

MAE 286: Hybrid Systems (W14)

Homework #3

Due on 1/28/14

1. (2 points) For the hybrid system \mathcal{H} with state $x \in \mathbb{R}^2$ and data

$$C := \{x \in \mathbb{R}^2 \mid \|x\| < 1\}, \quad f(x) := \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix}$$

$$D := \{x \in \mathbb{R}^2 \mid x_1 = 0, x_2 \in (-\frac{1}{2}, 0]\}, \quad g(x) := \frac{1}{2}x$$

Answer the following questions:

- (i) Does the hybrid system satisfy the basic conditions for existence of solutions?
 - (ii) Are there solutions to the hybrid system with state perturbation which are not a solution to the unperturbed hybrid system? Which ones?
 - (iii) Construct the Krasovskii regularization of the hybrid system and describe all its solutions
2. (3 points) (Switched server). Consider the switched server described in Figure 1. Buffer $i \in \{0, 1\}$

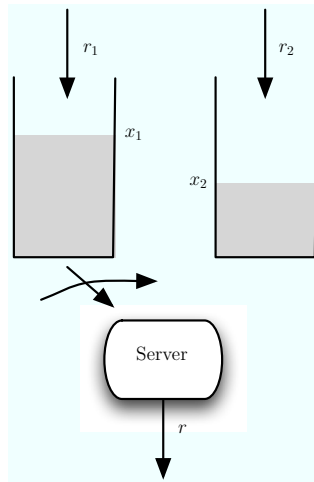


Figure 1: Switched server example.

has queue size x_i and is filled with incoming requests at a rate r_i . The server services requests at a rate r according to the following scheduling algorithm (called *cyclic scheduling*):

- it starts in buffer 0 and works on the queue until it is empty
- when the buffer i is empty, it moves to buffer $(i + 1) \bmod 2$.
- after a switch, the server needs to spend a setup time δ before it can start working on a buffer.

Assume that $\delta > 0$ and $r > r_1, r_2$. Answer the following questions:

- (i) Describe formally the server operation as a hybrid system
- (ii) For what values of r_1, r_2, r , and δ do the queues remain bounded?
- (iii) When is the boundedness property in (ii) robust against state perturbations?