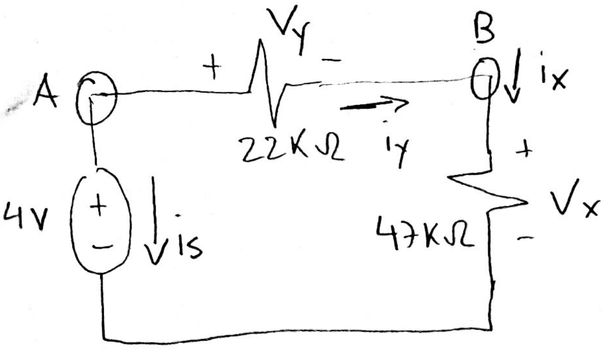
# Homework 2

2.29)




Connection Constraints:

KVL: $-24V + V_y + V_x = 0$ (1)

KCL: Node (A): $i_s + i_y = 0 \Rightarrow i_s = -i_y$

Node (B): $i_y - i_x = 0 \Rightarrow i_y = i_x$

Element Constraints:

$V_y = 22K\Omega\, i_y = 22K\Omega\, i_x$

$V_x = 47K\Omega\, i_x$ (2)

Using (1) and Replacing $V_y$ and $V_x$ in terms of $i_x$:

$$-24V + 22K\Omega\, i_x + 47K\Omega\, i_x = 0$$

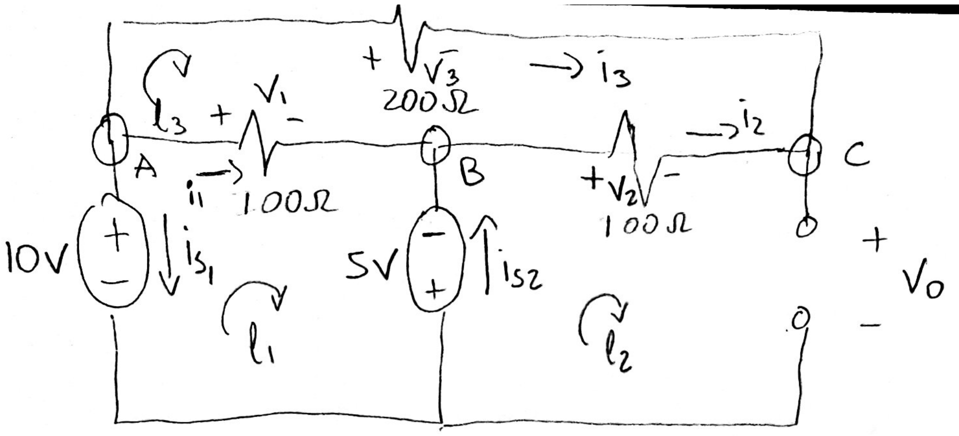
$$69K\Omega\, i_x = 24V$$

$$\boxed{i_x = \frac{24V}{69K\Omega} = 0.347\,mA}$$

Using (2):

$$\boxed{V_x = 47K\Omega\,(0.347mA) = 16.34\,V}$$

2.34)




Find $V_o$

**KVL**: $\ell_1$: $-10V + V_1 - 5V = 0 \Rightarrow V_1 = 15V$

$\ell_2$: $5V + V_2 + V_o = 0$

$\ell_3$: $-V_1 + V_3 - V_2 = 0$

**KCL**: A: $i_1 + i_3 + i_{s1} = 0$

B: $i_1 + i_{s2} - i_2 = 0$

C: $i_2 + i_3 = 0 \Rightarrow i_2 = -i_3$

Element Constraints: • $V_1 = 100\Omega\, i_1 \Rightarrow i_1 = \frac{15V}{100\Omega} = \frac{3}{20}A$

• $V_2 = 100\Omega\, i_2 = -100\, i_3$

• $V_3 = 200\Omega\, i_3$

In ($\ell_3$) replace $V_2$ and $V_3$ in terms of $i_3$:

$-V_1 + 200\Omega\, i_3 + 100\Omega\, i_3 = 0 \Rightarrow -15V + 300\Omega\, i_3 = 0$

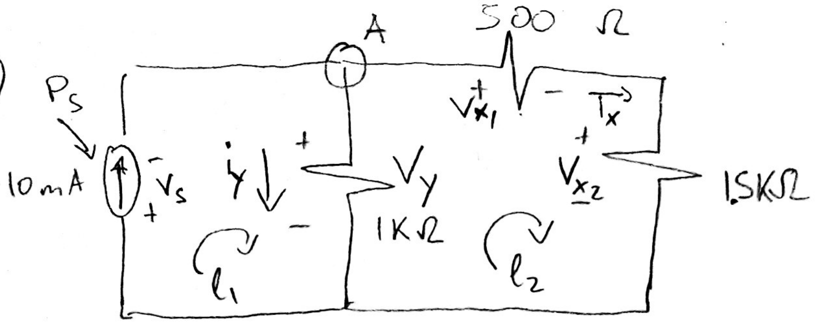
$i_3 = \frac{15V}{300\Omega} = \frac{1}{20}A$

Find $V_2$ and $V_3$:

$V_2 = -100\Omega\left(\frac{1}{20}A\right) = -5V$

$V_3 = 200\Omega\left(\frac{1}{20}A\right) = 10$

From ($\ell_2$): $5V + (-5V) + V_o = 0 \Rightarrow \boxed{V_o = 0}$

2.35) 

Find Power

KCL: A; $10mA - i_y - i_x = 0 \Rightarrow i_x = 10mA - i_y$ (1)

KVL: $\ell_1$: $V_s + V_y = 0 \Rightarrow V_s = -V_y$

$\ell_2$: $-V_y + V_{x_1} + V_{x_2} = 0$

Element constraints:

$V_y = 1K\Omega \, i_y$

$V_{x_1} = 500\Omega \times i_x = 500\Omega\,(10mA - i_y)$

$V_{x_2} = 1.5K\Omega \, i_x = 1.5K\Omega\,(10mA - i_y)$

Replace in ($\ell_2$) $V_y$, $V_{x_1}$, and $V_{x2}$ in terms of $i_y$:

$-1K\Omega\, i_y + 500\Omega\,(10mA - i_y) + 1.5K\Omega\,(10mA - i_y) = 0$

$-1\times10^3 i_y + 5000\times10^{-3} - 500\, i_y + 15 - 1.5\times10^3 i_y = 0$

$3000\, i_y = 20$

$i_y = \frac{20}{3000}\,A = \frac{1}{150}\,A$

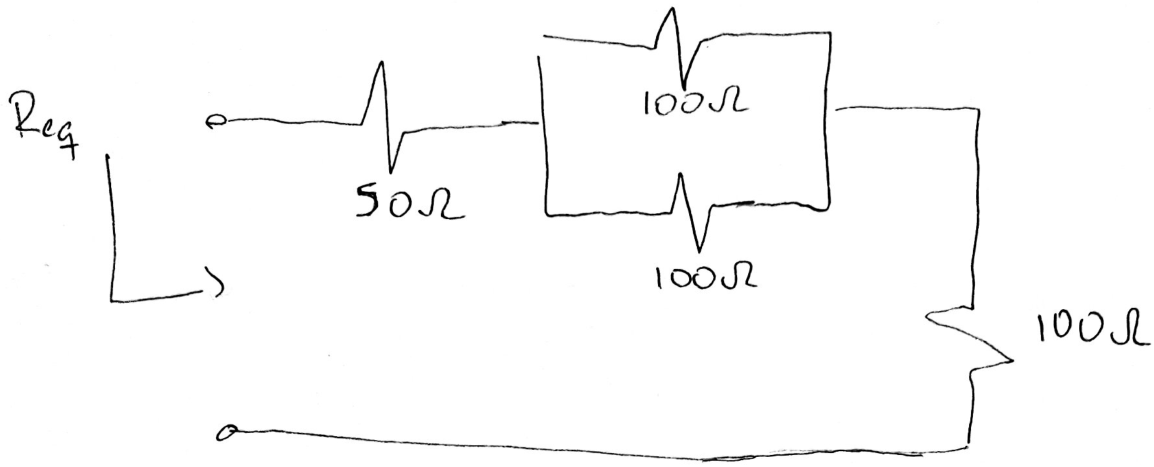
$V_y = 1\times10^3\left(\frac{1}{150}\right) = 6.66\,V$ , $i_s = 10mA$

