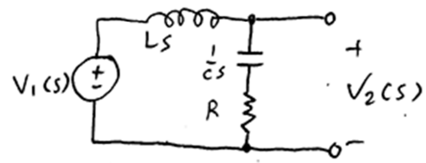
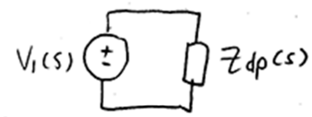
Homework 8 Solution

11-4 a) Find driving point impedance seen by the Volt. source, find $T_V(s) = V_2(s)/V_1(s)$



$$Z_{dp}(s) = Ls + \frac{1}{Cs} + R = \boxed{\frac{1 + RCs + LCs^2}{Cs}}$$

Voltage divider



$$\hookrightarrow V_2(s) = \frac{\frac{1}{Cs} + R}{Z_{dp}(s)} \cdot V_1(s) = \frac{1+RCs}{1+RCs+LCs^2} \cdot V_1(s)$$

$$\hookrightarrow \boxed{T_V(s) = \frac{1+RCs}{1+RCs+LCs^2}}$$

b) Select R, C, L such that the poles are $s = -9898$ rad/s, $s = -101$ rad/s. What's the zero?

So, we want $\frac{1}{LC}(1 + RCs + LCs^2) = (s+9898)(s+101)$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = s^2 + 9999s + 999698$$

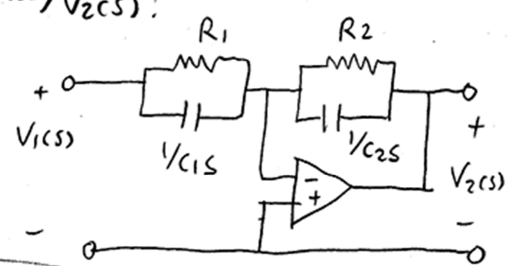
Using $\boxed{L = 1\,H \rightarrow R = 9999\,\Omega, \quad C = 1.0003\,\mu F}$

with zeros of:

$$\frac{1}{LC}(1+RCs) \Rightarrow s = -\frac{1}{RC}, \quad s = \infty$$

$$\boxed{\text{zeros} = \{-99.98, \infty\}}$$

11-8 a) Find DP Impedance seen by $V_1(s)$, find $T_V(s) = V_1(s)/V_2(s)$!



This is an inverting OP Amp configuration.

Input impedance $\rightarrow Z_1(s) = \left(\frac{1}{R_1} + C_1 s\right)^{-1}$

$$= \boxed{\frac{R_1}{1+R_1C_1s}}$$

Feedback impedance $\rightarrow Z_2(s) = \left(\frac{1}{R_2} + C_2 s\right)^{-1} = \boxed{\frac{R_2}{1+R_2C_2s}}$

$$V_2(s) = -\frac{Z_2}{Z_1} V_1(s) \rightarrow \boxed{T_V(s) = -\frac{R_2}{R_1} \cdot \frac{1+R_1C_1s}{1+R_2C_2s}}$$

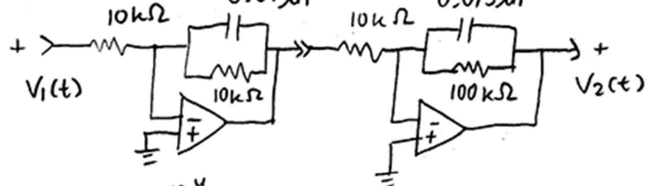
b) Select R_1, R_2, C_1, C_2 s.t. there is a pole at $s = -10000$ rad/s and zero at $s = -5000$ rad/s and input impedance at DC is $2k\Omega$.

DC input impedance $\rightarrow$ $Z_1(s)\Big|_{s=0} = \frac{R_1}{1} = 2k\Omega \rightarrow \boxed{R_1 = 2k\Omega}$

zero $\rightarrow$ $1 + R_1C_1s = 0 \rightarrow s = \frac{-1}{R_1C_1} = -5000 \rightarrow \boxed{C_1 = 100 \text{ nF}}$

pole $\rightarrow$ $1 + R_2C_2s = 0 \rightarrow s = \frac{-1}{R_2C_2} = -10000 \rightarrow \boxed{\begin{matrix} R_2 = 2k\Omega \\ C_2 = 50nF \end{matrix}}$

11-14 Find $T_V(s) = V_2(s)/V_1(s)$. Locate poles and zeros.



