Nonlinear Control - MAE281b

Midterm

Student name and number ______________________________

Please be accurate in the presentation of your solutions, and quote the results from class that you are using

1. (1.5 points) Consider the control system

$$\dot{x}_1 = x_1 + x_2$$
$$\dot{x}_2 = 3x_1^2 x_2 + x_1 + u$$

   (i) Determine the equilibria of the unforced system and its stability properties

   (ii) Using the output $y = -x_1^3 + x_2$, design an output feedback controller to stabilize the origin

2. (4 points) Consider the SISO system

$$\dot{x}_1 = x_2 + 2x_1^2$$
$$\dot{x}_2 = x_3 + u$$
$$\dot{x}_3 = x_1 - x_3$$
$$y = x_1$$

   Now,

   (i) What is the relative degree? Write the system in normal form

   (ii) What is the zero dynamics? Is the system minimum phase?

   (iii) Design a static state feedback controller such that x_1 asymptotically tracks $r(t) = \sin t$

   (iv) Is the system feedback linearizable? If it is, find a change of coordinates and a static state feedback controller that puts it into linear form

3. (2.5 points) Consider the following model of a DC motor

$$\dot{x}_1 = -\theta_1 x_1 - \theta_2 x_2 u + \theta_3$$
$$\dot{x}_2 = -\theta_4 x_2 + \theta_5 x_1 u$$
$$y = x_2$$

   where x_1 is the armature current, x_2 is the speed, u is the field current, and $\theta_1, \ldots, \theta_5 > 0$. The objective is to design a speed control system so that y asymptotically tracks a *constant* speed reference $r \in \mathbb{R}$. Assume that $r^2 < \theta_3^2 \theta_5 / 4\theta_1\theta_2\theta_4$ and the domain of operation is restricted to $x_1 > \theta_3/2\theta_1$. Now,

   (i) Find the steady-state input $u_{ss} \in \mathbb{R}$ and the steady-state $((x_1)_{ss}, (x_2)_{ss})$ needed to maintain the output r (recall that we only care for $x_1 > \theta_3/2\theta_1$)

   (ii) Using linearization, design a state feedback integral controller to achieve the desired speed regulation