MAE140 - Linear Circuits - Winter 19
Midterm, February 7

Instructions

(i) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook

(ii) You may use a hand calculator with no communication capabilities

(iii) You have 75 minutes

(iv) Do not forget to write your **name** and **student number**

(v) The exam has 3 questions, for a total of 30 points

Good luck!

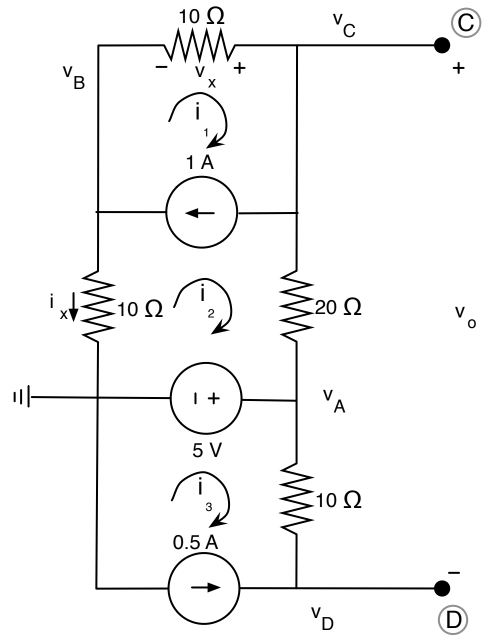


Figure 1: Circuit for all questions.

1. **Node voltage analysis**

Part I: [5 points] Formulate node-voltage equations for the circuit in Figure 1. Use the node labels provided in the figure. Clearly indicate how you handle the presence of a voltage source (if you have more than one choice to deal with it, use the simplest). The final equations must depend only on unknown node voltages. **Do not modify the circuit or the labels**. No need to solve any equations!

Solution: Part I: There are five nodes in this circuit. The ground node has already been chosen for us. Fortunately, with this choice, the ground node is directly connected to the voltage source, so we can use method 2 to take care of it. This readily tells us that $v_A = 5V$. **(+ 1 point)**
To write KCL for the other nodes, we proceed by inspection (making sure of not writing KCL for node A),

$$\begin{pmatrix} 0 & \frac{1}{10}+\frac{1}{10} & -\frac{1}{10} & 0 \\ -\frac{1}{20} & -\frac{1}{10} & \frac{1}{10}+\frac{1}{20} & 0 \\ -\frac{1}{10} & 0 & 0 & \frac{1}{10} \end{pmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \\ v_D \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 0.5 \end{pmatrix} \qquad \textbf{(+ 3 points)}$$

Substituting the value of v_A, we can write a system of 3 equations in 3 unknowns v_B, v_C, v_D,

$$\begin{pmatrix} \frac{1}{5} & -\frac{1}{10} & 0 \\ -\frac{1}{10} & \frac{3}{20} & 0 \\ 0 & 0 & \frac{1}{10} \end{pmatrix} \begin{pmatrix} v_B \\ v_C \\ v_D \end{pmatrix} = \begin{pmatrix} 1 \\ -0.75 \\ 1 \end{pmatrix} \qquad \textbf{(+ 1 point)}$$

Part II: [3 points] Provide expressions for the mesh currents i_1, i_2, and i_3 in terms of the node voltages.

Solution: Part II: The mesh currents can be expressed in terms of the node voltages by using Ohm's law. For instance, i_1 is the current passing through the 10Ω resistor at the top. Likewise, i_2 is the current passing through the 20Ω resistor (and also through the 10Ω resistor on the left). And finally, i_3 is the current passing through the 10Ω resistor at the bottom right. Therefore,

$$i_1 = \frac{1}{10}(v_B - v_C) \qquad \textbf{(+ 1 point)}$$

$$i_2 = \frac{1}{20}(v_C - v_A) = \frac{1}{10}(0 - v_B) \qquad \textbf{(+ 1 point)}$$

$$i_3 = \frac{1}{10}(v_A - v_D) \qquad \textbf{(+ 1 point)}$$

Part III: [2 points] Provide expressions for the voltage v_x and the current i_x in terms of node voltages.

Solution: In terms of the node voltages, we have

$$v_x = v_C - v_B \qquad \textbf{(+ 1 point)}$$

$$i_x = \frac{1}{10}(v_B - 0) \qquad \textbf{(+ 1 point)}$$

2. **Mesh current analysis**

Part I: [6 points] Formulate mesh-current equations for the circuit in Figure 1. Use the mesh currents shown in the figure and clearly indicate how you handle the presence of the two current sources. The final equations should only depend on the unknown mesh currents. **Do not modify the circuit or the labels** (meaning source transformation or circuit re-drawing are not allowed). No need to solve any equations!

Solution: Part I: There are three meshes in this circuit. The current source at the bottom belongs to a single mesh, so we can use method 2 to determine

$$i_3 = -0.5 \qquad \textbf{(+ 1 point)}$$

The current source at the top belongs to two meshes. It is in parallel with the top resistor, so we could use source transformation. However, the problem statement explicitly rules this out. So we are forced to use a supermesh.