Cascade of 2 inverting OP Amps (non-loading)

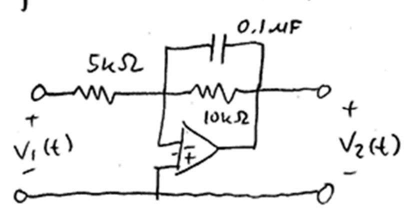
First OP Amp's gain

$$G_1(s) = \frac{-\left(\frac{1}{10k\Omega} + 0.01\mu F \cdot s\right)^{-1}}{10k\Omega} = \frac{-1}{1 + 10^{-4}s} = \frac{-10^4}{s + 10^4}$$

$$G_2(s) = \frac{-\left(\frac{1}{100k\Omega} + 0.015\mu F s\right)^{-1}}{10k\Omega} = \frac{-10}{1 + 0.0015 s} = \frac{-6666.7}{s + 666.67}$$

$$T_V(s) = G_1(s) \cdot G_2(s) = \boxed{\frac{-6.67 \cdot 10^7}{(s+1000)(s+666.7)}} \rightarrow \boxed{\begin{matrix} \text{poles} = \{-1000, -666.7\} \\ \text{zeros} = \{\infty, \infty\} \end{matrix}}$$

11-36 Steady state $V_1(t) = 5\cos(500t)$ V. Find $V_{2ss}(t)$. Repeat for $V_1(t) = 5\cos(1kt)$ V and $V_1(t) = 5\cos(10kt)$ V. Where is the pole?



Inverting OP Amp.

$$T_V(s) = \frac{V_2(s)}{V_1(s)} = -\frac{\left(\frac{1}{10k\Omega} + 0.1\mu F \cdot s\right)^{-1}}{5k\Omega} = \frac{-2}{1 + 0.001 s} = \frac{-2000}{s + 1000}$$

For sinusoidal input $V_1(t) = A\cos(\omega t)$, the steady state output $V_{2ss}(t)$ is:

$$V_{2ss}(t) = A \cdot |T_V(j\omega)| \cdot \cos(\omega t + \measuredangle T_V(j\omega))$$

$T_V(j\omega)$ is a complex number,

$V_1(t) = 5 \cdot \cos(500t) \rightarrow T_V(j500) = \frac{-2000}{500j + 1000} = \frac{-2000(1000 - 500j)}{1000^2 + 500^2} = \underline{\underline{-1.6 + j\,0.8}}$

$\hookrightarrow V_{2ss}(t) = 5 \cdot |-1.6 + j \cdot 0.8| \cos(500t + \measuredangle(-1.6 + 0.8j)) = \underline{\underline{5 \cdot \sqrt{3.2} \cos(500t + 2.68)}}$

$V_1(t) = 5 \cdot \cos(1kt) \rightarrow T_V(1000j) = -1 + j \rightarrow |T_V(1000j)| = \sqrt{2}, \quad \measuredangle T_V(1000j) = \frac{3}{4}\pi$

$\hookrightarrow V_{2ss}(t) = \underline{\underline{5 \cdot \sqrt{2} \cos(1000t + 3/4\,\pi)}}$

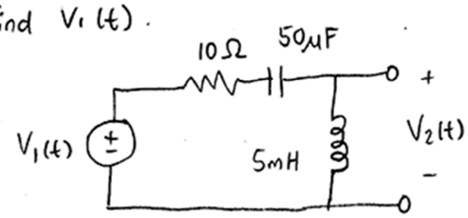
$V_1(t) = 5 \cdot \cos(10kt) \rightarrow T_V(10kj) = -0.0198 + j\,0.198 \rightarrow |T_V(10kj)| = 0.199$

$\measuredangle T_V(10kj) = 1.67$

$\hookrightarrow V_{2ss}(t) = \underline{\underline{5 \cdot 0.199 \cos(10kt + 1.67)}}$

The pole $\Rightarrow$ $s = -1000$ rad/s

11-38 If we have $V_{2ss}(t) = 25.5 \cos(10kt + 11.8°)$, find $V_1(t)$.



