MAE140 - Linear Circuits - Winter 18
Midterm, February 8

Instructions

(i) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities

(ii) You have 75 minutes

(iii) Do not forget to write your **name** and **student number**

Good luck!

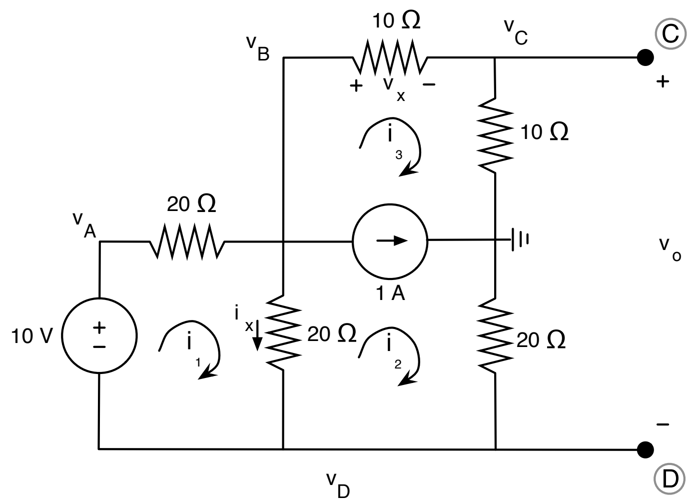


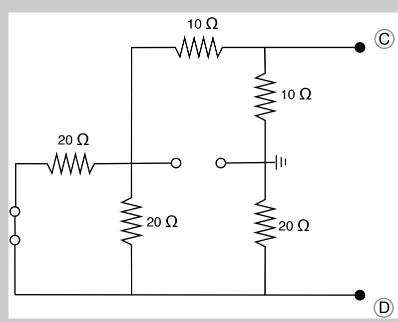
Figure 1: Circuit for all questions.

1. **Equivalent circuits**

Part I: [2 points] Turn off all the sources in the circuit of Figure 1 and find the equivalent resistance as seen from terminals C and D.

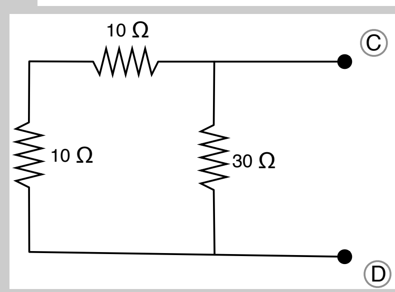
Solution: Part I: We start by switching off the sources.

We substitute the voltage source by a short circuit, and the current source by an open circuit. Then, we get the circuit on the right



(+ 0.5 point)

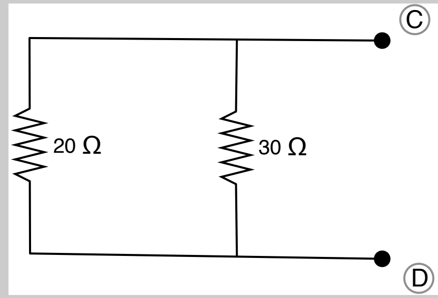
We combine the two 20Ohms resistors in parallel and the 10Ohms and 20Ohms resistors in series to get the circuit



(+ 0.5 point)

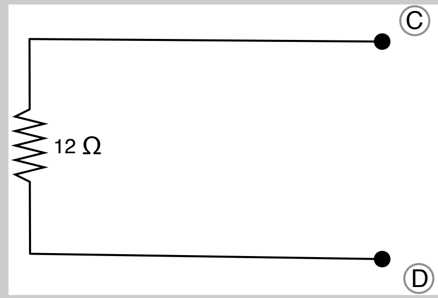
We combine now the two 10Ohms resistors in series to get the circuit

(+ 0.5 point)



Finally, we combine now the two resistors in parallel to get the circuit

(+ 0.5 point)

