Continuous-time Filtering: Modulation and AM Radio

Topics

Ideal filters

- distortion, signal power and bandwidth
- filter classifications
- impulse responses and causality

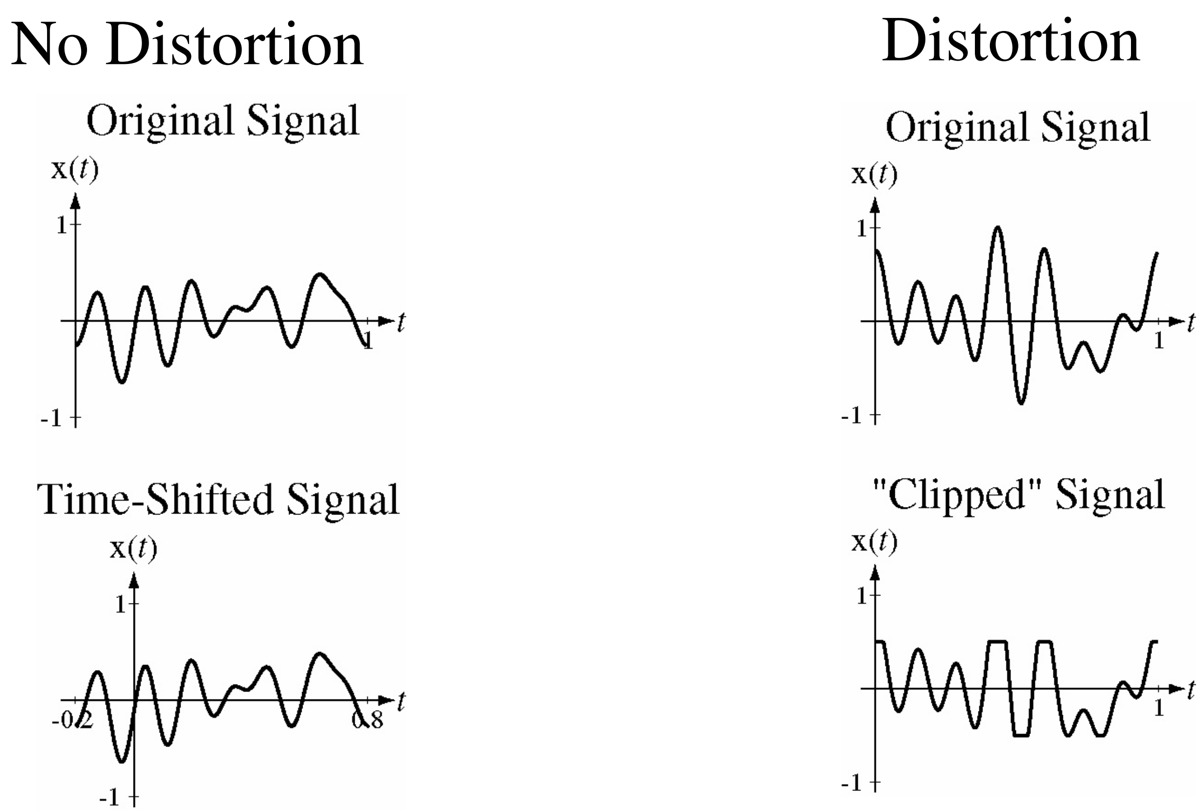
Communication systems

- radio transmission system: modulation
- demodulation of AM signals
- envelope detection of AM signals

Ideal filters

What is **distortion**? ‘Changing the shape’ of a signal

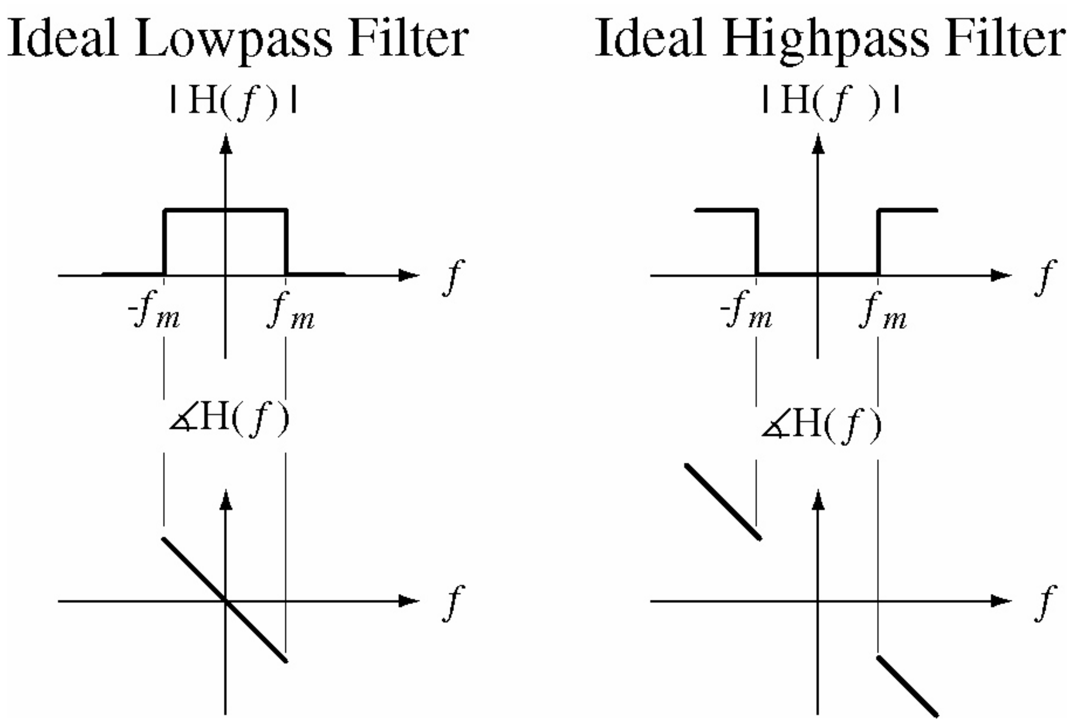
Multiplication of a signal by a constant (even a negative one) or shifting it in time do not change its shape