Voltage divider: $\frac{V_2(s)}{V_1(s)} = \frac{0.005 \cdot s}{10\Omega + \frac{1}{50 \cdot 10^{-6} s} + 0.005\,s}$

$$\boxed{T_V(s) = \frac{s^2}{s^2 + 2000s + 4 \cdot 10^6}}$$

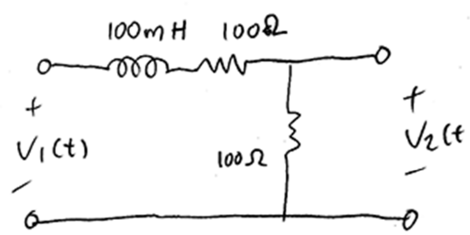
Then we have

$T_V(10kj) = \frac{(10000j)^2}{(10000j)^2 + 2000(10000j) + 4 \cdot 10^6} = 0.9983 + 0.208j$

$\hookrightarrow |T_V(10kj)| = 1.0198 \qquad \measuredangle T_V(10kj) = 11.8°$

So, $V_1(t) = \frac{25.5}{1.0198} \cos(10kt + 11.8° - 11.8°) \rightarrow \boxed{V_1(t) = 25 \cos(10kt)}$

12-4 Find $T_V(s) = V_2(s)/V_1(s)$.



$T_V(s) = \frac{100}{0.1s + 100 + 100} = \underline{\underline{\frac{1000}{s + 2000}}}$

using Voltage divider

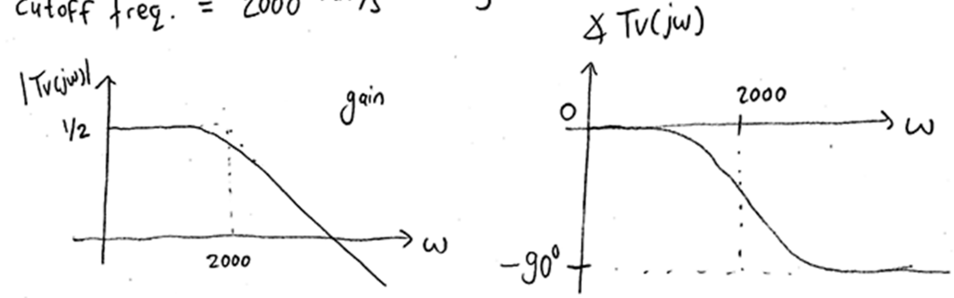
a) DC gain $= \lim_{s \to 0} T_V(s) = \frac{1}{2}$

∞ freq. gain $= \lim_{s \to \infty} T_V(s) = 0$

cutoff freq. $= 2000$ rad/s

$\Bigg\}$ This is a lowpass filter.

b)



c) Gain at $\omega = 0.1\,\omega_c$; $\omega = \omega_c$, $\omega = 10\,\omega_c$. $\omega_c = 2000$ rad/s.

↳ $\left| T_V(200j) \right| = \left| \dfrac{1000}{200j + 2000} \right| = 0.4975$

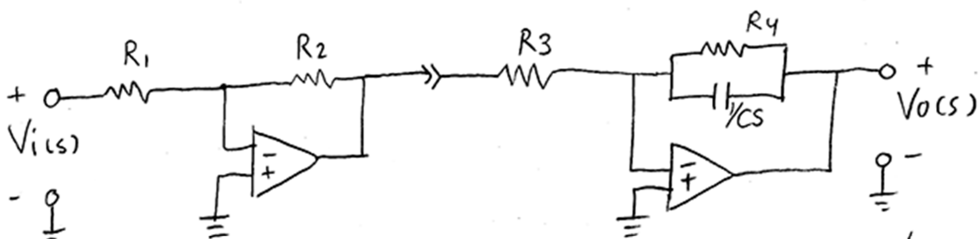
$\left| T_V(2000j) \right| = \left| \dfrac{1000}{2000j + 2000} \right| = 0.3536$

$\left| T_V(20000j) \right| = \left| \dfrac{1000}{20000j + 2000} \right| = 0.0498$

12-10 Design a low pass filter with 20 krad/s cutoff freq. and passband gain of 200.