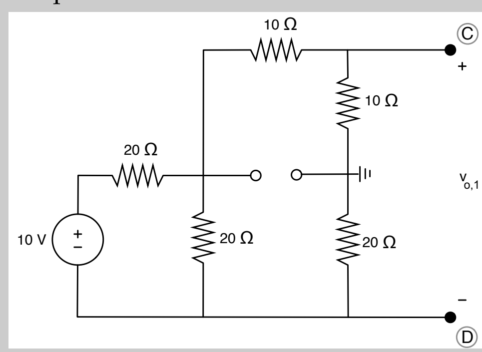


Part II: [5 points] Find the voltage v_0 using only superposition, source transformations, association of resistors, Ohm's law, voltage division, and current division.

Solution: Part II: To use superposition, we first turn off the independent current source

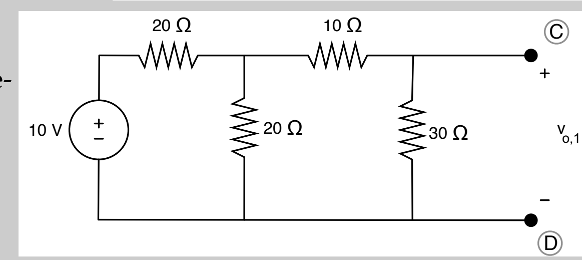
We substitute the current source by an open circuit. Then, we get the circuit on the right

(+ 0.5 point)



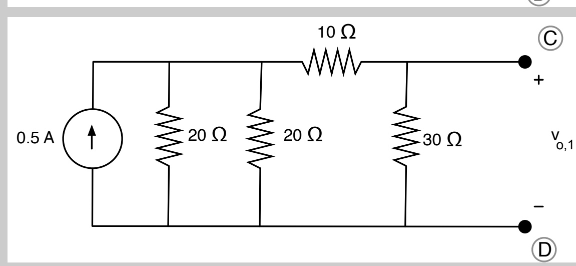
We combine the 10Ohms and 20Ohms resistors in series to obtain the circuit

(+ 0.25 point)



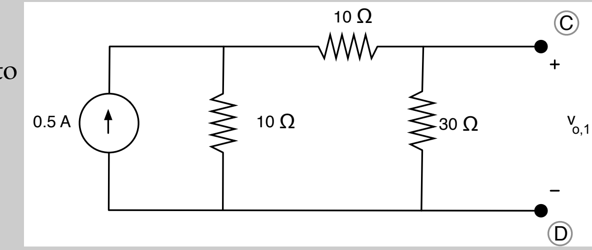
Next, we use source transformation to obtain

(+ 0.25 point)



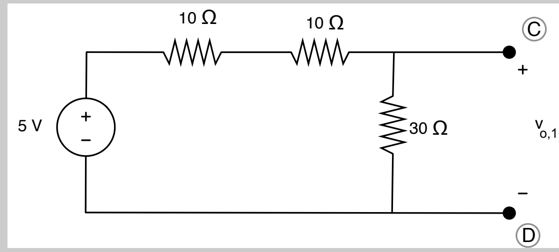
We combine the two 20Ohms resistors in parallel to obtain the circuit

(+ 0.25 point)



One more source transformation yields

(+ 0.25 point)



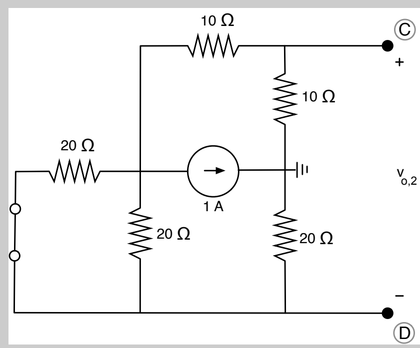
We can now compute $v_{o,1}$ by using voltage division. In fact,

$$v_{o,1} = \frac{30}{50}(5) = 3V$$ **(+ 0.5 point)**

Next, we turn off the independent voltage source.