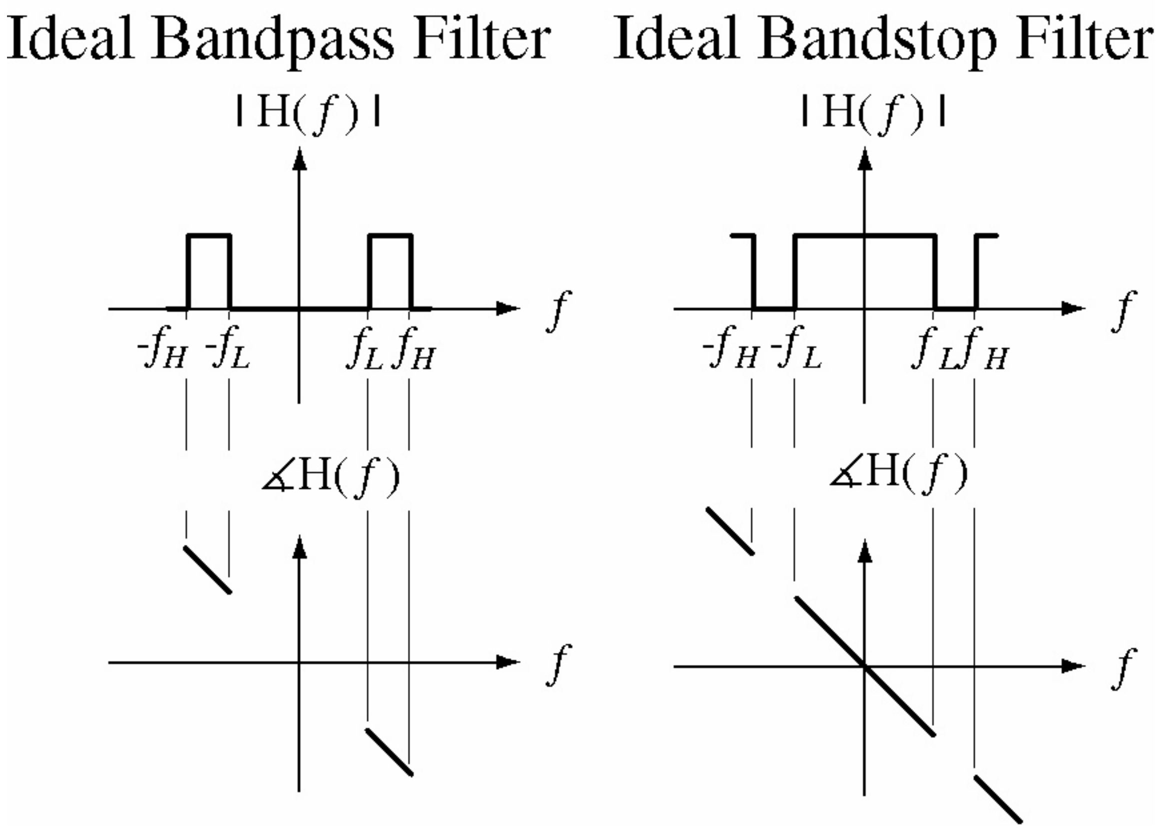
Ideal filters

Filters separate signals in one frequency range from signals in another frequency range

An **ideal filter** passes all signal power in its **passband** without distortion and completely blocks signal power outside its passband



Filter Classifications



FT analysis

The **Fourier Transform** of a signal is

$$f(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(j\omega)e^{j\omega t}d\omega \longleftrightarrow F(j\omega) = \int_{-\infty}^{+\infty} f(t)e^{-j\omega t}dt$$

For real-valued signals:

$$f(t) = \frac{1}{\pi}\int_{0}^{+\infty} |F(j\omega)|\cos(\omega t + \angle F(j\omega))dt$$

Fourier Transforms provide a **different perspective** on a signal. Some aspects become more evident in the **frequency domain**.

Signal energy

The **signal energy** can be computed in the time domain *or* in the frequency domain.

This is **Parseval's Theorem**:

$$\int_{-\infty}^{+\infty} \left| f(t) \right|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left| F(j\omega) \right|^2 d\omega$$

Most of the energy of physical signals is concentrated in parts of the spectrum.

Example*:* Humans cannot hear above 20 KHz. Hence the *useful* part of **audio signals** have frequencies below 20 KHz**.** We say that audio signals are **low-pass signals** and have a **bandwidth** of 20 KHz.

Signal bandwidth

Bandwidth means "range of frequencies." We use it denote range where most of signal energy is concentrated

E.g., for a **low-pass signal,** frequency Ω such that

$$\frac{1}{2\pi}\int_{-\Omega}^{+\Omega}\left|F(j\omega)\right|^2 d\omega = r\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|F(j\omega)\right|^2 d\omega$$

For r = 0.99, we say that Ω is the 99% bandwidth of the signal.

For **band-pass signal,** frequency range such that

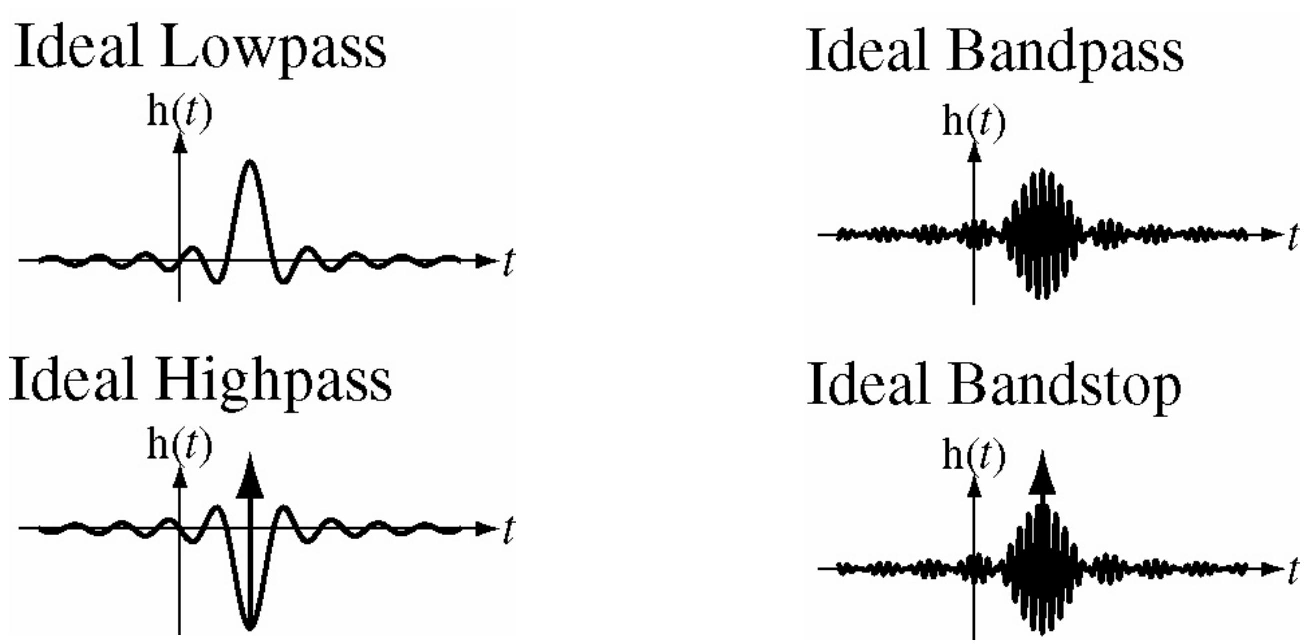
$$\frac{1}{\pi}\int_{+\Omega_l}^{+\Omega_u}\left|F(j\omega)\right|^2 d\omega = r\frac{1}{2\pi}\int_{-\infty}^{+\infty}\left|F(j\omega)\right|^2 d\omega$$

For r = 0.99 then we say that $\Omega = \Omega_u - \Omega_l$ is the 99% bandwidth of the signal.

Why are ideal filters called *ideal*?

