

MAE 286: Hybrid Systems (W14)

Homework #4

Due on 2/4/14

- (1 point) Consider a map $z : [0, \infty) \rightarrow \mathbb{R}^d$ (e.g., the complete solution of a differential equation). For $i \in \mathbb{N}$, define $S_i = \{z(t) \mid t \geq i\}$. Calculate $\limsup_{i \rightarrow \infty} S_i$ and explain its relationship with the ω -limit set of z .
- (1 point) Let dist^H denote the Hausdorff distance between two sets. Given S and a sequence $\{S_i\}_{i=1}^{\infty}$, consider the following notion of limit: $\lim_{i \rightarrow \infty}^H S_i = S$ if and only for all $\varepsilon > 0$, there exists i_0 such that if $i > i_0$, then $\text{dist}^H(S_i, S) < \varepsilon$. Give an example of a sequence $\{S_i\}_{i=1}^{\infty}$ of bounded sets such that $\lim_{i \rightarrow \infty} S_i = S$ but $\lim_{i \rightarrow \infty}^H S_i \neq S$.
- (1 point) Give an example of a set-valued map $M : \mathbb{R}^d \rightrightarrows \mathbb{R}^e$ which is upper semicontinuous at a point $x \in \mathbb{R}^d$ but not outer semicontinuous at $x \in \mathbb{R}^d$.
- (1 point) Give an example of a sequence $\{\phi_i\}_{i=1}^{\infty}$ of hybrid arcs that converges graphically to ϕ such that ϕ itself is not a hybrid arc.
- (2 points) Consider the hybrid system \mathcal{H} with state $x \in \mathbb{R}^2$ and data

$$\begin{aligned} C &:= \{x \in \mathbb{R}^2 \mid \|x\| < 1\}, & f(x) &:= \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix} \\ D &:= \{x \in \mathbb{R}^2 \mid x_1 = 0, x_2 \in (-\frac{1}{2}, 0]\}, & g(x) &:= \frac{1}{2}x \end{aligned}$$

Do the following:

- Give an example of a sequence of solutions of the hybrid system that graphically converges and whose limit is not a solution of the hybrid system.
- Is the limit of your sequence in (a) a Krasovskii solution of the hybrid system?