$V_s = -6.66\,V$

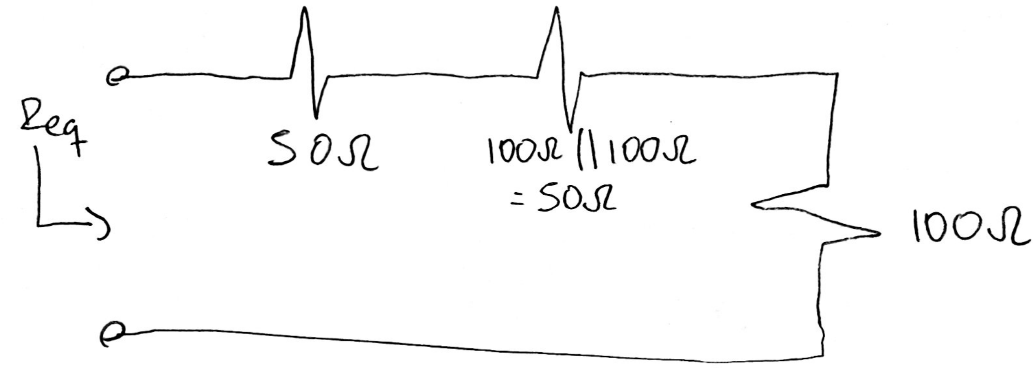
Power provided by the source: $P_s = V_s\, i_s = (-6.67V)(10\times10^{-3}A)$

$\boxed{P_s = -66.7\,mW}$

2.43) When the switch is open:

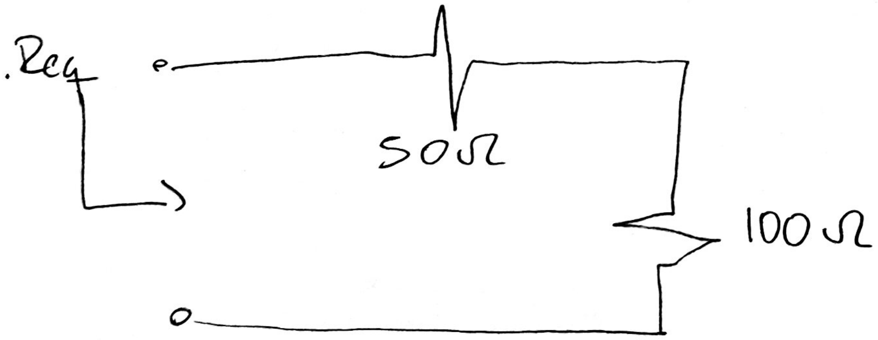



$\Leftrightarrow$



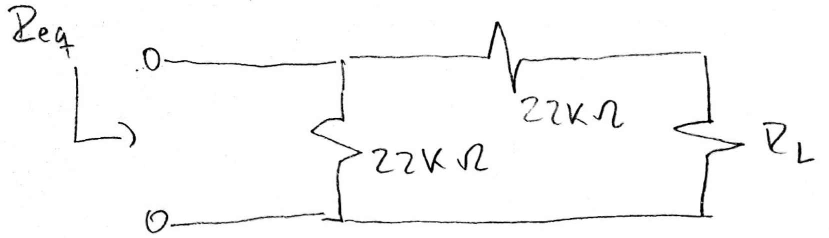

$$R_{eq} = 50\Omega + 50\Omega + 100\Omega = 200\Omega$$

When the switch is closed:




$$R_{eq} = 50\Omega + 100\Omega$$
$$= 150\ \Omega$$

2.48) a) When $R_{eq} = 15 K\Omega$, Find $R_L$




$$R_{eq} = (22K\Omega + R_L) \,||\, 22K\Omega$$

$$= \frac{(22K\Omega + R_L)\, 22K\Omega}{22K\Omega + R_L + 22K\Omega}$$

$$= \frac{484 \times 10^6 + 22 \times 10^3 R_L}{44 \times 10^3 + R_L} \ \Omega$$

Since it is given $R_{eq} = 15K\Omega$,

$$(15 \times 10^3)(44 \times 10^3 + R_L) = 484 \times 10^6 + 22 \times 10^3 R_L \qquad (1)$$

$$660 \times 10^6 + 15 \times 10^3 R_L = 484 \times 10^6 + 22 \times 10^3 R_L$$

$$7 \times 10^3 R_L = 176 \times 10^6$$

$$R_L = \frac{176}{7} \times 10^3 \ \Omega$$

$$\boxed{R_L = 25.14 \ K\Omega}$$

b) when $R_{eq} = 11 K\Omega$

In (1) we replace $15 \times 10^3$ by $11 \times 10^3$, i.e,

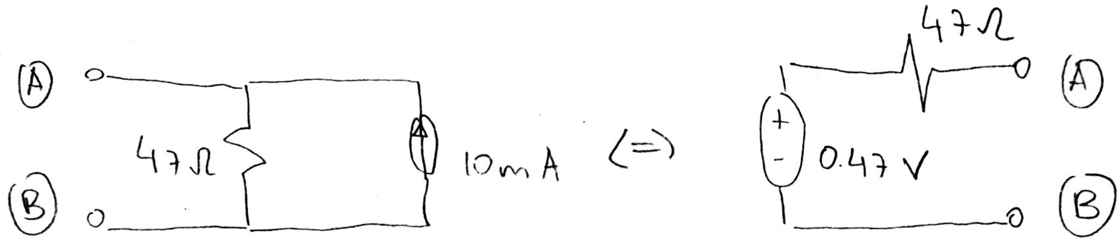
$$(11 \times 10^3)(44 \times 10^3 + R_L) = 484 \times 10^6 + 22 \times 10^3 R_L$$

$$\cancel{484 \times 10^6} + 11 \times 10^3 R_L = \cancel{484 \times 10^6} + 22 \times 10^3 R_L$$

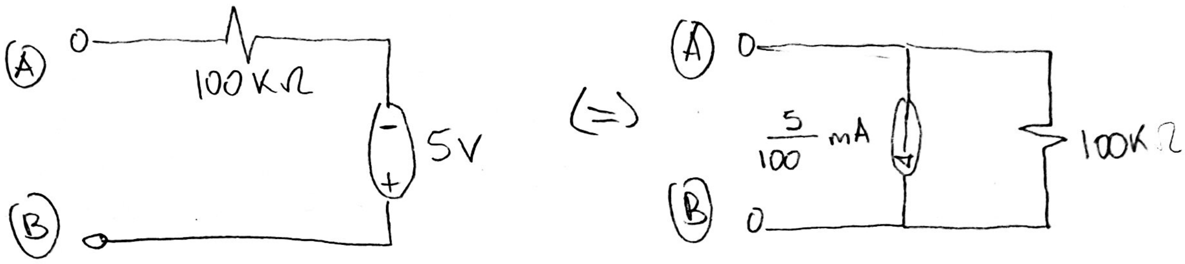
$$\boxed{R_L = 0}$$

2.50)