One reason is that perfect circuit components with ideal characteristics do not exist

Another, more fundamental, reason can be seen in the corresponding impulse responses



All responses have nonzero response *before the impulse is applied* at $t = 0$

Ideal filters are non-causal

For example, ideal lowpass filter has a frequency response

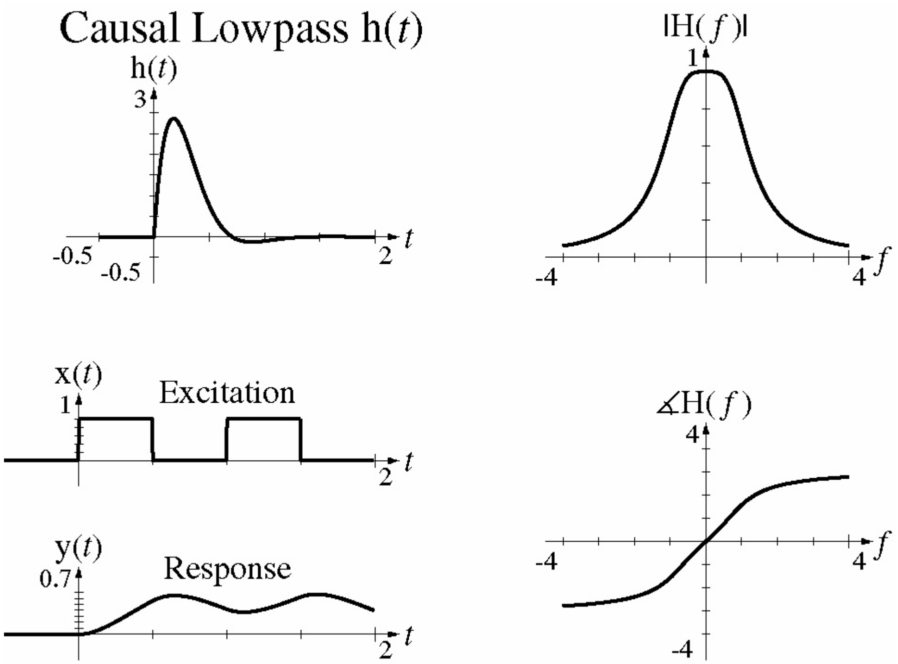
$$H(f) = A\, rect(f/2f_m)e^{-j2\pi f t_0}$$

The inverse Fourier transform is the impulse system response

$$h(t) = 2Af_m \mathrm{sinc}(2f_m(t - t_0))$$

Non-causal system!

Ideal filters cannot be physically realized, but they can be closely approximated



Topics

Ideal filters

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Communication systems

- radio transmission system: modulation
- demodulation of AM signals
- envelope detection of AM signals

What is modulation?

Modulation is the process of shifting the frequency of a signal so that the resulting signal is in a desired frequency band

Why would one be interested in modulation?

1. Antenna length requirements: for efficient radiation, antenna length is 1/2 or 1/4 of the wavelength to be radiated

$$\lambda_c = \frac{c}{f_c}$$

Here, c is the speed of light. At a frequency of 1 KHz (human voice), a 1/4 wavelength antenna would measure

$$L = \frac{c}{4f_c} = \frac{3 \times 10^8}{4 \times 10^3} = \frac{3}{4}10^5 = 75km$$

However, had the signal higher frequency, smaller antennas would do

Why modulation?

Why would one be interested in modulation?

2. Interference: two communications, at the same time and over the same geographical area, using the same frequency, would interfere with each other.

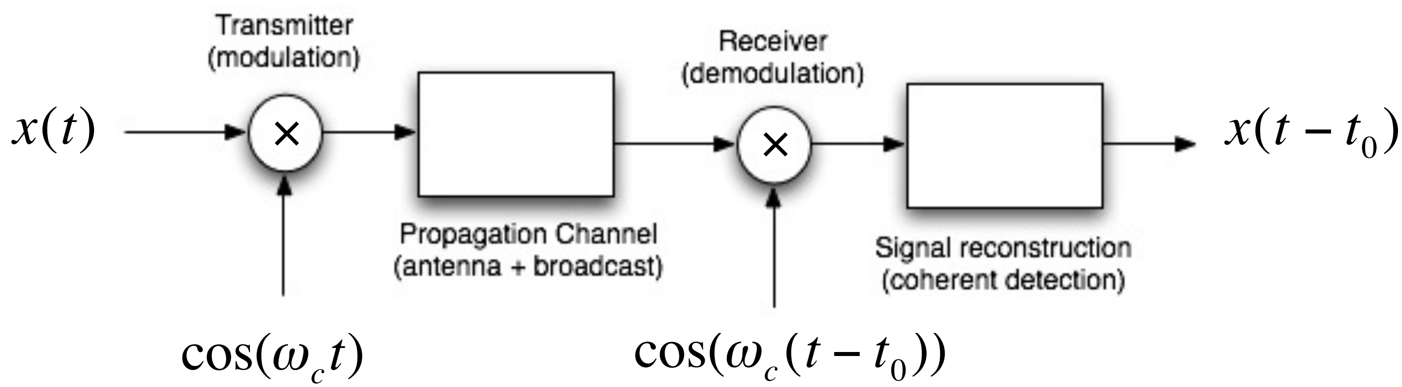
   This can be solved by shifting information signals to different frequencies

Frequency band	Designation	Typical users
3-30 kHz	Very low frequency (VLF)	Long-range navigation
30-300 kHz	Low frequency (LF)	Marine communications
300-3000kHz	Medium frequency (MF)	AM radio broadcasts
3-30 MHz	High frequency (HF)	Amateur radio; telephone
30-300 MHz	Very high frequency (VHF)	VHF TV; FM radio
0.3-3 GHz	Ultrahigh frequency (UHF)	UHF TV; radar
3-30 GHz	Superhigh frequency (SHF)	Satellite communications

FCC Frequency Band Assignments

Modulation and Radio

A simplified radio communication system can be the following:



Music, people talking produces the **input signal** $x(t)$

In the **transmitter**, the signal is **modulated**, here as an amplitude modulated **(AM)** signal

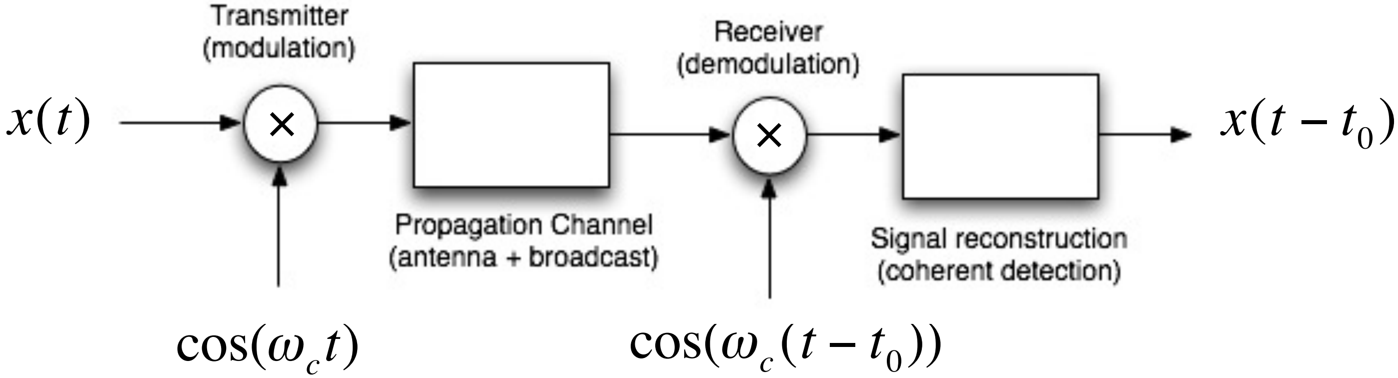
$$x(t)\cos(\omega_c t)$$

The frequency ω_c is the **carrier frequency** of the radio station.

