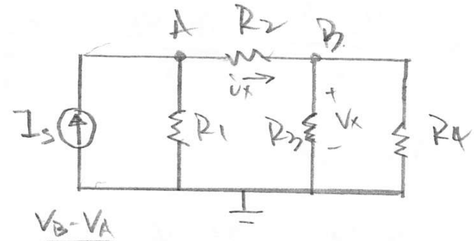
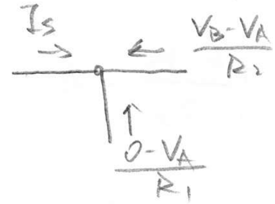
HW3 Solution

3.2

a) Note: Assume all current flows into



A: $I_s + \dfrac{0 - V_A}{R_1} + \dfrac{V_B - V_A}{R_2} = 0$



B: $\dfrac{V_A - V_B}{R_2} + \dfrac{0 - V_B}{R_3} + \dfrac{0 - V_B}{R_4} = 0$

Rearrange into matrix form:

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} \end{bmatrix} \begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} I_s \\ 0 \end{pmatrix}$$

* You could directly jump into matrix form and receive full credit

b) $V_A = \dfrac{I_s}{\frac{1}{R_1} + \frac{1}{R_2} - \frac{\frac{1}{R_2^2}}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}} = \dfrac{R_1R_3R_4 + R_1R_2R_4 + R_1R_2R_3}{R_3R_4 + R_2R_4 + R_2R_3 + R_1R_4 + R_1R_3} I_s$

or $= \dfrac{I_s}{G_1 + G_2 - \frac{G_2^2}{G_2 + G_3 + G_4}} = \dfrac{G_2 + G_3 + G_4}{G_1G_2 + G_1G_3 + G_1G_4 + G_2G_3 + G_3G_4} I_s$

$V_B = \dfrac{\frac{\frac{1}{R_2}}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}}{\frac{1}{R_1} + \frac{1}{R_2} - \frac{\frac{1}{R_2^2}}{\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}}} I_s = \dfrac{R_1R_3R_4}{R_3R_4 + R_2R_4 + R_2R_3 + R_1R_4 + R_1R_3} I_s$

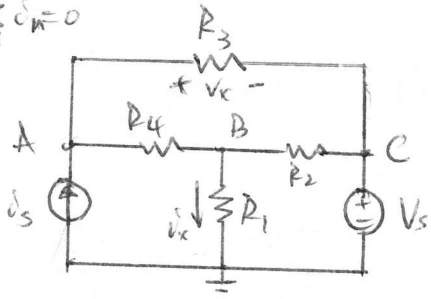
or $= \dfrac{\frac{G_2}{G_2 + G_3 + G_4}}{G_1 + G_2 - \frac{G_2^2}{G_2 + G_3 + G_4}} I_s = \dfrac{G_2}{G_1G_2 + G_1G_3 + G_1G_4 + G_2G_3 + G_2G_4} I_s$

c) $V_x = V_B$

$i_x = \dfrac{V_A - V_B}{R_2} = \dfrac{R_1R_4 + R_1R_3}{R_3R_4 + R_2R_4 + R_2R_3 + R_1R_4 + R_1R_3} I_s$ or $\dfrac{G_2G_3 + G_2G_4}{G_1G_2 + G_1G_3 + G_1G_4 + G_2G_3 + G_3G_4} I_s$

3.6 KCL:

assume current flows into node $\rightarrow \sum_{n=1}^{M} i_n = 0$



a) $A: \; i_s + \frac{V_B - V_A}{R_4} + \frac{V_C - V_A}{R_3} = 0$

$B: \; \frac{V_A - V_B}{R_4} + \frac{0 - V_B}{R_1} + \frac{V_C - V_B}{R_2} = 0$

$C: \; V_C = V_s$ (because of the way we choose ground → Method 2 in Lecture slide)

rearrange:

$$\boxed{\begin{bmatrix} \frac{1}{R_3}+\frac{1}{R_4} & -\frac{1}{R_4} & -\frac{1}{R_3} \\ -\frac{1}{R_4} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4} & -\frac{1}{R_2} \end{bmatrix} \begin{pmatrix} V_A \\ V_B \\ V_C \end{pmatrix} = \begin{pmatrix} i_s \\ 0 \end{pmatrix}}$$ w/ $V_C = V_s$.

