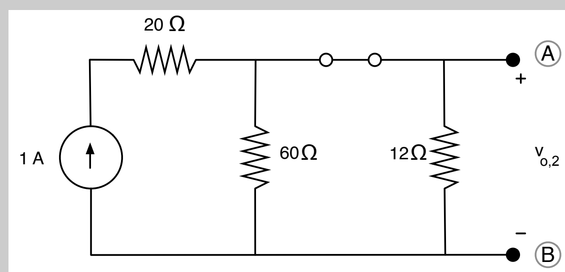
Now, using voltage division, we obtain

$$v_{o,1} = \frac{12}{12+60}(-12) = -2V \qquad \textbf{(+ .5 point)}$$

Next, we turn off the independent voltage source.

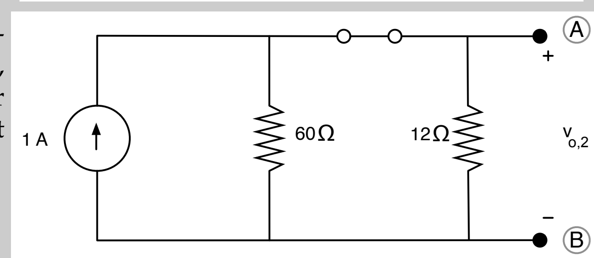
We substitute the voltage source by a closed circuit. Then, we get the circuit on the right

(+ .5 point)



The current source is in series with the 20Ohms resistor. From the point of view of the rest of the circuit, there is no difference whether the resistor is there or not. Therefore, we use source transformation to get the circuit on the right

(+ .5 point)



Using now current division and Ohm's law, we obtain

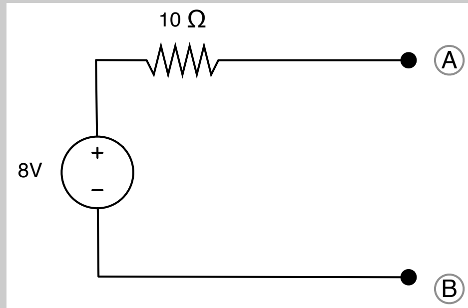
$$v_{0,2} = 12\frac{1/12}{1/12+1/60}(1) = 10V \qquad \textbf{(+ .5 point)}$$

By superposition, we conclude that

$$v_0 = v_{0,1} + v_{0,2} = -2 + 10 = 8V \qquad \textbf{(+ 1 point)}$$

Part III: [1 point] What is the Thévenin equivalent of the circuit as seen from terminals A and B?

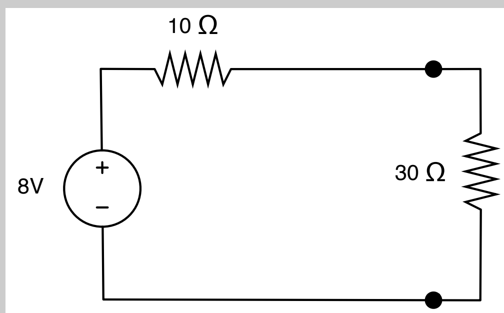
Solution: Part III: We have computed the equivalent resistance from terminals A and B with all sources turned off in Part I, and the open-circuit voltage in Part II. Therefore, the Thévenin equivalent of the circuit is simply



(+ 1 point)

Part IV: [1 point] Find the power absorbed by a $30\,\Omega$ resistor that is connected to terminals A and B.

Solution:
Part IV: We use the Thévenin equivalent of the circuit to obtain the answer in an easy way. Connecting the 30Ohms resistor gives rise to the circuit



(+ .5 point)

By voltage division, the voltage drop across the load is

$$v = \frac{30}{30+10}(8) = 6V$$

Therefore, the power absorbed by the resistor load is

$$P = v^2G = (6)^2\frac{1}{30} = 1.2W$$ **(+ .5 point)**

2. **Node voltage analysis**

Part I: [5 points] Formulate node-voltage equations for the circuit in Figure 1(b). Use the node labels A through E provided in the figure and clearly indicate how you handle the presence of a voltage source. The final equations must depend only on unknown node voltages and the resistor values R_1 through R_5. **Do not modify the circuit or the labels**. No need to solve any equations!

Solution: Part I: There are five nodes in this circuit. The ground node E (hence $v_E = 0$) has already been chosen for us. Unfortunately, with this choice, the ground node is not directly connected to the voltage source, so we cannot use method 2 to take care of it. The voltage source is also not in series with any resistor, so method 1 is discarded too. Therefore, we are left with method 3, a supernode (A+D).