Usually $\omega_c \geq \Omega$, where Ω is the bandwidth of $x(t)$

Modulation and Radio

A simplified radio communication system can be the following:



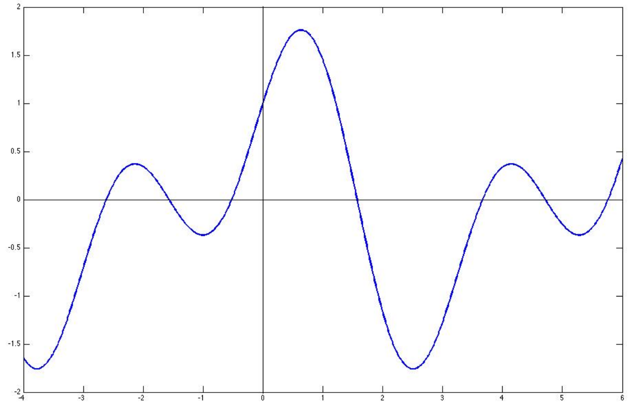
The radio signal is **broadcast** and picked up by the antenna on the **receiver**.

The **receiver** converts down the broadcast signal (**demodulates**) into a signal proportional to $x(t - t_0)$ with t_0 a small time delay (less than a millisecond)

Amplitude Modulation in the Time Domain

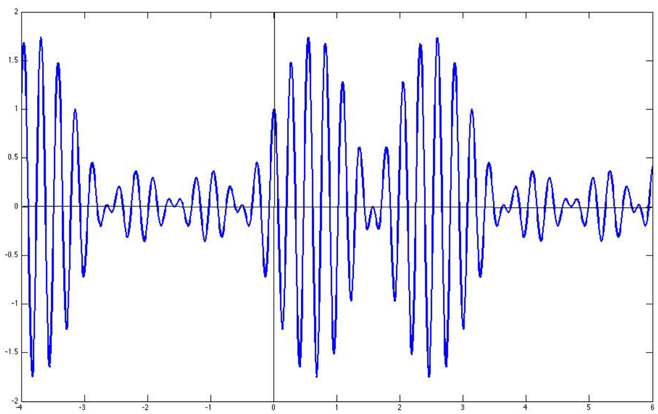
Input signal:

$x(t)$



AM modulated signal:

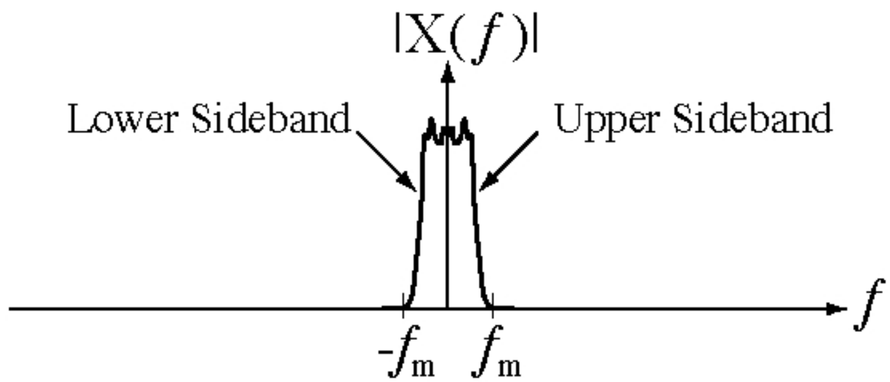
$x(t)\cos(\omega_c t)$



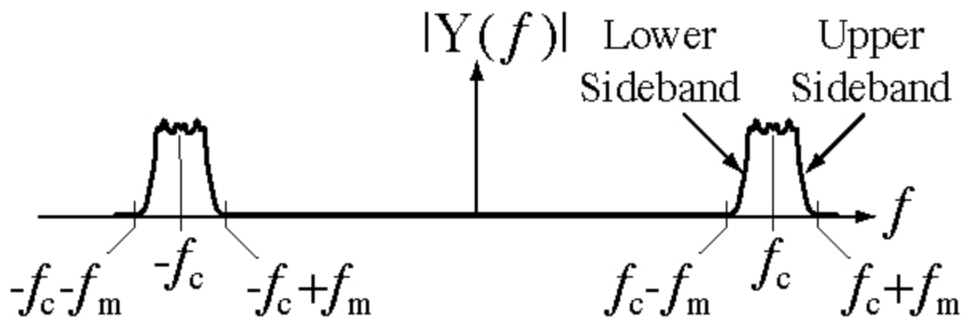
Amplitude Modulation in the Frequency Domain

Input signal:

$$x(t) \longleftrightarrow X(f)$$



AM modulated signal:

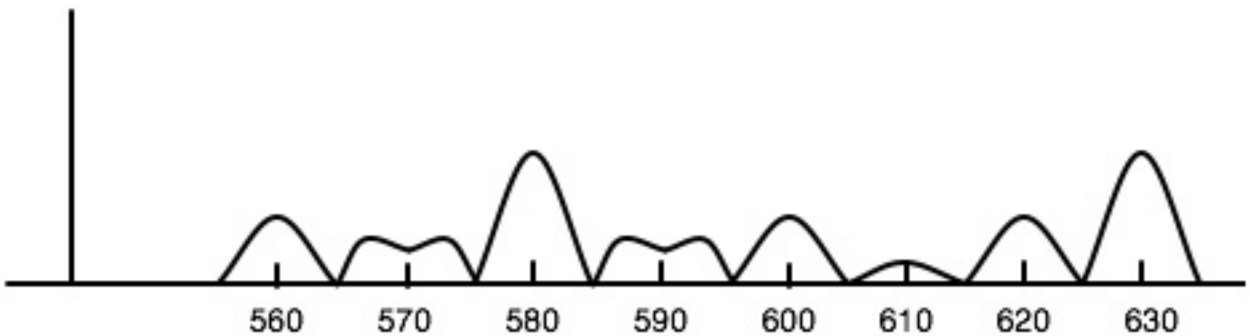


$$x(t)\cos(2\pi f_c t) \longleftrightarrow Y(f) = X(f) * \left(\tfrac{1}{2}\delta(f - f_c) + \tfrac{1}{2}\delta(f + f_c)\right) = \tfrac{1}{2}X\left(f - f_c\right) + \tfrac{1}{2}X\left(f + f_c\right)$$

Modulation and Fourier Transform

Different AM stations have different carrier frequencies

In the US, the carrier frequency is chosen from a set spaced 10 KHz apart. These are taken within the frequency range that starts at 540 KHz and ends at 1700 KHz.

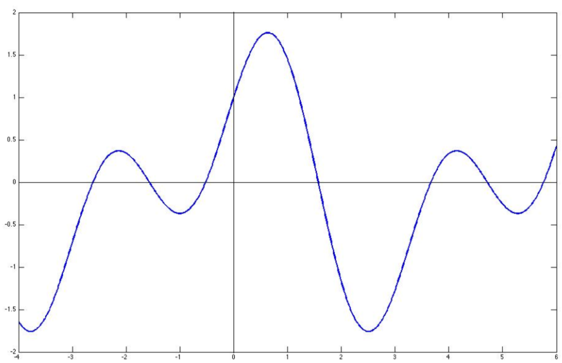


With a 10 KHz operation bandwidth, each AM station can broadcast a signal of 5 KHz bandwidth.

