MAE143 A - Signals and Systems - Winter 11
Final

Instructions

(i) This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities

(ii) You have 3 hours

(iii) Do not forget to write your name, student number, and instructor

1. **Filtering and sampling.** The continuous-time signal $x(t) = 100\,\text{sinc}^2(100t)$ goes through an ADC block that samples signals at a frequency $f_s = 190$ Hz. Answer the following questions
   (a) (1 point) Compute the Fourier transform of $x(t)$. What is its bandwidth?
   (b) (1 point) Would you be able to reconstruct the original signal out of the samples taken by the ADC block? Why?
   (c) (2 points) Consider a system whose impulse response is given by
   $$h(t) = 150\,\text{sinc}(150t)$$
   What kind of filter is this? What is its cutoff frequency in Hz? Plot the magnitude and phase of the transfer function.
   (d) Suppose the signal $x(t)$ passes first through the system in (c) to produce the signal $y(t)$ and then goes through the ADC block to produce the samples, see Figure 1.

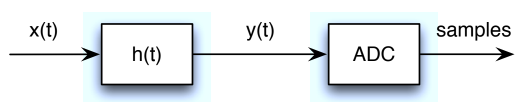

Figure 1: Block diagram for question 3, part (d).

   i. (1 point) What is the bandwidth of $y(t)$? Would you be able to reconstruct the signal $y(t)$ out of the samples taken by the ADC block? Justify your answer.
   ii. (1 point) Why would one call the system in (c) an antialiasing filter?

   (e) (1 point (bonus)) With knowledge of $y(t)$, would you be able to recover the original signal $x(t)$? Why?

2. **Continuous-time Fourier Series (CTFS) and system response.** Consider a periodic function described over one fundamental period ($T_0 = 4$) by
$$x(t) = \begin{cases} 1 - t, & t \in [0, 1], \\ 0, & t \in [-2, 0] \cup [1, 2]. \end{cases}$$
Do the following
   (a) (.5 points) Plot $x(t)$ over the interval $[-6, 6]$. Could this signal correspond to the impulse response of a causal system? Why?

(b) (2 points) Show that the harmonic numbers of $x(t)$ with representation time $T_F = T_0$ are

$$X[0] = \frac{1}{8},$$
$$X[k] = \frac{1}{\pi^2 k^2}\left(1 - j\frac{\pi}{2}k - e^{-j\frac{\pi}{2}k}\right), \qquad k \in \mathbb{Z} \setminus \{0\}.$$

(c) (1.5 points) Compute the transfer function of the LTI system

$$\dot{y}(t) + y(t) = x(t)$$

What is the impulse response? Is the system BIBO stable?

(d) (2 points) Show, using the CTFS from (b) and the transfer function from (c), that the response $y(t)$ of the LTI system to the input signal $x(t)$ is

$$y(t) = X[0] + \sum_{k=1}^{\infty} \frac{4}{\sqrt{4+\pi^2 k^2}} |X[k]| \cos\left(\frac{\pi}{2}kt - \arctan(\frac{\pi k}{2}) + \angle X[k]\right)$$

3. **Continuous-time Fourier Transform (CTFT).** Consider the periodic signal

$$z(t) = \mathrm{tri}(t-1) * \delta_2(t)$$

Do the following

(a) (1 point) Plot $z(t)$ over the interval $[-6, 6]$. What is the fundamental period of $z(t)$?

(b) (2 points) Show, using only the tables of basic CTFT transforms and basic CTFT properties, that the Fourier transform of $z(t)$ is

$$Z(f) = \frac{1}{2}\delta(f) - \sum_{k=-\infty}^{\infty} \frac{2}{\pi^2(2k-1)^2}\delta\left(f - \frac{2k-1}{2}\right)$$

and sketch a plot of its magnitude $|Z(f)|$.

(c) (3 points) Consider now the signal

$$u(t) = z(t)\,\mathrm{rect}\left(\frac{t-1}{4}\right)$$

Plot the signal $u(t)$ on the interval $[-6, 6]$. Compute the Fourier transform of $u(t)$ using the tables of basic CTFT transforms and basic CTFT properties together with the solution to (b).

(d) (1 point (bonus)) The effect in the time domain that the multiplication by the rectangle function has on $z(t)$ is called 'windowing.' Can you explain what is the effect of windowing in the spectrum of $z(t)$?