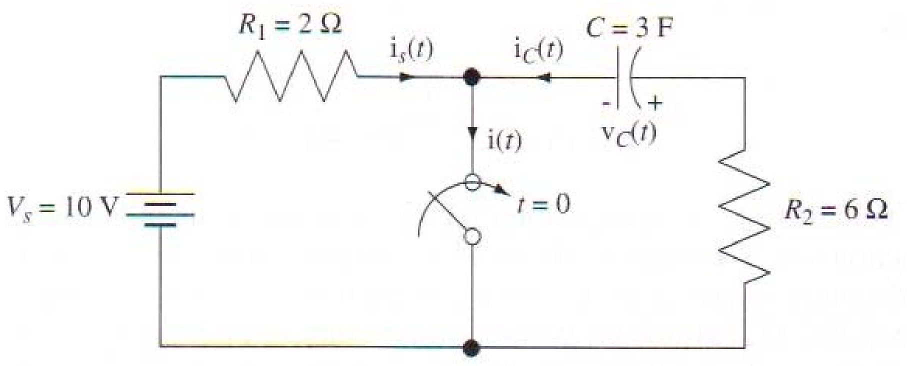
2.



① $i(t) = i_s(t) + i_c(t)$ ② $i_s(t) = \frac{V_s}{R_1}$

③ $i_c(t) = C\frac{d}{dt}(V_c(t))$ ④ $V_c(t) + i_c(t)R_2 = 0$

③, ④ $\Rightarrow$ ⑤ $V_c(t) + C\frac{d}{dt}(V_c(t))R_2 = 0$

The eigenvalue of the solution is $\lambda = -\frac{1}{CR_2}$, so the solution is $Ke^{\lambda t} = Ke^{-\frac{t}{CR_2}}$, where K is a constant we will get from initial condition.

Initial condition: The power switch is open before $t = 0$, so it's just a simple series circuit containing V_s, R_1, C and R_2. The current of this series circuit is 0 and the capacitor store charges coming from V_s. Positive charges go to the left side of the capacitor while negative charges go to its right. $\Rightarrow$ $V_c(0^+) = V_c(0^-) = -V_s = -10V$

$$\begin{cases} V_c(t) = Ke^{-\frac{t}{CR_2}} = Ke^{-\frac{t}{18}} \\ V_c(0) = -10V \end{cases} \Rightarrow K = -10,\ V_c(t) = -10e^{-\frac{t}{18}}$$

② $\Rightarrow$ $i_s(t) = 5$

③ $\Rightarrow$ $i_c(t) = C\frac{d}{dt}(V_c(t)) = 3\frac{d}{dt}(V_c(t)) = \frac{5}{3}e^{-\frac{t}{18}}$

① $\Rightarrow$ $i(t) = i_s(t) + i_c(t) = 5 + \frac{5}{3}e^{-\frac{t}{18}}$

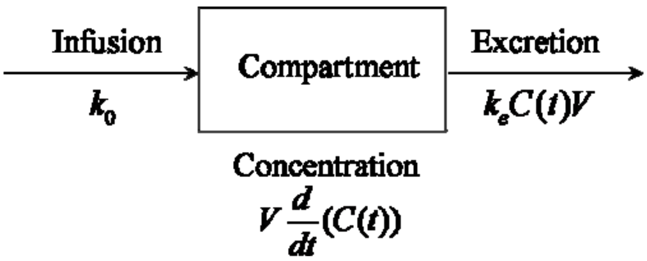
13. (a) V : the volume with unit l;

$C(t)$: the total amount of drug concentration, with unit mg/l;

k_0 : the amount of drug enters the compartment per hour, with unit mg/hr;

k_e : the percentage of total concentrated drug that runs out of the compartment per hour, with unit hr^{-1}.

At some time point, drug enters the compartment is k_0; While total concentration amount is $C(t)V$, concentration created at this time point is $\frac{d}{dt}(C(t)V)$, with unit mg/hr; Excretion is the percentage per hour k_e multiply by the total concentrated amount $C(t)V$.



Then,

$$\text{Concentration} = \text{Infusion} - \text{Excretion}$$

$$V\frac{d}{dt}(C(t)) = k_0 - k_e C(t)V$$

(b) Substitute with parameters $\Rightarrow$ $20\frac{d}{dt}(C(t)) + 8C(t) = 200$

① Homogeneous solution: $20\frac{d}{dt}(C(t)) + 8C(t) = 0$

The eigenvalue of the solution is $\lambda = -\frac{8}{20} = -0.4$, so the homogeneous solution should be in the form of $C_h(t) = K_1 e^{-0.4t}$, where K_1 is a constant.