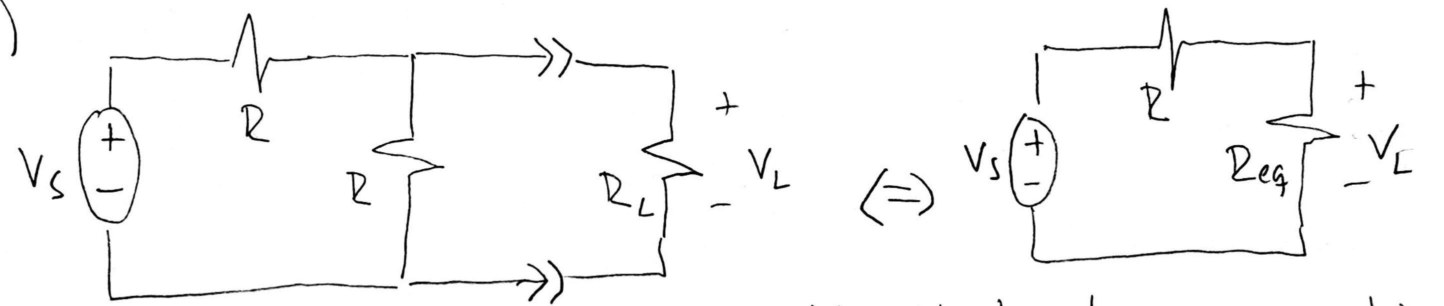
a)




b)




2.59)




$$R_{eq} = R \parallel R_L = \frac{R R_L}{R + R_L}$$

Note that when we combine R and $R_L$ in parallel, $V_L$ remains unchanged in $R_{eq}$.

Using voltage division:

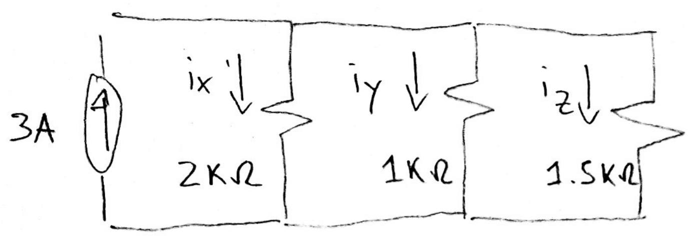
$$V_L = \frac{R_{eq}}{R + R_{eq}} V_s$$

$$= \frac{\frac{R R_L}{R + R_L}}{R + \frac{R R_L}{R + R_L}} V_s$$

$$= \frac{R R_L}{R(R + R_L) + R R_L} V_s$$

$$= \frac{R_L}{R + 2R_L} V_s$$

2.60)




Using current division:

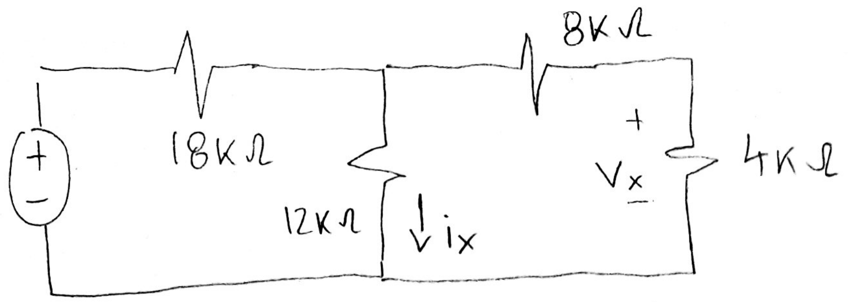
$$i_x = \frac{\frac{1}{2K\Omega}}{\left(\frac{1}{2K\Omega}+\frac{1}{1K\Omega}+\frac{1}{1.5K\Omega}\right)}(3A) = \frac{\frac{1}{2}\times 3A}{\frac{13}{6}} = \frac{3}{13}\times 3A = \frac{9}{13}A = 0.69A$$

$$i_y = \frac{\frac{1}{1K\Omega}}{\frac{1}{2K\Omega}+\frac{1}{1K\Omega}+\frac{1}{1.5K\Omega}}(3A) = \frac{1}{\frac{13}{6}}\times 3A = \frac{18}{13}A = 1.38A$$

$$i_z = \frac{\frac{1}{1.5K\Omega}}{\frac{1}{2K\Omega}+\frac{1}{1K\Omega}+\frac{1}{1.5K\Omega}}(3A) = \frac{\frac{1}{1.5}\times 3A}{\frac{13}{6}} = \frac{12}{13}A = 0.92A$$