b) $V_x = V_A - V_C$, $\; i_x = \frac{V_B - 0}{R_1} = \frac{V_B}{R_1}$

plug in $R_1 = R_2 = R_3 = R_4 = 5\,k\Omega$, $V_s = 12\,V$, $i_s = 2\,mA$

$$\begin{bmatrix} \frac{2}{5} & -\frac{1}{5} & -\frac{1}{5} \\ -\frac{1}{5} & \frac{3}{5} & -\frac{1}{5} \end{bmatrix} \begin{pmatrix} V_A \\ V_B \\ 12 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$ ⇒ solve for: $V_A = 15.6\,V$ $\; V_B = 9.2\,V$

⇒ $V_x = 3.6\,V$ $\qquad i_x = 1.84\,mA$

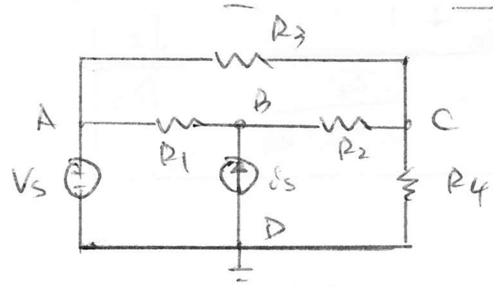
3.7.

1st eqn: ~~node~~ A only V_s btw node A & ground

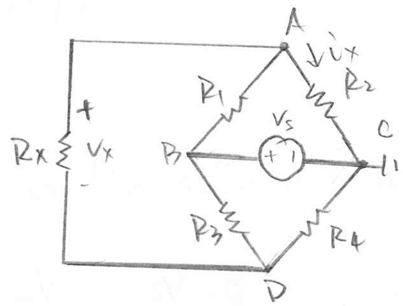
2nd eqn: R_1 is btw A and B, R_2 is btw B & C, i_s flow into node B

3rd eqn: R_3 is btw A & C, R_2 is btw B & C, R_4 is btw C & ground

4th eqn: D is grounded



3.11.



a) $A: \dfrac{V_D - V_A}{R_x} + \dfrac{V_B - V_A}{R_1} + \dfrac{0 - V_A}{R_2} = 0$

$B: \quad V_B = V_s$

$D: \quad \dfrac{V_A - V_D}{R_x} + \dfrac{V_B - V_D}{R_3} + \dfrac{0 - V_D}{R_4} = 0$

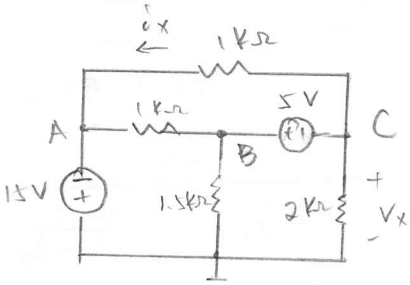
$$\Rightarrow \begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_x} & -\frac{1}{R_1} & -\frac{1}{R_x} \\ -\frac{1}{R_x} & -\frac{1}{R_3} & \frac{1}{R_x} + \frac{1}{R_3} + \frac{1}{R_4} \end{bmatrix} \begin{pmatrix} V_A \\ V_B \\ V_D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

b) plug in values and solve for V_A & V_D:

$$\begin{bmatrix} 11.6667 & -1 & -10 \\ -10 & -2 & 12.5 \end{bmatrix} \begin{pmatrix} V_A \\ 15 \\ V_D \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow V_A = 10.6364\ V \quad V_D = 10.9091\ V$$

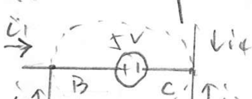
$i_x = \dfrac{V_A}{R_2} = \boxed{7.0909\ mA} \qquad V_x = V_A - V_D = \boxed{-0.2727\ V}$

3.12



choosing bottom node as ground $\Rightarrow V_A = -15\ V$

form supernode between B and C, assume all current flows into supernode.