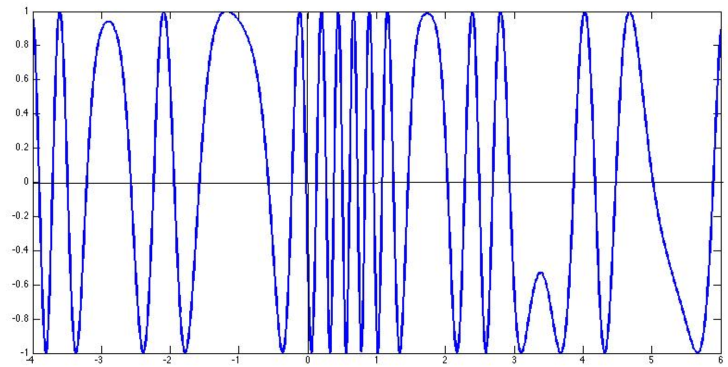
Comparison with Frequency Modulation (FM)

Comparison of a signal and an FM version of it:

$x(t)$



$\cos\left(\left(\omega_c + \beta x(t)\right)t\right)$



FM transforms a signal into another with higher-frequency band but unit magnitude

Stations range from 88 MHz to 108MHz at 200 KHz apart

- Bandwidth of each station is only 15 KHz
- FM Antenna is only

$$L = \frac{c}{4f_c} = \frac{3\times10^8}{4\times10^8} = \frac{3}{4} = 0.75m$$

Other types of Modulation

In general, **modulation** is a process that changes one or more properties of a periodic signal (its amplitude, its frequency or its phase) by another signal

In the previous examples, the modulated, periodic signal is a cos function, while the modulating signal is the radio wave signal that we would like to transmit

Instead of cos, other periodic signals can be used. For example, using a pulse wave instead of a cosine wave results into **pulse modulation.**

Pulse modulation is used to transfer analog signals as digital signals (that is, as a sampled and quantized signal) by using a digital transmission system

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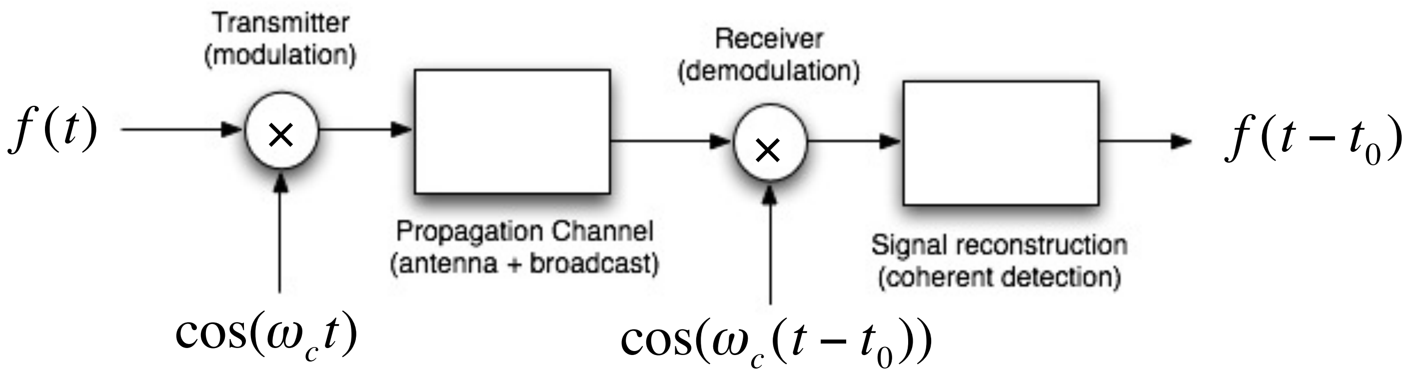
Coherent Demodulation of AM signals

An antenna for an AM radio receiver captures signals from tens of radio stations. How does it select the right signal?



The receiver uses a **band-pass filter** with an **adjustable** pass-band to **tune in** $f(t)\cos(\omega_c t)$

The band-pass output is then converted into an audio signal that is proportional to $f(t-t_0)$. Here, t_0 is the time delay of the propagation channel

Coherent demodulation of AM signals

The **receiver input** will be a signal

$$r(t) = kf(t - t_0)\cos(\omega_c(t - t_0))$$

The **receiver output** will be a signal

$$\begin{aligned} m(t) &= r(t)\cos\left(\omega_c(t - t_0)\right) \\ &= \frac{k}{2} f(t - t_0)\{1 + \cos(2\omega_c(t - t_0))\} \end{aligned}$$

The Fourier Transform of *m(t)* is

$$M(j\omega) = \frac{k}{2} F(j\omega) e^{-j\omega t_0} + \frac{k}{4}\left\{F\left(j\left(\omega - 2\omega_c\right)\right) + F\left(j\left(\omega + 2\omega_c\right)\right)\right\} e^{-j\omega t_0}$$

