MAE 286: Hybrid Systems (W14)
Homework #1

Due on 1/14/14

1. (2 points) Consider the discontinuous differential equation

$$\dot{x}_1 = -\operatorname{sign}(x_1) + 2\operatorname{sign}(x_2)$$
$$\dot{x}_2 = -2\operatorname{sign}(x_1) - \operatorname{sign}(x_2)$$

where $x_1(0) \neq 0$ and $x_2(0) \neq 0$ and

$$\operatorname{sign}(z) = \begin{cases} 1 & z > 0 \\ -1 & z < 0 \\ \text{undefined} & z = 0 \end{cases}$$

This system defines a hybrid automaton with four discrete modes having invariants corresponding to the four quadrants. Do the following

- Specify a non-blocking and deterministic hybrid automaton H modeling the system.
- Does H accept Zeno executions for every initial state?

2. (2 points) Consider the balls depicted in Figure 1. Assume they all have unit mass, and that they are

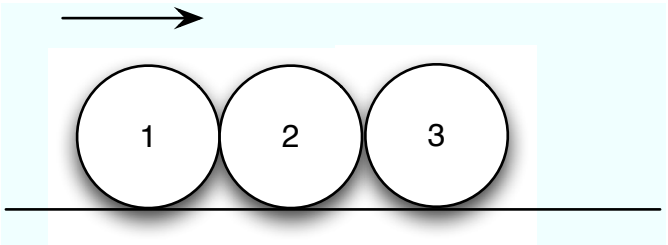

Figure 1: Three balls colliding.

touching at time $t = 0$. The initial velocity of ball 1 is $v_1(0) = 1$, while balls 2 and 3 are at rest. Assume the impact is a sequence of simple inelastic impacts. The first inelastic collision occurs between balls 1 and 2, resulting in $v_1(0+) = v_2(0+) = 0.5$ and $v_3(0+) = 0$. Since $v_2(0+) > v_3(0+)$, ball 2 hits ball 3 instantaneously giving $v_1(0++) = 0.5$ and $v_2(0++) = v_3(0++) = 0.25$. Now $v_1(0++) > v_2(0++)$, so ball 1 hits ball 2 again resulting in a new inelastic collision. This leads to an infinite sequence of collisions.

(i) Model the inelastic collisions of the three ball system as a hybrid automaton H with a single discrete mode and three continuous variables (x_1, x_2, x_3) representing the velocities of the balls.

(ii) Show that H accepts a Zeno execution corresponding to the sequence of collisions described above.