then, $V_B - V_C = 5\ V$ ①

$i_1 + i_2 + i_3 + i_4 = 0 \Rightarrow \dfrac{V_A - V_B}{1\ k\Omega} + \dfrac{0 - V_B}{1.5\ k\Omega} + \dfrac{0 - V_C}{2\ k\Omega} + \dfrac{V_A - V_C}{1\ k\Omega} = 0$ ②

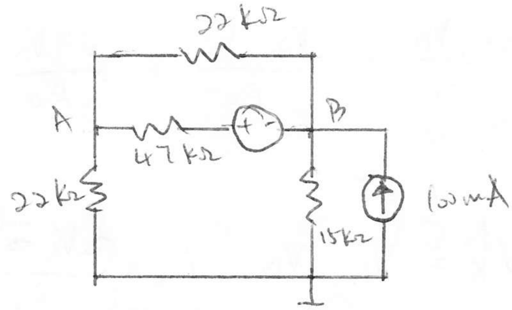
2 equations, 2 unknowns, V_B, V_C $\Rightarrow$ solve for: $V_B = -7.1053\ V \quad V_C = -12.1053\ V$.

$V_x = V_C = \boxed{-12.1053\ V.} \qquad i_x = \dfrac{V_C - V_A}{1\ k\Omega} = \boxed{-2.8947\ mA}$

rearrange ①, ②:

$$\begin{bmatrix} 1 & -1 \\ 1.6667 & 1.5 \end{bmatrix} \begin{pmatrix} V_B \\ V_C \end{pmatrix} = \begin{pmatrix} 5 \\ \frac{2V_A}{1\ k\Omega} \end{pmatrix}$$

3.26

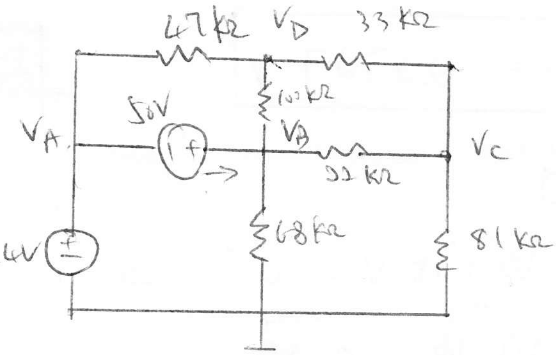


b) A: $\dfrac{0 - V_A}{22k\Omega} + \dfrac{V_B - V_A}{22k\Omega} + \dfrac{V_B + 50V - V_A}{47k\Omega} = 0$

B: $\dfrac{0 - V_B}{15k\Omega} + 100\,mA + \dfrac{V_A - V_B}{22k\Omega} + \dfrac{V_A - 50V - V_B}{47k\Omega} = 0$

$$\begin{bmatrix} \frac{1}{22k\Omega} + \frac{1}{47k\Omega} + \frac{1}{22k\Omega} & -\frac{1}{47k\Omega} - \frac{1}{22k\Omega} \\ -\frac{1}{47k\Omega} - \frac{1}{22k\Omega} & \frac{1}{22k\Omega} + \frac{1}{15k\Omega} + \frac{1}{47k\Omega} \end{bmatrix} \begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} \frac{50V}{47k\Omega} \\ -\frac{50V}{47k\Omega} + 100mA \end{pmatrix} \Rightarrow \boxed{\begin{pmatrix} V_A \\ V_B \end{pmatrix} = \begin{pmatrix} 641.5\ V \\ 1062.6\ V \end{pmatrix}}$$

3.27



c) A: $\boxed{V_A = 24\ V}$

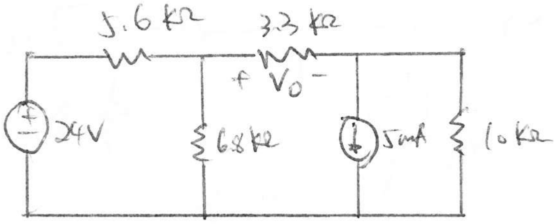
B: $V_B - V_A = 50V \Rightarrow \boxed{V_B = 74\ V}$

