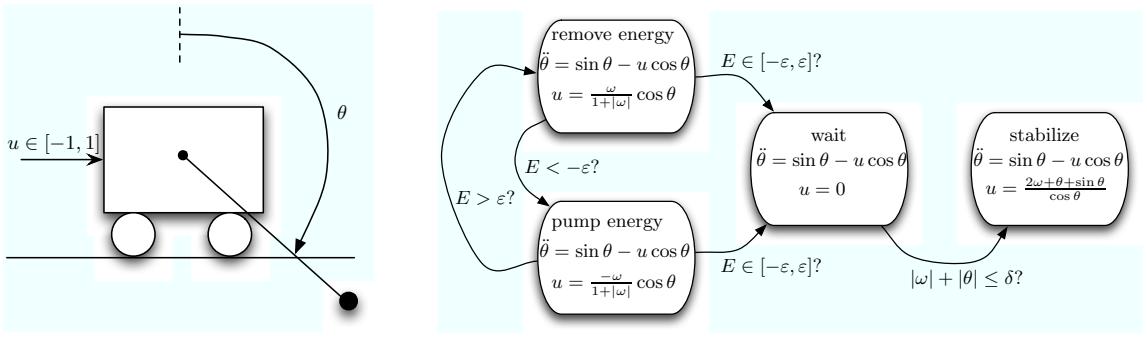
MAE 286: Hybrid Systems (F10)
Homework #2

Due on 10/12/10

1. (3 points) Consider the inverted pendulum swing-up controller shown in Figure 1. The angular



(a) (b)

Figure 1: (a) Swing-up pendulum and (b) hybrid controller.

position of the pendulum is θ, and its angular velocity is $\omega = \dot{\theta}$. The motion of the pendulum is described by the equation

$$\ddot{\theta} = \sin\theta - u\cos\theta$$

The energy function is a combination of kinetic plus potential energy $E = \frac{1}{2}\omega^2 + (\cos\theta - 1)$.

Do the following:

(i) Provide a formal description of the closed-loop system by specifying its discrete and continuous state-space as well as the activities and jump maps.

(ii) What is required of δ so that $u \in [-1, 1]$ in the discrete mode "stabilize"? Select an appropriate value for this parameter.

(iii) What is required of ε so that the guard to transition from mode "wait" to mode "stabilize" is enabled at some point in time? Select an appropriate value for this parameter.

(iv) The controller will not work if the pendulum starts exactly from the down position at rest ($\theta = \pi, \omega = 0$). How can you fix this?

2. (3 points) For the hybrid system $\mathcal{H}$ with state $x \in \mathbb{R}^2$ and data

$$\begin{array}{rclcrcl} C & := & \left\{x \in \mathbb{R}^2 \mid \|x\| < 1\right\}, & & f(x) & := & \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix} \\ D & := & \left\{x \in \mathbb{R}^2 \mid x_1 = 0, x_2 \in (-\frac{1}{2}, 0]\right\}, & & g(x) & := & \frac{1}{2}x \end{array}$$

Do the following:

(i) classify all its solutions (i.e., which ones are nontrivial, which ones complete, Zeno,... all the way to maximal)

(ii) indicate the points in $\overline{C} \cup D$ from which nontrivial solutions exist

(iii) indicate the properties of maximal solutions of $\mathcal{H}$