Syllabus for MAE286
Hybrid Systems - Fall 2021

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This course covers the modeling, analysis, and design of hybrid dynamical systems. Topics include: Basic notion of hybrid system. Examples from mechanics, vision, and multi-agent systems. Modeling approaches to hybrid systems. Switching systems. Solutions of hybrid systems. Chattering, Zeno phenomena. Graphical convergence. Stability analysis. Robustness. Lyapunov functions. Arbitrary switching: common Lyapunov functions. Slow switching: dwell time, average dwell-time. State-dependent switching: multiple Lyapunov functions. Invariance Principle. Hybrid control design. Applications.

Instructor

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Teaching Assistant

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Course Objectives

By the end of the course, you would/should have:

1. learned and used various modeling and analysis techniques for hybrid systems
2. appreciated the complexities in the evolution of hybrid systems and its trajectories
3. learned and used tools for the stability analysis and stabilization of hybrid systems
4. known and played around with hybrid systems in a variety of scenarios

Prerequisites

Knowledge of ordinary differential equations and maturity with the fundamentals of nonlinear dynamical systems are assumed. Familiarity with simulation software of your choice (e.g., Matlab/Python/Mathematica/Maple).

Text

We will use a variety of sources for reference. Our main references are

(M1) R. Goebel, R. G. Sanfelice, and A. R. Teel. *Hybrid Dynamical Systems: Modeling, Stability, and Robustness*. Princeton University Press, Princeton, NJ, 2012

(M2) R. G. Sanfelice. *Hybrid Feedback Control*. Princeton University Press, Princeton, NJ, 2021

Additional recommended readings

Depending on the specific topic we are dealing with, we will complement the books above with the following material

[A1] A. J. van der Schaft and H. Schumacher. *An Introduction to Hybrid Dynamical Systems*, volume 251 of *Lecture Notes in Control and Information Sciences*. Springer, 2000

[A2] J. Lygeros, C. Tomlin, and S.S. Sastry. *Hybrid Systems: Modeling, Analysis and Control*. 2008

[A3] D. Liberzon. *Switching in Systems and Control*. Systems & Control: Foundations & Applications. Birkhäuser, 2003

Useful software

You will find useful the HyEQ Toolbox in Matlab. All the info is available at https://hybrid.soe.ucsc.edu/software. The current version is v2.04, and you can download it here. There is a brief webinar that covers the basics about how to use the toolbox.

Course webpage

http://carmenere.ucsd.edu/jorge/teaching/mae286/f21

The webpage contains this syllabus and the list of homework due. Please check it periodically for updates and other announcements related to the course.

Calendar

Introduction and examples (M1, Ch 1. See also M2, Ch 1; A1, Ch 2; A2, Ch 1)

Modeling

- Modeling approaches to hybrid systems (M1, Ch 1. See also A1, Ch 1; A2, Ch 3)
- Trajectories: time domains, notion of solution, degeneracies (M1, Ch 2. See also A2, Ch 4)
- Systems with state perturbations, hybrid regularizations (M1, Ch 4)
- Well-posed hybrid systems, graphical convergence (M1, Ch 5)

Analysis

- Stability and asymptotic stability (M1, Ch 3&7. See also A1, Ch 5; A2, Ch 5)
- Invariance principle (M1, Ch 8. See also A2, Ch 5)
- Stability under arbitrary switching (A3, Ch 2)
- Stability under constrained switching: slow switching, state-dependent switching (A3, Ch 3)

Design

- Uniting control (M2, Ch 4)
- Event-triggered control (M2, Ch 5)
- Feedback design via control Lyapunov functions (M2, Ch 10)
- Invariance-based control (M2, Ch 11)
- Temporal logic (M2, Ch 12)

Exams

The midterm will be on Thursday, October 28, 2021

The final will be on Wednesday, December 8, 2021

Homework

There will be a set of homework problems per week. Homework assignments are due weekly, on Fridays at midnight (specific dates for your reference are included in the webpage). *You need to complete all exercises, although only* **two***, randomly selected, will be corrected from each assignment.* No late homework will be accepted. Collaboration is allowed, but your homework should reflect your own original work and be the result of your understanding of the material.

We use an all electronic homework submission and grading process through Canvas. Homework and instructions will be posted there. You can handwrite legibly or type, then scan your homework as a PDF file for submission. Please check the quality of your PDF file before submission. If we cannot read it, we cannot grade it! Please turn in a readable and organized homework. Here is a suggestion: include your name and your ID # on top of each and every page, answer questions in logical order, and start answering a question always on the top of the page.

To efficiently address questions related to homework, we use Piazza at https://piazza.com/ucsd/fall2021/mae286. Anybody can take a stab at posting answers to questions and we will monitor it regularly, but do not expect immediate turnarounds!

Grading policy

Homework: 30%
Midterm: 30%
Final exam: 40%

Canvas

Your grades will be available via Canvas. Check out https://canvas.ucsd.edu/courses/30166 for instructions on how to register and log in. Please **check it regularly** to make sure your homework scores are being transcribed correctly.

Academic honesty

No form of academic dishonesty will be tolerated. We take this very seriously. For the definition of academic dishonesty and its (ominous) consequences, refer to the UCSD General Catalog 2021-2022 at http://www.ucsd.edu/catalog/

Prior to starting the course, you should visit https://academicintegrity.ucsd.edu/forms/form-pledge.html and take the UCSD Academic Integrity Pledge.

Virtual lectures and hours

Lectures will be virtual throughout the quarter. They will be held on Zoom (specific access information is provided on Canvas in the announcement "Zoom Link for Lectures"). Lectures take place live on Tuesdays and Thursdays, from 11:00am to 12:20pm, and will be recorded. Lecture recordings will be available on Canvas.

Virtual office hours

Specific access information is provided on Canvas in the announcement "Virtual Office Hours"

Jorge: Mondays, from 2:30pm to 3:30pm, on Zoom.

Shenyu: Thursdays, from 2:30pm to 3:30pm, on Zoom.