C: $\dfrac{V_B - V_C}{22k\Omega} + \dfrac{V_D - V_C}{33k\Omega} + \dfrac{0 - V_C}{81k\Omega} = 0$

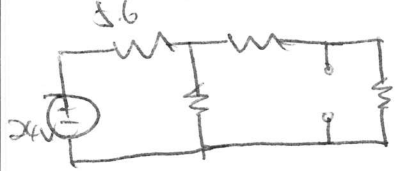
D: $\dfrac{V_A - V_D}{47k\Omega} + \dfrac{V_B - V_D}{100k\Omega} + \dfrac{V_C - V_D}{33k\Omega} = 0$

$$\Rightarrow \begin{bmatrix} \frac{1}{22k\Omega} + \frac{1}{33k\Omega} + \frac{1}{81k\Omega} & -\frac{1}{33k\Omega} \\ -\frac{1}{33k\Omega} & \frac{1}{47k\Omega} + \frac{1}{100k\Omega} + \frac{1}{33k\Omega} \end{bmatrix} \begin{pmatrix} V_C \\ V_D \end{pmatrix} = \begin{pmatrix} \frac{V_B}{22k\Omega} \\ \frac{V_B}{100k\Omega} + \frac{V_A}{47k\Omega} \end{pmatrix} \Rightarrow \boxed{V_C = 54.3653\ V} \quad \boxed{V_D = 47.0622\ V}$$

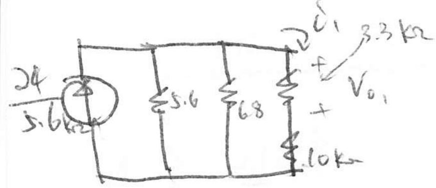
3.42



Turn off current source → ① V_{o1}



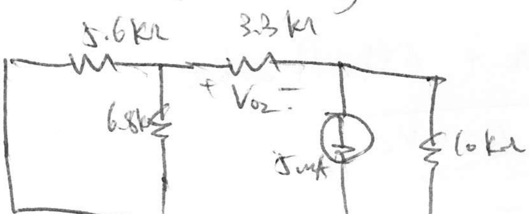
and use source transformation:



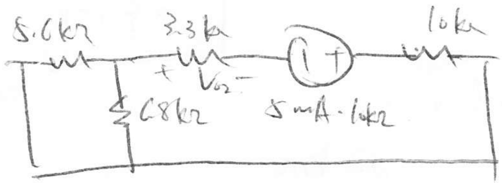
$$i_1 = \frac{\frac{1}{13.3\,k\Omega}}{\frac{1}{13.3k\Omega}+\frac{1}{5.6k\Omega}+\frac{1}{6.8k\Omega}} \cdot \frac{24}{5.6k\Omega} = 0.8039 \text{ mA}$$

$$V_{o1} = i_1 \cdot 3.3\,k\Omega = \boxed{2.653\,V}$$

Turn off Voltage source & open current source → V_{o2}.



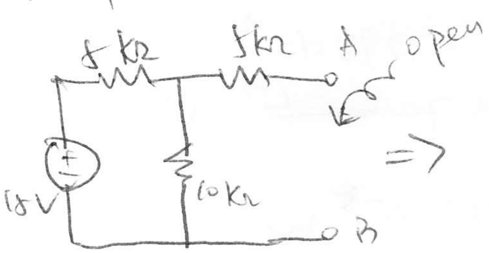
source transform →



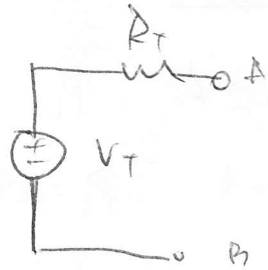
$$V_{o2} = \frac{3.3\,k\Omega}{3.3k\Omega + 10k\Omega + 5.6k\Omega \| 6.8k\Omega}(50\,V) = \boxed{10.078\,V}$$