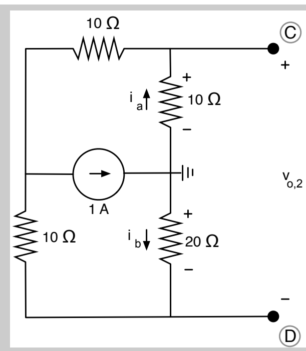
We substitute the voltage source by a closed circuit. Then, we get the circuit on the right

(+ 0.5 point)



We combine the two 20Ohms resistors in parallel to get the circuit on the right

(+ 0.5 point)



Note that $v_{o,2}$ is the sum of the voltage drops across two rightmost resistors. To determine each voltage drop, we can use current division to find the current flowing through each of them, then use Ohm's law. By current division, we have

$$i_a = \frac{1/20}{1/20 + 1/30}(1) = 0.6A$$ **(+ 0.25 point)**

$$i_b = \frac{1/30}{1/20 + 1/30}(1) = 0.4A$$ **(+ 0.25 point)**

Therefore, we obtain

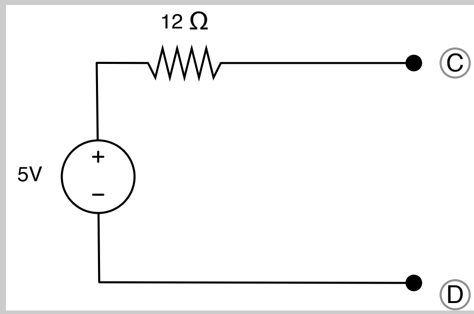
$$v_{o,2} = 10(-0.6) + 20(0.4) = 2V$$ **(+ 0.5 point)**

By superposition, we conclude that

$$v_0 = v_{0,1} + v_{0,2} = 3 + 2 = 5V$$ **(+ 1 point)**

Part III: [1 point] What is the Thévenin equivalent of the circuit as seen from terminals Ⓒ and Ⓓ?

Solution: Part III: We have computed the equivalent resistance from terminals Ⓒ and Ⓓ with all sources turned off in Part I, and the open-circuit voltage in Part II. Therefore, the Thévenin equivalent of the circuit is simply



(+ 1 point)

Part IV: [1 bonus point] If we short-circuit terminals Ⓒ and Ⓓ, what would be the short-circuit current?

Solution: The short-circuit current would precisely correspond to the Norton equivalent current. Given our answer in Part III, we know

$$i_{SC} = i_N = \frac{5}{12}A$$ **(+ 1 bonus point)**

would flow top down.

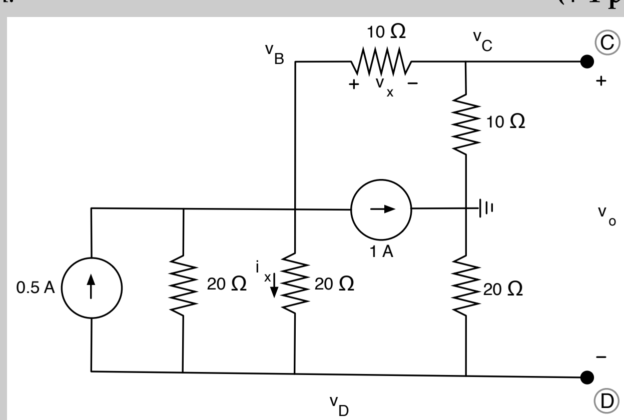
2. **Node voltage analysis**

Part I: [5 points] Formulate node-voltage equations for the circuit in Figure 1. Use the node labels provided in the figure. Clearly indicate how you handle the presence of a voltage source (if you have more than one choice to deal with it, use the simplest). The final equations must depend only on unknown node voltages. **Do not modify the circuit or the labels**. No need to solve any equations!