(+ 1 point)

The equation for the supernode is

$$v_A - v_D = v_S \qquad \textbf{(+ 1 point)}$$

KCL at the supernode takes the form

$$G_2(v_A - v_B) + G_3(v_D - v_B) + i_S = 0 \qquad \textbf{(+ 1 point)}$$

where, as we do usually, we denote $G_i = 1/R_i$.
We also need to write KCL at nodes B and C. These take the form

$$G_2(v_B - v_A) + G_3(v_B - v_D) + G_4(v_B - v_C) = 0 \qquad \textbf{(+ 1 point)}$$
$$G_4(v_C - v_B) + G_5 v_C = 0 \qquad \textbf{(+ 1 point)}$$

This gives us a total of 4 equations in the 4 node voltage unknowns v_A, v_B, v_C, v_D. In matrix form, we can write this as

$$\begin{pmatrix} 1 & 0 & 0 & -1 \\ G_2 & -G_2 - G_3 & 0 & G_3 \\ -G_2 & G_2 + G_3 + G_4 & -G_4 & -G_3 \\ 0 & -G_4 & G_4 + G_5 & 0 \end{pmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \\ v_D \end{pmatrix} = \begin{pmatrix} v_S \\ -i_S \\ 0 \\ 0 \end{pmatrix}$$

Part II: [1 point] Provide expressions for the voltage v_x and the current i_x in terms of node voltages.

Solution: Part II: In terms of the node voltages, v_x and i_x can be expressed as

$$v_x = v_A - v_B \qquad \textbf{(+ .5 point)}$$
$$i_x = G_3(v_B - v_D) \qquad \textbf{(+ .5 point)}$$

Part III: [1 bonus point] If it was up to us, would you have chosen ground differently? Justify your answer.

Solution: Yes! Either node A or D are better choices, because they are directly connected to the voltage source, and any of those choices would allow us to use method 2 (instead of having to resort to method 3 – supernode).

(+ 1 bonus point)

3. **Mesh current analysis**

Part I: [5 points] Formulate mesh-current equations for the circuit in Figure 1(b). Use the mesh currents shown in the figure and clearly indicate how you handle the presence of the current source. The final equations should only depend on the unknown mesh currents and the resistor values R_1 through R_5. **Do not modify the circuit or the labels**. No need to solve any equations!

Solution: Part I: There are three meshes in this circuit. The current source, i_S, belongs to only one mesh, so we can use method 2 to deal with it.

(+ 1 point)

Consequently, we set

$$i_3 = -i_S \qquad \textbf{(+ 1 point)}$$

We write KVL for each of the other 2 meshes. For mesh 1, KVL reads like

$$R_1(i_1 - i_2) - v_S = 0 \qquad \textbf{(+ 1.5 point)}$$

For mesh 2, KVL reads like

$$R_1(i_2 - i_1) + R_2 i_2 + R_3(i_2 - i_3) = 0 \qquad \textbf{(+ 1.5 point)}$$

This gives us a total of 3 equations in the 3 mesh current unknowns i_1, i_2, i_3. In matrix form, we can write this as

$$\begin{pmatrix} 0 & 0 & 1 \\ R_1 & -R_1 & 0 \\ -R_1 & R_1 + R_2 + R_3 & -R_3 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} -i_S \\ v_S \\ 0 \end{pmatrix}$$

Alternatively, one may substitute the value of i_3 into the KVL equation for mesh 2, and set up a system of 2 equations with 2 unknowns i_1 and i_2.

Part II: [1 point] Provide expressions for the voltage v_x and the current i_x in terms of mesh currents.

Solution: Part II: In terms of the node voltages, v_x and i_x can be expressed as

$$v_x = R_2 i_2 \qquad \textbf{(+ .5 point)}$$

$$i_x = i_2 - i_3 \qquad \textbf{(+ .5 point)}$$

Part III: [1 bonus point] Would changing the value of the resistors R_4 and R_5 have any effect on the mesh currents? Is this consistent with what we know about source transformations?

Solution: The value of these resistors does not have any effect on the mesh currents. This is because, from what we have learned about source transformations, the portion of the circuit composed by R_4, R_5, and i_S is equivalent, from the point of view of the rest of the circuit, to just the current source i_S. This can also be seen in the mesh current equations we wrote in Part I, where R_4 and R_5 do not appear. Changing the value of those resistors would definitely change the voltage drop seen by the current source.

(+ 1 bonus point)