② Particular solution: As the right side of the equation is 200, the particular solution should in a form of a constant K_2.

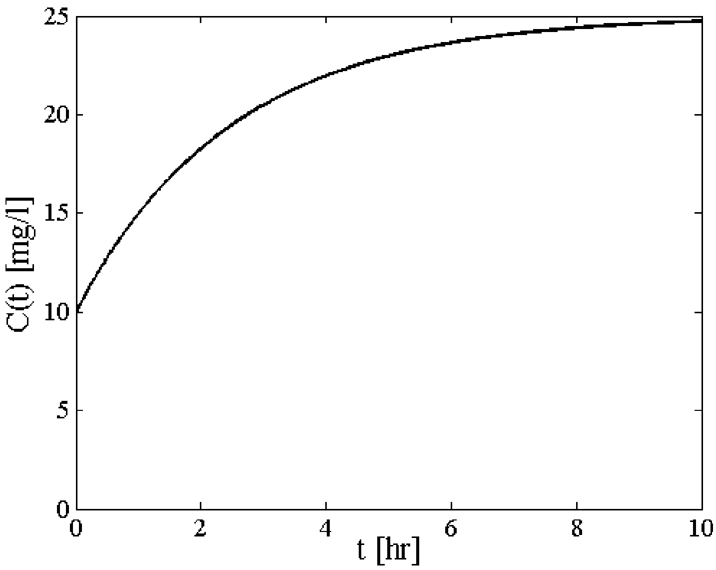
Put into the equation $\Rightarrow$ $C_p(t) = K_2 = \dfrac{200}{8} = 25$

③ Final solution: $C(t) = C_h(t) + C_p(t) = K_1 e^{-0.4t} + 25$

Initial conditions: $C(0) = 10\text{mg/l}$

$$\Rightarrow \quad K_1 = -15$$

$$C(t) = (-15e^{-0.4t} + 25)\text{mg/l}$$



16. (a) $f(t) - mg\sin(\theta) - k_f \upsilon(t) = ma(t)$,

where $\upsilon(t) = \dfrac{d}{dt}(y(t))$, $a(t) = \dfrac{d^2}{dt^2}(y(t))$

$$\Rightarrow \quad m\frac{d^2}{dt^2}(y(t)) + k_f \frac{d}{dt}(y(t)) + mg\sin(\theta) = f(t)$$

(b) No excitation force applied $\Rightarrow$ $f(t) = 0$

$$m\frac{d^2}{dt^2}(y(t)) + k_f \frac{d}{dt}(y(t)) = -mg\sin(\theta)$$

① Homogeneous solution:

The eigenvalues of the solution are: $\lambda_1 = 0$, $\lambda_2 = -\frac{k_f}{m}$

So the homogeneous solution should in the form of:

$$y_h(t) = K_{h1}e^{\lambda_1 t} + K_{h2}e^{\lambda_2 t} = K_{h1} + K_{h2}e^{-\frac{k_f}{m}t}$$

② Particular solution:

The right side of the equation is a constant $-mg\sin(\theta)$ and the left side contains second order and first order derivatives, so the particular solution should in the form of a linear function of t , set as $y_p = K_p t$. Put into the equation $\Rightarrow$ $K_p = -\frac{mg}{k_f}\sin(\theta)$, $y_p(t) = -\frac{mg}{k_f}\sin(\theta)t$

③ Summation of homogeneous solution and particular solution:

$$y(t) = y_h(t) + y_p(t) = K_{h1} + K_{h2}e^{-\frac{k_f}{m}t} - \frac{mg}{k_f}\sin(\theta)t$$

Initial conditions:

$$y(0) = 0 = K_{h1} + K_{h2} ,$$

$$y'(0) = 10 = -\frac{k_f}{m}K_{h2} - \frac{mg}{k_f}\sin(\theta)$$

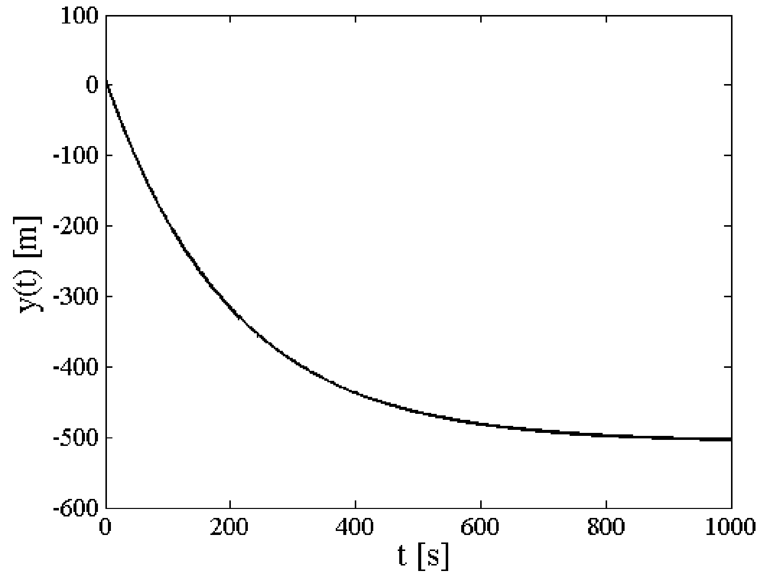
Substituting for the constants given in the problem statement: $m = 1000\text{kg}$, $k_f = 5\text{N}\cdot\text{s/m}$, $\theta = \pi/12$ and using $g = 9.8\text{m/s}^2$

$$\Rightarrow \quad K_{h2} = -(\frac{m}{k_f})^2 g\sin(\theta) - 10\frac{m}{k_f} = -1.0346\times10^5$$

$$K_{h1} = -K_{h2} = 1.0346\times10^5$$

$$y(t) = 1.0346\times10^5(1 - e^{-\frac{t}{200}}) - 507.28t$$

$$y'(t) = 517.28e^{-\frac{t}{200}} - 507.28$$



(c) Constant force $f(t) = F = 200\text{N}$ is applied,

$$m\frac{d^2}{dt^2}(y(t)) + k_f \frac{d}{dt}y(t) = F - mg\sin(\theta)$$

① Homogeneous solution: same as part (b) ①

$$y_h(t) = K_{h1}e^{x_1 t} + K_{h2}e^{x_2 t} = K_{h1} + K_{h2}e^{-\frac{k_f}{m}t}$$

② Particular solution: similar to part (b) ②

The right side of the equation is a constant $F - mg\sin(\theta)$. Set particular solution as linear function of t, as $y_p = K_p t$. Put into the equation

$$\Rightarrow \quad K_p = \frac{1}{k_f}[F - mg\sin(\theta)],$$

$$y_p(t) = \frac{t}{k_f}[F - mg\sin(\theta)]$$

③ Summation:

$$y(t) = y_h(t) + y_p(t) = K_{h1} + K_{h2}e^{-\frac{k_f}{m}t} + \frac{t}{k_f}[F - mg\sin(\theta)]$$

$$\frac{d}{dt}(y(t)) = (-\frac{k_f}{m})K_{h2}e^{-\frac{k_f}{m}t} + \frac{1}{k_f}[F - mg\sin(\theta)]$$

$$\lim_{t\to\infty}(\frac{d}{dt}(y(t))) = \frac{1}{k_f}[F - mg\sin(\theta)] = -467.3\,\mathrm{m}/\mathrm{s}$$

A negative terminal velocity indicates the force of 200N is insufficient to move the car forward on that inclined plane, so the car is rolling backward down the hill.

17. System equation: $4y''(t) + y'(t) + 3y(t) = x(t)$

Linearity

$x_1(t) = g(t) \;\Rightarrow\; 4y_1''(t) + y_1'(t) + 3y_1(t) = x_1(t) = g(t)$ ①

$x_2(t) = h(t) \;\Rightarrow\; 4y_2''(t) + y_2'(t) + 3y_2(t) = x_2(t) = h(t)$ ②

$x_3(t) = Kg(t) + h(t) \;\Rightarrow\; 4y_3''(t) + y_3'(t) + 3y_3(t) = x_3(t) = Kg(t) + h(t)$ ③

①$\times K +$② $\Rightarrow$

$4Ky_1''(t) + 4y_2''(t) + Ky_1'(t) + y_2'(t) + 3Ky_1(t) + 3y_2(t) = Kg(t) + h(t)$ ④

Comparing ③ and ④ $\Rightarrow$ $y_3(t) = Ky_1(t) + y_2(t)$

So this system is linear.

Time Invariance

$x_1(t) = g(t) \;\Rightarrow\; 4y_1''(t) + y_1'(t) + 3y_1(t) = x_1(t) = g(t)$ ①

$x_2(t) = g(t - t_0) \;\Rightarrow\; 4y_2''(t) + y_2'(t) + 3y_2(t) = x_2(t) = g(t - t_0)$ ②

Substituting t with $t - t_0$ in ①

$\Rightarrow\; 4y_1''(t - t_0) + y_1'(t - t_0) + 3y_1(t - t_0) = g(t - t_0)$ ③

Comparing ② and ③

$\Rightarrow\; y_2(t) = y_1(t - t_0)$

So this system is time invariant.