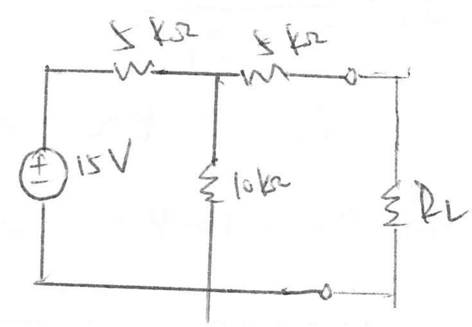
$$V_o = V_{o1} + V_{o2} = \boxed{12.732\,V}$$

3.54

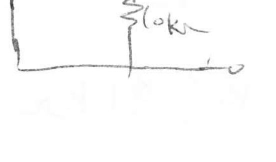


⇒





zero out source → ~~R_T~~



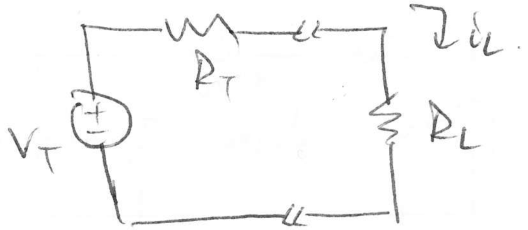
$$R_T = 5k\Omega + 5k\Omega \| 10k\Omega$$
$$= \boxed{8.333\,k\Omega}$$

using voltage divider :

$$V_T = \frac{10k\Omega}{5k\Omega + 10k\Omega} \cdot 15V = \boxed{10\,V}$$

* Note : 5kΩ on the right sees no current because of open connection btw A, B.

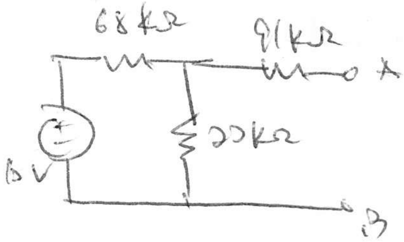
b)



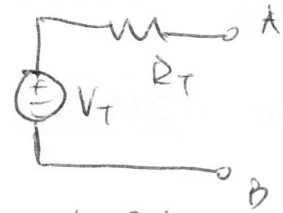
$$i_L = \frac{V_T}{R_T + R_L} = \frac{10\,V}{8.1333\,k\Omega + 15\,k\Omega}$$

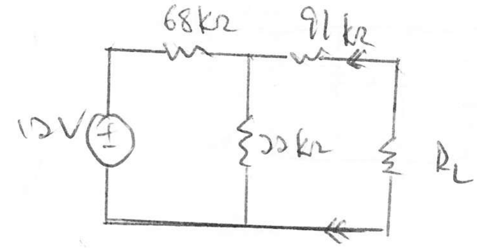
$$= \boxed{0.4286\ mA}$$

3.59

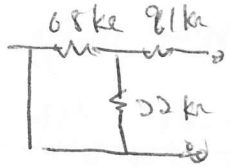


$\Rightarrow$





zero out source $\rightarrow$



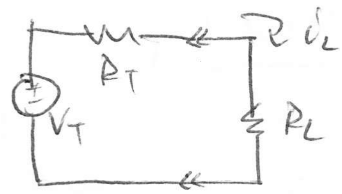
$$R_T = 91\,k\Omega + 68\,k\Omega \,||\, 22\,k\Omega$$

$$= \boxed{107.6222\ k\Omega}$$

voltage divider :

$$V_T = \frac{22\,k\Omega}{22\,k\Omega + 68\,k\Omega}(12\,V) = \boxed{2.9333\ V\cancel{mA}}$$

$$P_L = VI = I_L^2 R_L$$



$$I_L = \frac{V_T}{R_T + R_L}$$

for $R_L = 50\,k\Omega \rightarrow I_L = 0.0186\ mA$. $P_L = \cancel{3.7273 \times 10^{-8}}\ 1.7298 \times 10^{-5}\ W$

for $R_L = 100\,k\Omega \rightarrow I_L = 0.0141\ mA$ $P_L = 1.9961 \times 10^{-5}\ W$

$\boxed{P_{50k\Omega} = 17.298\ \mu W}$ $\boxed{P_{100k\Omega} = 19.961\ \mu W}$

*Note : when computing V_T, 91 kΩ resistor sees not current because open connection btw A & B.