Sum of the currents: $i_x + i_y + i_z = \frac{9}{13}A + \frac{18}{13}A + \frac{12}{13}A = 3A$

2.76)




First we find $i_x$. For that we apply source transformation

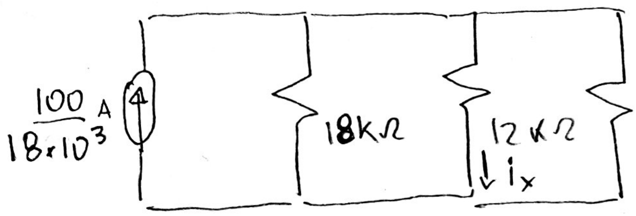



Fig 1

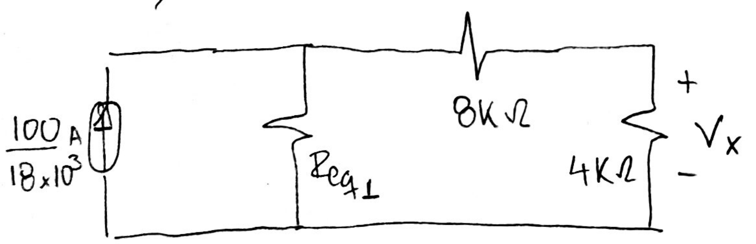
$$R_{eq} = 8K\Omega + 4K\Omega = 12K\Omega$$

Using current division:

$$i_x = \frac{\frac{1}{12}}{\frac{1}{18} + \frac{1}{12} + \frac{1}{12}} \cdot \frac{100}{18} mA = \frac{\frac{1}{12}}{\frac{2+3+3}{36}} \cdot \frac{100}{18} mA$$

$$= \frac{3}{8} \cdot \frac{100}{18} mA = 2.08 mA$$

Next, we find $V_x$. From Fig 1 we have:

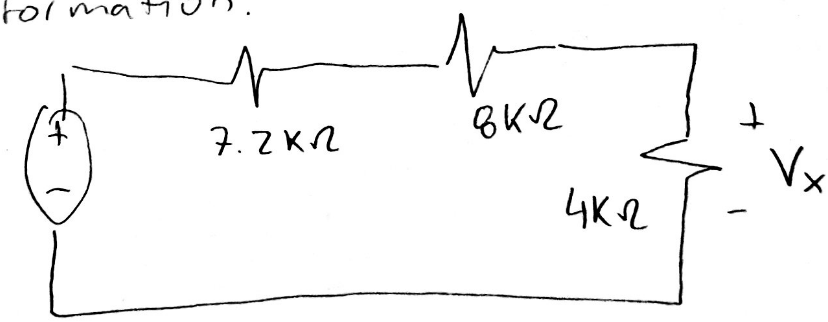



$$R_{eq1} = 18K\Omega \parallel 12K\Omega$$

$$= 7.2K\Omega$$

We apply source transformation:

$$\frac{100}{18 \times 10^3} \cdot 7.2 \times 10^3 V = 40V$$




Using Voltage division: $V_x = \frac{4}{19.2} \cdot 40V = 8.33 V$

2.7B)