Stability

The eigenvalues of the solution of this system are:

$$\lambda_1 = -0.125 + j0.484$$

$$\lambda_2 = -0.125 - j0.484$$

Real parts of all eigenvalues are negative, so the system is stable.

Causality and Memory

The response at any time $t = t_0$ depends on the excitation at times $t < t_0$, not on any future values, so this system is causal and has memory.

Invertibility

The excitation of this system is expressed in terms of the response and its derivatives, so the excitation is uniquely determined by the response. This system is invertible.

18. System equation: $y(t) = x^3(t)$

Linearity

$$x_1(t) = g(t) \quad \Rightarrow \quad y_1(t) = x_1^{\,3}(t) = g^3(t) \qquad ①$$

$$x_2(t) = h(t) \quad \Rightarrow \quad y_2(t) = x_2^{\,3}(t) = h^3(t) \qquad ②$$

$$x_3(t) = Kg(t) + h(t) \quad \Rightarrow \quad y_3(t) = x_3^{\,3}(t) = [Kg(t) + h(t)]^3 \qquad ③$$

$$① \times K + ② \quad \Rightarrow \quad Ky_1(t) + y_2(t) = Kg^3(t) + h^3(t) \qquad ④$$

Comparing ③ and ④ $\Rightarrow \quad y_3(t) \neq Ky_1(t) + y_2(t)$

So this system is non-linear.

Time Invariance

$$x_1(t) = g(t) \quad \Rightarrow \quad y_1(t) = x_1^{\,3}(t) = g^3(t) \qquad ①$$

$$x_2(t) = g(t - t_0) \quad \Rightarrow \quad y_2(t) = x_2^{\,3}(t) = g^3(t - t_0) \qquad ②$$

Substituting t with $t - t_0$ in ①

$$\Rightarrow \quad y_1(t-t_0) = x_1^3(t-t_0) = g^3(t-t_0) \qquad ③$$

Comparing ② and ③ $\Rightarrow \quad y_2(t) = y_1(t-t_0)$

So this system is time invariant.

Stability

If $x(t)$ is bounded then its response $y(t)$ is bounded, so this system is stable.

Causality and Memory

The response at any time $t = t_0$ depends only on the excitation at time $t = t_0$, not on any future value or any past values, so this system is causal and memoryless.

Invertibility

The excitation can be written in terms of its response as: $x(t) = y^{\frac{1}{3}}(t)$

Two cases need to be considered:

① When excitation $x(t)$ is real, the cube root produces a unique excitation for a given response, so the system is invertible for real $x(t)$.

② When excitation $x(t)$ is complex, the cube root operation is multiple valued, so the excitation cannot be uniquely determined by a given response. This system is non-invertible for complex $x(t)$.

22. System equation: $y(t) = \mathrm{Re}(x(t))$

Homogeneity:

$x_1(t) = g(t) + jh(t)$, where $g(t)$ and $h(t)$ are both real-valued functions.

$$\Rightarrow \quad y_1(t) = \mathrm{Re}[g(t) + jh(t)] = g(t)$$

$x_2(t) = (K_r + jK_i)[g(t) + jh(t)]$, where K_r, K_i are both real constants.

$$\Rightarrow \quad y_1(t) = \mathrm{Re}[(K_r + jK_i)[g(t) + jh(t)]] = K_r g(t) - K_i h(t)$$

$$\neq (K_r + jK_i)g(t) = (K_r + jK_i)y_1(t)$$

So the system is not homogeneous.

Additivity:

$x_1(t) = g_1(t) + jh_1(t)$, where $g_1(t)$ and $h_1(t)$ are both real-valued functions.

$$\Rightarrow \quad y_1(t) = \mathrm{Re}[g_1(t) + jh_1(t)] = g_1(t)$$

$x_2(t) = g_2(t) + jh_2(t)$, where $g_2(t)$ and $h_2(t)$ are both real-valued functions.

$$\Rightarrow \quad y_2(t) = \mathrm{Re}[g_2(t) + jh_2(t)] = g_2(t)$$

$x_3(t) = x_1(t) + x_2(t) = (g_1(t) + g_2(t)) + j(h_1(t) + h_2(t))$

$$\Rightarrow \quad y_3(t) = \mathrm{Re}[x_3(t)] = g_1(t) + g_2(t) = y_1(t) + y_2(t)$$

So the system is additive.