(+ 1 point)

Consequently, we set

$$i_1 - i_2 = 1 \qquad \textbf{(+ 2 points)}$$

And we write KVL for the supermesh as

$$10i_1 + 20i_2 + 5 + 10i_2 = 0 \qquad \textbf{(+ 2 points)}$$

This gives us a total of 2 equations in the 2 mesh current unknowns i_1, i_2. In matrix form, we can write this as

$$\begin{pmatrix} 1 & -1 \\ 10 & 30 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$$

Part II: [2 points] A former student of MAE140 said that, if the value of the bottom right resistor was 5Ω (instead of 10Ω), then (a) the value of the mesh currents would not change and (b) the power delivered by the $0.5A$ current source would be the same. Determine the validity of each assertion, justifying your answer.

Solution: Part II: Assertion (a) is correct. The value of the mesh currents would not change because we know that, from the point of view of the rest of the circuit, a current source in series with a resistor is equivalent to a current source by itself.

(+ 1 point)

However, the presence of the resistor matters for the amount of power delivered by the source (having to provide more power with larger resistance). This means that assertion (b) is incorrect.

(+ 1 point)

Part III: [2 points] If you could modify the circuit instead (meaning circuit re-drawing and source transformations were allowed), how many other methods could you have used to deal with the $1\ A$ current source?

Solution: Two other methods. We could use source transformation (method 1) to make the current source disappear.

(+ 1 point)

We could also have simply redrawn the circuit, interchanging the places of the current source and the 10Ω resistor at the top. This would have the current source only belonging to one mesh, and to deal with this we could use method 2.

(+ 1 point)

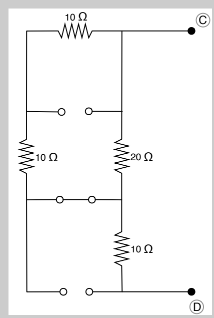
3. **Equivalent circuits**

Part I: [4 points] Turn off all the sources in the circuit of Figure 1 and find the equivalent resistance as seen from terminals C and D.

Solution: Part I: We start by switching off the sources.

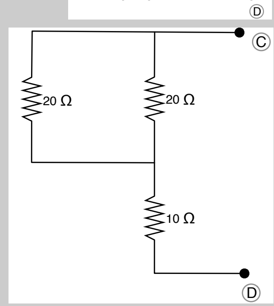
We substitute the voltage source by a short circuit, and the current sources by open circuits. Then, we get the circuit on the right

(+ 1 point)



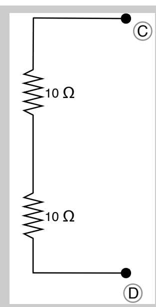
We combine the two 10Ohms resistors in series to get the circuit

(+ 1 point)



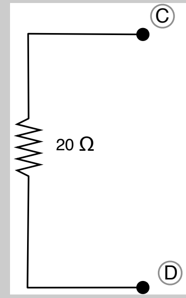
We combine now the two 20Ohms resistors in parallel to get the circuit

(+ 1 point)



Finally, we combine the two resistors in series to get the circuit

(+ 1 point)



Part II: [2 points] If you solved for the node voltages in Problem 1, you would find that $v_B = 3.75V$, $v_C = -2.5V$, and $v_D = 10V$. Use this information to find the open-circuit voltage v_0.

Solution: Part II: The open-circuit voltage can be expressed in terms of the node voltages as

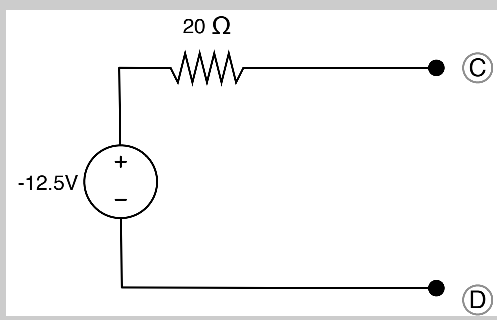
$$v_o = v_C - v_D$$ **(+ 1 point)**

Therefore, we have that $v_o = -12.5V$.

(+ 1 point)

Part III: [2 points] Use your answers to Parts I and II to determine the Thévenin equivalent of the circuit as seen from terminals C and D.

Solution: Part III: We have computed the equivalent resistance from terminals C and D with all sources turned off in Part I, and the open-circuit voltage in Part II. Therefore, the Thévenin equivalent of the circuit is simply



(+ 2 points)

Part IV: [2 points] If we short-circuit terminals C and D, what would be the value of the short-circuit current?

Solution: The short-circuit current would precisely correspond to the Norton equivalent current. Given our answer in Part III, we know

$$i_{SC} = i_N = \frac{-12.5}{20} = -0.625\,A = -625\,mA$$ **(+ 2 points)**

would flow top down.