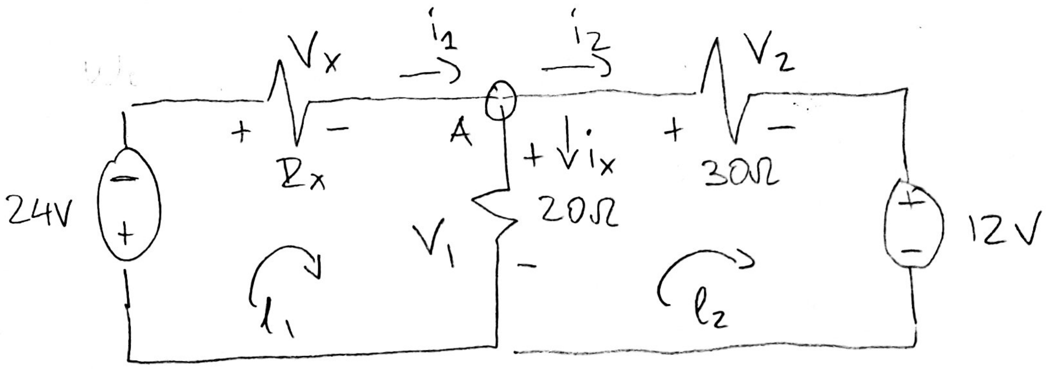



Fig 1

KVL: $l_1$: $24 + V_x + V_1 = 0$

$l_2$: $-V_1 + V_2 + 12V = 0$

Element constraints: $V_1 = 20\Omega\, i_x = 0$ (Recall that the problem gives $i_x = 0$)

$V_2 = 30\Omega\, i_2$ (1)

KCL: Node A: $i_1 - i_2 - i_x = 0$

$i_1 = i_2$ (since $i_x = 0$)

Using ($l_1$), we have $V_x = -24V$ (2)

Using ($l_2$), we have $V_2 = V_1 - 12V = -12V$

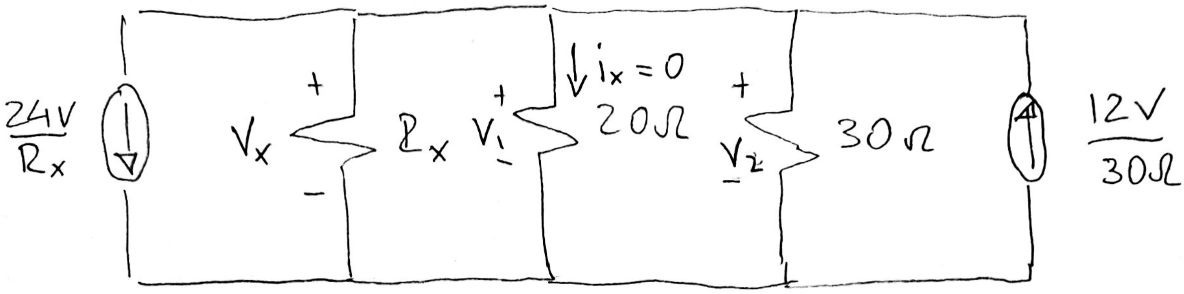
From (1): $V_2 = 30\Omega\, i_2 = -12V \Rightarrow i_2 = \frac{-12}{30} A = \frac{-2}{5} A$

$V_x = R_x i_1 = R_x i_2 = R_x\left(\frac{-2}{5} A\right) \Rightarrow$ Using (2): $-24V = \frac{-2}{5} A\, R_x$

$\boxed{R_x = 60\Omega}$

Another way to do it:

We use source transformation to the circuit in P2-78




$V_x = V_1 = V_2$ (Since all resistors are in parallel)

$V_1 = 20\Omega\ i_x = (20)(0)V = 0V \Rightarrow V_x = 0,\ V_2 = 0$

Since there is not voltage drop in any resistor, then there is not current flowing through the resistors.

It follows:

$$\underbrace{\frac{24V}{R_x}}_{\text{current source 1}} = \underbrace{\frac{12V}{30\Omega}}_{\text{current source 2}}$$

$$R_x = \frac{(24)(30)}{12} = 60\Omega$$