Nonlinear Control - MAE281b

Midterm

Student name and number _____

**Please be accurate in the presentation of your solutions, and quote the results from class that you are using**

1. (2 points) Design a dynamic output feedback controller to locally stabilize the origin of

$$\begin{aligned}\dot{x}_1 &= -x_1 + x_2\\ \dot{x}_2 &= x_1 - x_2 - x_1x_3 + u\\ \dot{x}_3 &= x_1 + x_1x_2 - 2x_3\\ y &= x_2\end{aligned}$$

2. (3 points) Consider the system of question 1. Answer the following questions

   (i) what is its relative degree? Where is it valid?

   (ii) use your answer to (i) to put the system in normal form

   (iii) what is the zero dynamics? Is the system minimum-phase?

3. (3 points) Consider the unicycle dynamics

$$\begin{aligned}\dot{x}_1 &= u_1\cos\theta\\ \dot{x}_2 &= u_1\sin\theta\\ \dot{\theta} &= u_2\\ y_1 &= x_1\\ y_2 &= x_2\end{aligned}$$

   Answer the following questions

   (i) Does the system have a vector relative degree? Justify your answer

   (ii) If the answer to (i) is negative, find a regularizing dynamic state feedback controller that endows the system with a vector relative degree. Over what region is your controller valid?

4. (2 points) Let $\Delta = \text{span}\{f_1, \ldots, f_k\}$ be a nonsingular distribution on $\mathbb{R}^n$. Show that $\Delta$ is involutive if and only if $[f_i, f_j] \in \Delta$ for all $i, j \in \{1, \ldots, k\}$.