Solution: Part I: There are five nodes in this circuit. The ground node has already been chosen for us. Unfortunately, with this choice, the ground node is not directly connected to the voltage source, so we cannot use method 2 to take care of it. However, the voltage source is in series with the 20Ohms resistor, so we can use method 1, source transformation. **(+ 1 point)**

Doing source transformation has the added advantage of eliminating a node. After source transformation, we obtain

(+ 1 point)



We can now write node-voltage equations by inspection

$$\begin{pmatrix} \frac{1}{20}+\frac{1}{20}+\frac{1}{10} & -\frac{1}{10} & -\frac{1}{20}-\frac{1}{20} \\ -\frac{1}{10} & \frac{1}{10}+\frac{1}{10} & 0 \\ -\frac{1}{20}-\frac{1}{20} & 0 & \frac{1}{20}+\frac{1}{20}+\frac{1}{20} \end{pmatrix} \begin{pmatrix} v_B \\ v_C \\ v_D \end{pmatrix} = \begin{pmatrix} 0.5-1 \\ 0 \\ -0.5 \end{pmatrix} \qquad \textbf{(+ 3 points)}$$

These are 3 equations in 3 unknowns v_B, v_C, v_D. After simplification, this looks like

$$\begin{pmatrix} \frac{1}{5} & -\frac{1}{10} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{1}{5} & 0 \\ -\frac{1}{10} & 0 & \frac{3}{20} \end{pmatrix} \begin{pmatrix} v_B \\ v_C \\ v_D \end{pmatrix} = \begin{pmatrix} -0.5 \\ 0 \\ -0.5 \end{pmatrix}$$

Part II: [1 point] Provide expressions for the voltage v_x and the current i_x in terms of node voltages.

Solution: Part II: In terms of the node voltages, v_x and i_x can be expressed as

$$v_x = v_B - v_C \qquad \textbf{(+ .5 point)}$$

$$i_x = \frac{1}{20}(v_B - v_D) \qquad \textbf{(+ .5 point)}$$

Part III: [1 bonus point] Could you have used another method to deal with the voltage source? How can you determine v_A if you knew v_B, v_C, and v_D? Justify your answer.

Solution: We could have used a supernode (method 3) instead, combining nodes A and D. Because we used source transformation, which was simpler, we eliminated node A. However, solving the equations in Part II would give us v_B, v_C, and v_D, and then v_A can easily be determined from $v_A = 10 + v_D$.

(+ 1 bonus point)

3. **Mesh current analysis**

Part I: [5 points] Formulate mesh-current equations for the circuit in Figure 1. Use the mesh currents shown in the figure and clearly indicate how you handle the presence of the current source. The final equations should only depend on the unknown mesh currents. **Do not modify the circuit or the labels**. No need to solve any equations!

Solution: Part I: There are three meshes in this circuit. The current source belongs to two meshes and is not in parallel with any resistor, so we are forced to use method 3, a supermesh, to deal with it.

(+ 1 point)

Consequently, we set

$$i_2 - i_3 = 1 \qquad \textbf{(+ 1 point)}$$

And we write KVL for the supermesh as

$$10i_3 + 10i_3 + 20i_2 + 20(i_2 - i_1) = 0 \qquad \textbf{(+ 1.5 point)}$$

For mesh 1, KVL reads like

$$20i_1 + 20(i_1 - i_2) - 10 = 0 \qquad \textbf{(+ 1.5 point)}$$

This gives us a total of 3 equations in the 3 mesh current unknowns i_1, i_2, i_3. In matrix form, we can write this as

$$\begin{pmatrix} 0 & 1 & -1 \\ -20 & 40 & 20 \\ 40 & -20 & 0 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \\ i_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 10 \end{pmatrix}$$

Part II: [1 point] Provide expressions for the voltage v_x and the current i_x in terms of mesh currents.

Solution: Part II: In terms of the node voltages, v_x and i_x can be expressed as

$$v_x = 10i_3 \qquad \textbf{(+ .5 point)}$$
$$i_x = i_1 - i_2 \qquad \textbf{(+ .5 point)}$$