MAE143 A - Signals and Systems - Winter 11
Final Project, Due on March 11th

Instructions

(i) Download the file `project.wav` from the course website;

(ii) Write a report answering the following questions;

(iii) Your response to each question should include step-by-step derivations but no matlab code;

(iv) Use matlab to compute what you are asked and generate plots that help you answer the questions;

(v) Include the matlab files you generate as an appendix.

(vi) Do not forget to write your name, student number, and instructor.

The file `project.wav` contains samples from an audio message that was incorrectly demodulated from an AM radio transmission. The original signal was modulated and transmitted at frequency f_c, but the signal you now have in hand was demodulated coherently using the incorrect frequency $\widetilde{f}_c \neq f_c$. The bandwidth of the originally transmitted low-pass signal is $f_B = 4$ kHz.

The file `project.wav` was obtained by sampling the continuous signal which was demodulated incorrectly using the frequency $\widetilde{f}_c$ at a uniform sampling frequency f_s. The sampling frequency f_s is much higher than f_B or the difference $|f_c - \tilde{f}_c|$.

Start by loading this file in matlab and playing the associated audio (the commands `wavread` and `audioplayer` will be helpful here). In the rest of the project you will write a number of matlab programs to help you answer the following questions.

Other commands that you might find useful (i.e., you can use them if you want, but there are many other ways of doing what you want without them) are `freqresp`, `reshape`, `fftshift`, `circshift`, `colon`, `.*`. Also, remember that all the matlab files used in class are posted in the webpage and you can resort to them for specific syntax.

Q1: **The spectrum of the message**

Before you start: type 'help `fft`' in matlab and read the description of what this command does.

Let $x(t)$ be the message you downloaded and N be its number of samples.

(a) (2 points) Use the matlab function `fft` to compute the spectrum $X(f)$, more specifically the DFT of $x(t)$ at a finite number of frequency points $X_k = X(f)|_{f=kf_s/N}$, $k = 0, \ldots, N-1$. Plot $|X(f)|$ at those frequency points.

(b) (2 points) Based on the plot you obtained, estimate the frequency $f_D = |f_c - \tilde{f}_c|$. Justify your answer. Do you have enough information to tell whether $f_c > \tilde{f}_c$ or $f_c < \tilde{f}_c$? Does it matter?

Q2: **Filter design**

Before you start: type 'help `butter`' in matlab and read the description of what this command does.

Now that you have estimated f_D, you will design two filters.

(a) (2 points) The first filter is a high-pass *continuous-time* (analog) filter with cuttoff frequency f_D. Use the matlab function `butter` to design a filter of order 8. Use the function `tf` to construct a linear system that represents this filter. What is the transfer function you obtained?

(b) (2 points) The second filter is a low-pass *continuous-time* (analog) filter with cuttoff frequency f_B. Use the matlab function `butter` to design a low-pass filter of order 8. Use the function `tf` to construct a linear system that represents this filter. What is the transfer function you obtained?

(c) (2 points) Evaluate the frequency response of both filters at the frequency points of the FFT you produced in Q1. Plot the magnitude of these frequency responses.

Q3: **Corrective Demodulation in the Time-domain**

Before you start: type 'help `lsim`' in matlab and read the description of what this command does.

You will now use the filters you designed in Q2 to correctly demodulate the signal.

(a) (2 points) Use the function `lsim` to run the signal $x(t)$ through the high-pass filter you designed in Q2 and produce the signal $x_h(t)$;

(b) (2 points) Create the signal $x_b(t)$ by multiplying $x_h(t)$ by the signal $\cos(2\pi f_D t)$;

(c) (2 points) Use the function `lsim` to run the signal $x_b(t)$ through the low-pass filter you designed in Q2(b) and produce the signal $x_l(t)$;

(d) (2 points) Draw a block diagram with the operations you just performed, indicating clearly where each of the above signals is generated. Classify the properties of each block concerning linearity, time-variance and causality.

(e) (2 points) Use the function `fft` to produce plots of the magnitude of the spectra $X_h(f)$, $X_b(f)$ and $X_l(f)$ at frequencies kf_s/N, $k = 0, \ldots, N-1$.

(f) (2 points) At this point you might want to play the signal $x_l(t)$. Explain why these operations recover the signal that was originally transmitted. What was the message originally transmitted?

(g) (1 point (bonus)) Can you skip the high-pass filtering step? Can you apply it after multiplying by the $\cos(2\pi f_D t)$ function?

Q4: **Corrective Demodulation in the Frequency-domain**

Before you start: type 'help `ifft`' in matlab and read the description of what this command does.

You will now manipulate the Discrete Fourier Transform (DFT) of the signal $x(t)$ to correctly demodulate the original signal. You should obtain an answer similar to the one you obtained in Q3. Start with the DFT of $x(t)$ that you obtained in Q1(a), that is, $X_k = X(f)|_{f=kf_s/N}$, $k = 0, \ldots, N-1$.

(a) (2 points) Compute $X_h(f) = H_h(f)X(f)|_{f=kf_s/N}$ where $H_h(f)$ is the frequency response of an ideal high-pass filter with cutoff frequency f_D. Plot the magnitude of $X_h(f)$.

(b) (2 points) Compute $X_b(f) = X_h(f - f_D) + X_h(f + f_D)$. Plot the magnitude of $X_b(f)$.

(c) (2 points) Compute $X_l(f) = H_l(f)X_b(f)|_{f=kf_s/N}$ where $H_l(f)$ is the frequency response of an ideal low-pass filter with cutoff frequency f_B. Plot the magnitude of $X_l(f)$. Compare it with the spectrum of $X_l(f)$ you computed in Q3.

(d) (2 points) Use the matlab function `ifft` to compute a sampled signal $x_l(t)$

(e) (2 points) Draw a block diagram with the operations you just performed, indicating clearly where each of the above signals is generated. Classify the properties of each block concerning linearity, time-variance and causality.

(f) (2 points) At this point you might want to play the signal $x_l(t)$. Explain why these operations recover the signal that was originally transmitted.

(g) (1 point (bonus)) Can you modify the above procedure so that you do not need the low-pass filtering step in the frequency domain?