The first term of *M(jw)* is the **signal we would like to recover**

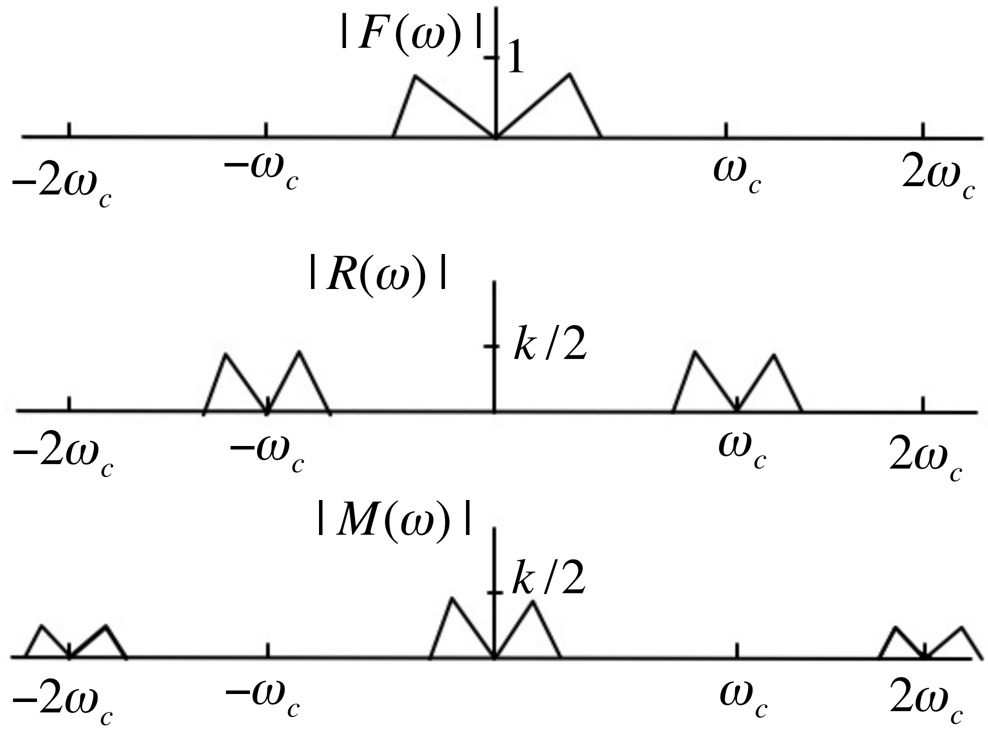
Coherent demodulation of AM signals

Graphically, we have

$f(t) \leftrightarrow F(\omega)$

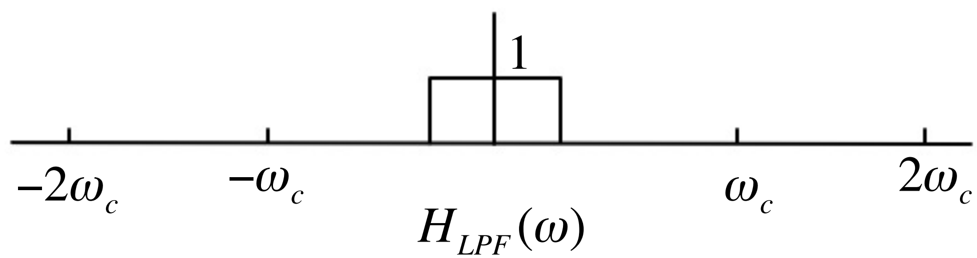
$r(t) \leftrightarrow R(\omega)$

$m(t) \leftrightarrow M(\omega)$



In the final step,
we take

$$\boxed{Y(\omega) = H_{LPF}(\omega)M(\omega)}$$



Coherent demodulation of AM signals

After taking $Y(\omega) = H_{LPF}(\omega) M(\omega)$, we obtain

$$y(t) \leftrightarrow Y(\omega) \qquad\qquad y(t) = \frac{k}{2} f(t - t_0)$$

What happens if the receiver multiplies the broadcast signal

$$r(t) = kf(t - t_0)\cos(\omega_c(t - t_0))$$

By $\cos(\omega_c(t - t_1))$ with $t_1 \neq t_0$?

$$m(t) = r(t)\cos\big(\omega_c(t - t_1)\big)$$

$$= \frac{k}{2} f(t - t_0)\{\cos(\omega_c(t_0 - t_1)) + \cos(2\omega_c(t - t_0 + t_1))\}$$

For optimal demodulation the receiver must **know exactly** the value t_0 , hence the term **coherent demodulation**.

To avoid the estimation of an accurate t_0 , another process called **envelope detection** is used in practice

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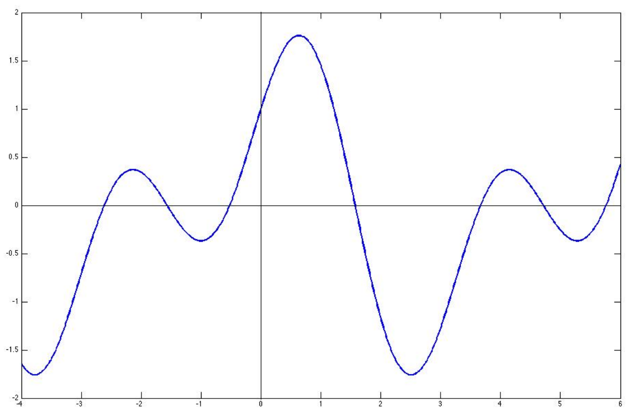
Recall AM Signal in Time Domain

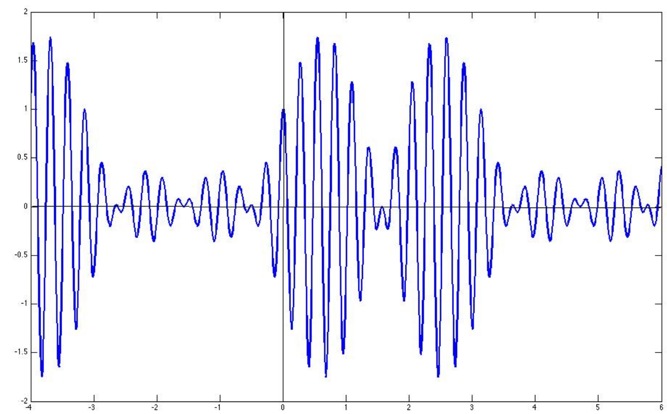
Input Signal:

$f(t)$



AM Modulated Signal:

$f(t)\cos(\omega_c t)$

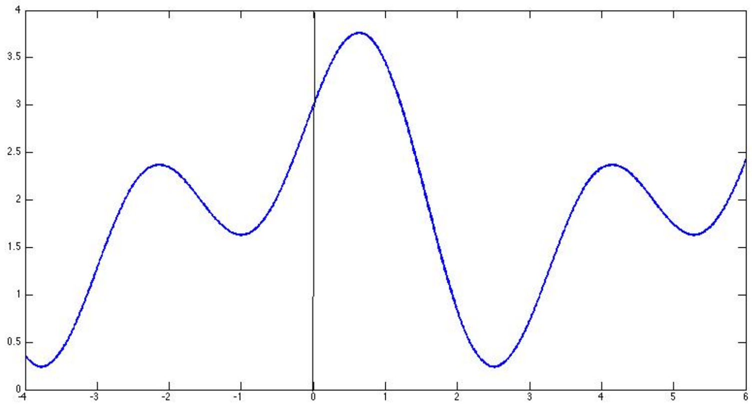


Envelope of signal is proportional to $|f(t)|$

Recall AM Signal in Time Domain

Input Signal:

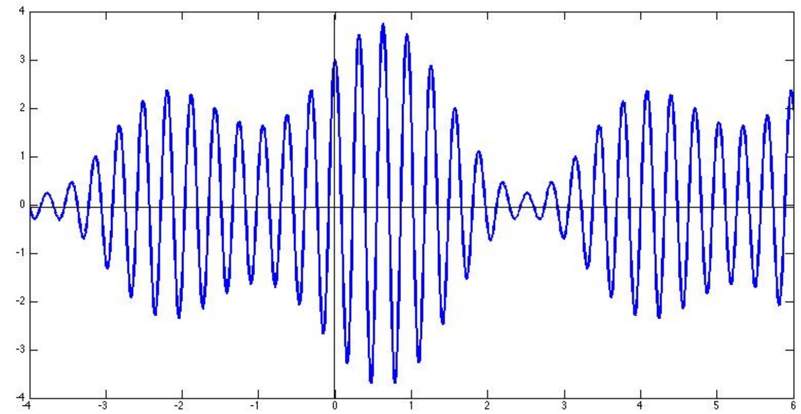
$$f(t)+\alpha = |f(t)+\alpha|$$



$$\alpha > \max |f(t)|$$

AM Modulated Signal:

$$(f(t)+\alpha)\cos(\omega_c t)$$

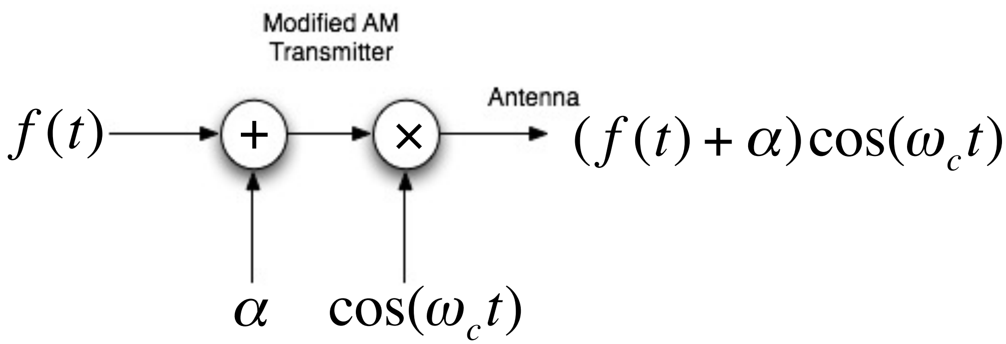


$$f(t)+\alpha = |f(t)+\alpha|$$

Envelope of signal is proportional to

Envelope detection of AM signals

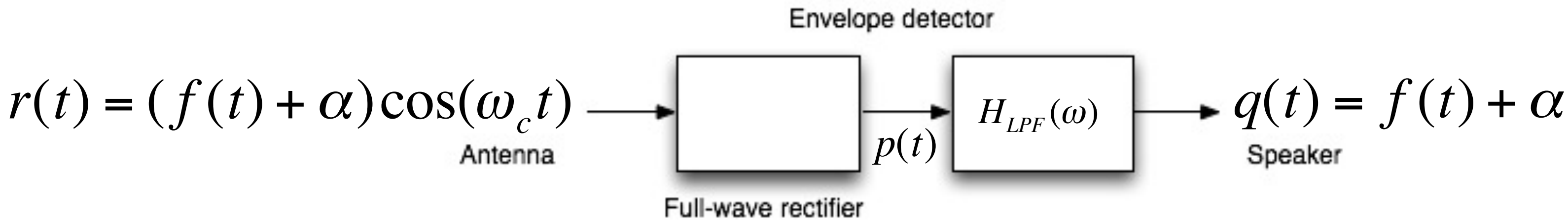
In order to implement envelope detection at the AM receiver, the AM transmitter is modified as follows



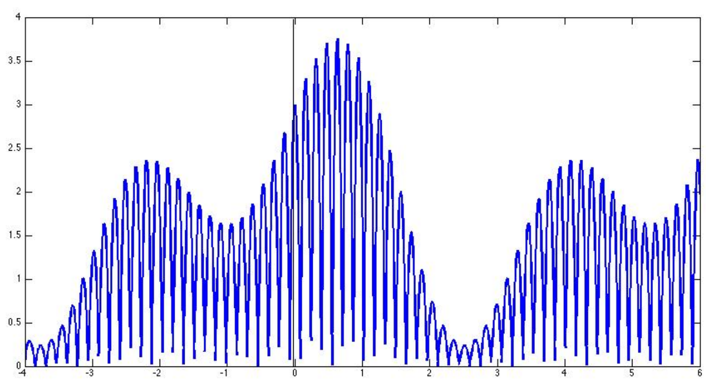
$$\alpha > \max |f(t)|$$

Envelope detection of AM signals

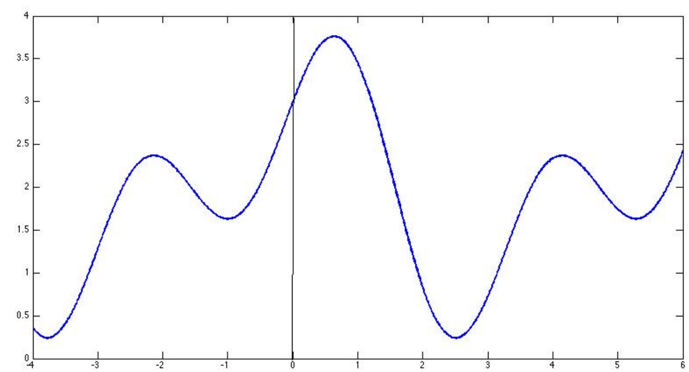
The AM receiver will extract the signal as follows



$$p(t) = |r(t)|$$



$$q(t) = f(t) + \alpha$$



Envelope detection of AM signals

To see how the signal extraction works, let us look at the signal/system interaction in the FD

First, $|\cos(\omega_c t)|$ is a periodic signal of frequency $2\omega_c$, thus it can be expanded as

$$|\cos(\omega_c t)| = \frac{a_0}{2} + \sum_{n=1}^{+\infty} a_n \cos(n2\omega_c t)$$

In this way, $p(t) = |r(t)| = (f(t)+\alpha)|\cos(\omega_c t)|$ can be expressed as $p(t) = p_1(t) + p_2(t)$

$$p_1(t) = f(t)\left|\cos(\omega_c t)\right| = \frac{a_0}{2} f(t) + \sum_{n=1}^{+\infty} a_n f(t)\cos(n2\omega_c t)$$

$$p_2(t) = \alpha\left|\cos(\omega_c t)\right| = \frac{a_0}{2}\alpha + \sum_{n=1}^{+\infty} a_n \alpha \cos(n2\omega_c t)$$

Envelope detection of AM signals

Now, consider that $H_{LPF}(\omega)$ is designed in such a way that

$$H_{LPF}(0) = \frac{2}{a_0} \qquad H_{LPF}(n2\omega_0) = 0 \qquad n \geq 1$$

Then, the response of $H_{LPF}(\omega)$ to $p_2(t)$ is just $q_2(t) = \alpha$

To determine the filter response to $p_1(t)$, first we obtain

$$P_1(\omega) = \frac{a_0}{2} F(\omega) + \sum_{n=1}^{+\infty} \frac{a_n}{2} \{F(\omega - n2\omega_c) + F(\omega + n2\omega_c)\}$$

Because only the first term is within the pass-band of the filter H_{LPF} we just obtain