↳ want : $\boxed{T_V(s) = \dfrac{200 \cdot 20000}{s + 20000}}$

Simplest way is to use 2 inverting OP Amps, with gains of -200 and $\dfrac{-20000}{s+20000}$



$\boxed{\text{gain} = -\dfrac{R_2}{R_1}} = -200$

We can use:

$\boxed{\begin{array}{l} R_2 = 200\,k\Omega \\ R_1 = 1\,k\Omega \end{array}}$

$\text{gain} = -\dfrac{\left(\frac{1}{R_4} + Cs\right)^{-1}}{R_3} = \dfrac{-R_4}{R_3} \dfrac{1}{1 + R_4 C s}$

$\boxed{= \dfrac{-1}{R_3 C} \cdot \dfrac{1}{s + 1/R_4 C}} = -\dfrac{20000}{s+20000}$

we can use:

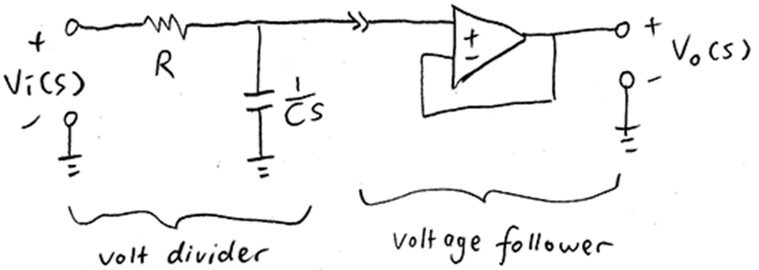
$\boxed{\begin{array}{l} C = 100\,nF \\ R_4 = 500\,\Omega \\ R_3 = 500\,\Omega \end{array}}$

12-14 Design a lowpass filter w/ $\omega_c = 100\ Hz$ and passband gain of 1.

$\omega_c = 100\ Hz = 628.3\ rad/s$

Want : $\boxed{T_V(s) = \dfrac{628.3}{s+628.3}}$

We can use this circuit below :



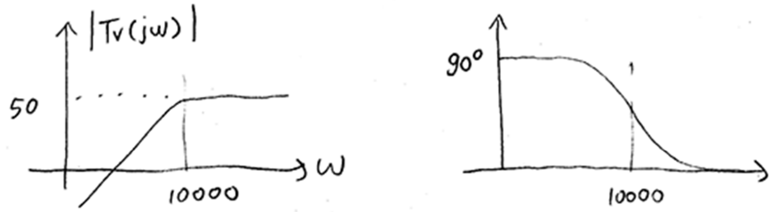
$$\text{gain} = \frac{1/Cs}{R + 1/Cs} = \frac{1}{RCs+1} = \frac{1/RC}{s + 1/RC}$$

Use $\boxed{\begin{array}{l} C = 1\mu F \\ R = 1592\,\Omega \end{array}}$

12-23 $T_V(s) = \dfrac{50s}{s+10000}$

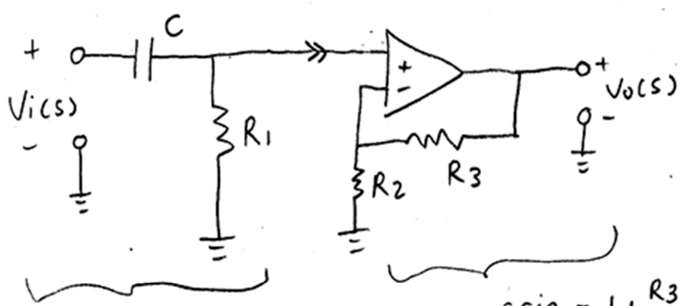
a) This is a highpass filter, $\omega_c = 10000\ rad/s$, passband gain $= \lim_{s\to\infty} T_V(s) = 50$

b)



c) Design the circuit.

