

MAE 286: Hybrid Systems (W14)

Homework #2

Due on 1/21/14

1. (3 points) Consider the inverted pendulum swing-up controller shown in Figure 1. The angular

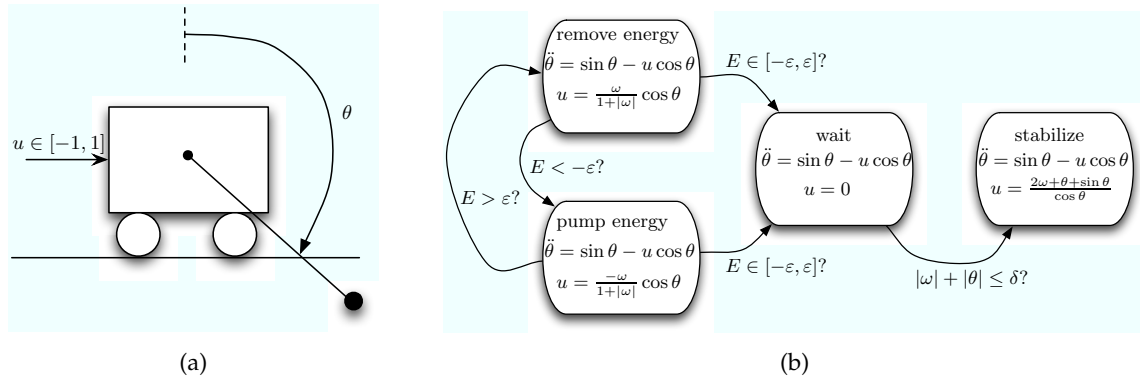


Figure 1: (a) Swing-up pendulum and (b) hybrid controller.

position of the pendulum is θ , and its angular velocity is $\omega = \dot{\theta}$. The motion of the pendulum is described by the equation

$$\ddot{\theta} = \sin \theta - u \cos \theta$$

The energy function is a combination of kinetic plus potential energy $E = \frac{1}{2}\omega^2 + (\cos \theta - 1)$.

Do the following:

- (i) Provide a formal description of the closed-loop system by specifying its discrete and continuous state-space as well as the activities and jump maps.
 - (ii) What is required of δ so that $u \in [-1, 1]$ in the discrete mode "stabilize"? Select an appropriate value for this parameter.
 - (iii) What is required of ε so that the guard to transition from mode "wait" to mode "stabilize" is enabled at some point in time? Select an appropriate value for this parameter.
 - (iv) The controller will not work if the pendulum starts exactly from the down position at rest ($\theta = \pi, \omega = 0$). How can you fix this?
2. (3 points) For the hybrid system \mathcal{H} with state $x \in \mathbb{R}^2$ and data

$$C := \{x \in \mathbb{R}^2 \mid \|x\| < 1\}, \quad f(x) := \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix}$$

$$D := \{x \in \mathbb{R}^2 \mid x_1 = 0, x_2 \in (-\frac{1}{2}, 0]\}, \quad g(x) := \frac{1}{2}x$$

Do the following:

- (i) classify all its solutions (i.e., which ones are nontrivial, which ones complete, Zeno,... all the way to maximal)
- (ii) indicate the points in $\overline{C} \cup D$ from which nontrivial solutions exist
- (iii) indicate the properties of the maximal solutions of \mathcal{H}