$$Q_1(\omega) = \frac{a_0}{2} H(0) F(\omega) \leftrightarrow q_1(t) = f(t)$$

Therefore, using superposition, we find the output of $H_{LPF}(\omega)$

$$\boxed{q(t) = q_1(t) + q_2(t) = f(t) + \alpha}$$

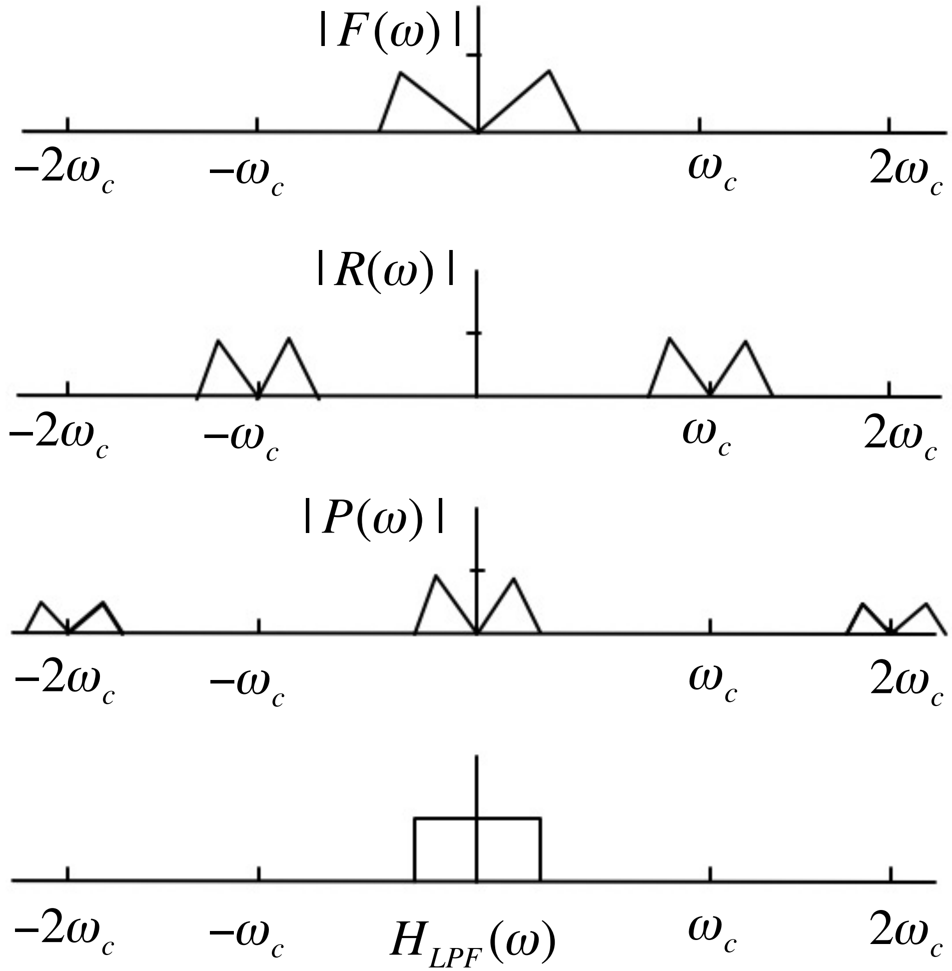
Envelope detection of AM signals

Graphically, we have

$f(t) \leftrightarrow F(\omega)$

$r(t) \leftrightarrow R(\omega)$

$p(t) \leftrightarrow P(\omega)$



In the final step, we take

$$\boxed{Q(\omega) = H_{LPF}(\omega)P(\omega)}$$

Envelope detection of AM signals

The process of envelope detection is insensitive to time shifts in the carrier wave

In other words, if the input to the detector is

$$r(t) = (f(t - t_0) + \alpha)\cos(\omega_c(t - t_0))$$

The detector **output will still be** $f(t - t_0) + \alpha$ because the coefficients of the CTFS of $|\cos(\omega_c(t - t_0))|$ do not change with a shift in time

Modulation and AM Radio Summary

Radio and Communication Systems can be better understood in the Frequency Domain and by looking at signals spectra:

Before transmitting, signals need to be transformed from low-pass to band-pass signals.

-This is done by; e.g. AM modulation:

$$f(t) \longrightarrow f(t)\cos(\omega_c t) \text{ or } (f(t)+\alpha)\cos(\omega_c t)$$

At the receiver, signals are detected using demodulation and a low-pass filter. We discussed here two approaches:

- Coherent detection

$$r(t) \longrightarrow m(t) = r(t)\cos(\omega_c(t-t_0)) \longrightarrow \boxed{H_{LPF}(\omega)} \longrightarrow \frac{k}{2} f(t-t_0)$$

- Envelope detection

$$r(t) \longrightarrow p(t) = |r(t)| \longrightarrow \boxed{H_{LPF}(\omega)} \longrightarrow f(t-t_0)+\alpha$$

Applications to radio-controlled devices

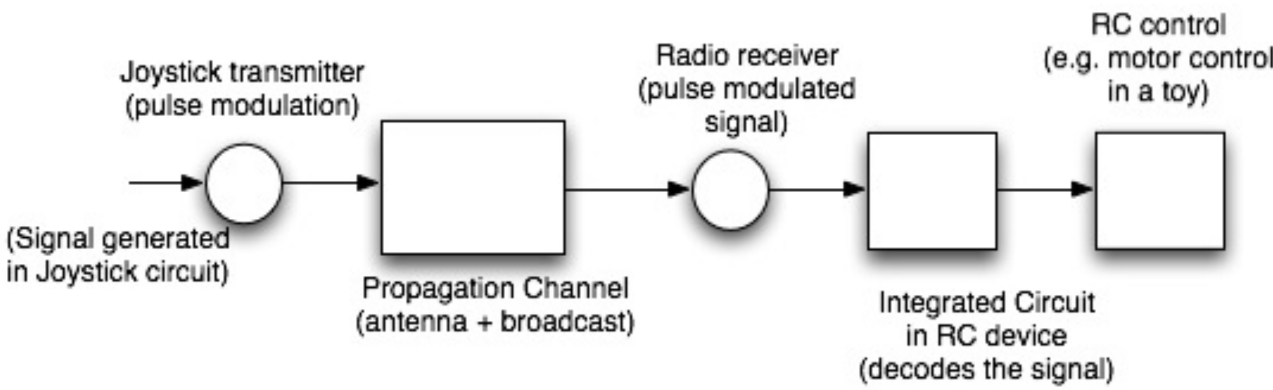
At its core, we have seen that radio is a very simple technology, yet the impact of radio in our society is tremendous. Some of the technologies that depend on radio include the following:

AM and FM radio
Cordless phones
Garage door openers
TV remote control
Wireless networks
Radio-controlled toys
GPS receivers
Satellite communications…

Note that radio waves not only can be used to transmit information, but also that **this information can be used for remote control**

Applications to radio-controlled devices

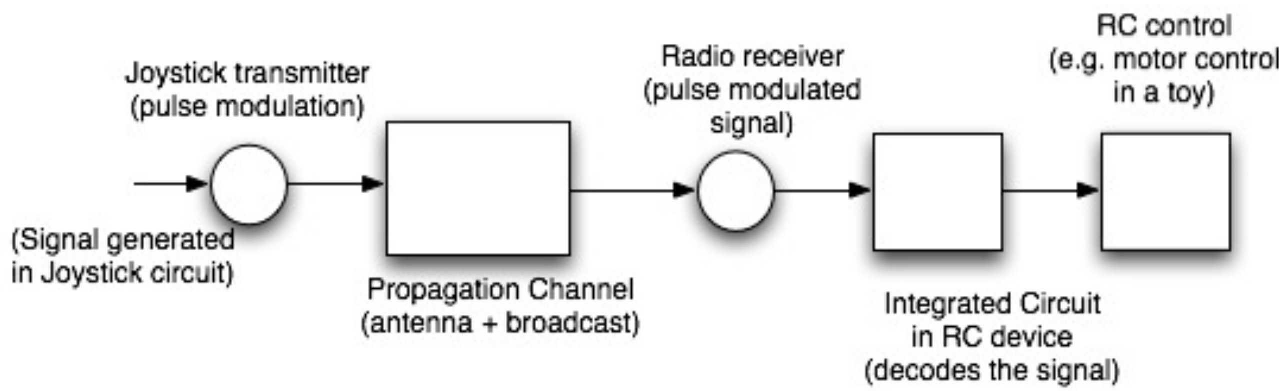
A simplified block diagram of a remotely controlled toy is the following:



By turning a knob in the joystick, an electric circuit is closed. This circuit is connected to an Integrated Circuit which generates the signal to be transmitted. It is possible to generate different signals

Before transmission, the signal is pulsed-modulated, to place the signal frequencies in a range anywhere between $27MHz$ to $49MHz$

Applications to radio-controlled devices



At the other end, the truck receiver is constantly monitoring for incoming signals

The picked signals are passed through a band-pass filter to block out the frequencies outside the chosen frequency range

The obtained pulse sequence is then sent to an Integrated Circuit in the truck. The IC decodes the sequence to generate the right control for the motor of the truck!