$$T_V(s) = \frac{50s}{s+10000} = 50\left(\frac{s}{s+10000}\right)$$



$$\text{gain} = \frac{R_1}{R_1 + 1/Cs} = \frac{R_1Cs}{1+R_1Cs} = \frac{s}{s+\frac{1}{R_1C}}$$

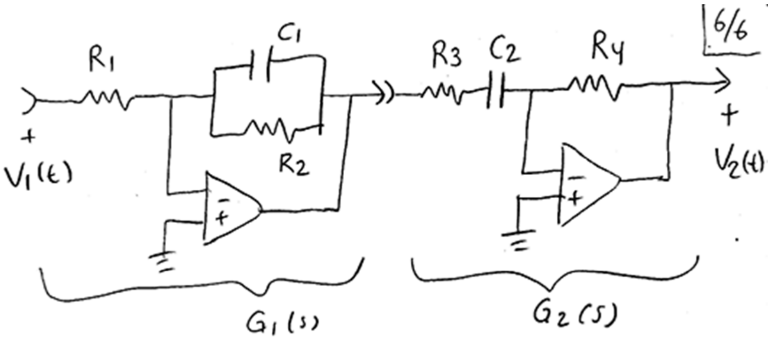
$$\text{gain} = 1 + \frac{R_3}{R_2}$$

$\boxed{\text{Total gain} = \left(1+\frac{R_3}{R_2}\right)\left(\frac{s}{s+1/R_1C}\right)}$

we can use :

$\boxed{\begin{array}{l} C = 1\mu F \\ R_1 = 100\,\Omega \\ R_2 = 1k\Omega \\ R_3 = 49\ k\Omega \end{array}}$

12-32 Bandpass circuit.



Identify the elements that control the 2 cutoff frequencies

↳ $\boxed{C_1, R_2 \quad \text{and} \quad C_2, R_3}$

Want: passband gain 100, cutoff frequencies: 1000 rad/s, 40 krad/s

Cascade of 2 inverting OP Amps.

$$G_1(s) = -\frac{\left(\frac{1}{R_2} + C_1 s\right)^{-1}}{R_1} = -\frac{R_2}{R_1}\frac{1}{1+R_2C_1 s} = -\frac{1/R_1C_1}{s + 1/R_2C_1}$$

$$G_2(s) = -\frac{R_4}{R_3 + 1/C_2 s} = -\frac{R_4C_2 s}{1+R_3C_2 s} = -\frac{(R_4/R_3)\,s}{s + 1/R_3C_2}$$

Then we have: $T_V(s) = \frac{V_2(s)}{V_1(s)} = G_1(s)\cdot G_2(s)$:

$$\boxed{T_V(s) = \frac{R_4}{R_1R_3C_1}\left(\frac{s}{\left(s+\frac{1}{R_2C_1}\right)\left(s+\frac{1}{R_3C_2}\right)}\right)}$$

Bandpass gain of $\frac{s}{(s+1000)(s+40000)}$ → evaluate around $\sqrt{1000\cdot 40000} \simeq 6300$ rad/s

$$\left|\frac{6300j}{(6300j+1000)(6300j+40000)}\right| = 2.44\cdot 10^{-5}$$

$$\boxed{T_V(s) = 4.1\cdot 10^6 \frac{s}{(s+1000)(s+40000)}}$$

So, we want:

$$\frac{1}{R_2C_1} = 1000 \text{ rad/s}$$

$$\frac{1}{R_3C_2} = 40000 \text{ rad/s}$$

$$\frac{R_4}{R_1R_3C_1} = \frac{100}{2.44\cdot 10^{-5}}$$

→

We can use:

$C_1 = 1$ nF

$C_2 = 1$ nF

$R_1 = 10$ kΩ

$R_2 = 1$ MΩ

$R_3 = 25$ kΩ

$R_4 = 1.025$ MΩ