MAE 286: Hybrid Systems (W14)
Homework #5

Due on 2/18/14

1. Consider the sequence of hybrid arcs $\{\phi_i\}_{i=1}^{\infty}$ where $\phi_i : [0, \infty) \times \{0\} \to \mathbb{R}$, $\phi_i(t, j) = t^i$.
   - Is the sequence locally eventually bounded?
   - Is the graphical limit a hybrid arc?

2. Consider the set-valued map $F : \mathbb{R} \rightrightarrows \mathbb{R}$ given by

$$F(x) = \begin{cases} -1 & x > 0, \\ [-1, 1] & x = 0, \\ 1 & x < 0. \end{cases}$$

   Is F outer semicontinuous? Is it locally bounded? Does it take nonempty and convex values? Do solutions exist to the differential inclusion

$$\dot{x} \in F(x) \tag{1}$$

   starting from any initial condition? Which ones are they? Are they unique? What is the connection of (1) with $\dot{x} = -\operatorname{sgn}(x)$?

3. Consider the hybrid system with data $C = \mathbb{R}$, $D = \emptyset$,

$$F(x) = \begin{cases} 1 & x < 0 \\ 2 & x \geq 0 \end{cases}$$

   and any G you want. Does this system satisfy the basic assumptions? Is it nominally well-posed?

4. Consider the hybrid system $\mathcal{H}$ with state $x \in \mathbb{R}^2$ and data

$$\begin{aligned} C &:= \left\{ x \in \mathbb{R}^2 \mid \|x\| < 1 \right\}, & f(x) &:= \begin{pmatrix} -x_2 \\ x_1 \end{pmatrix} \\ D &:= \left\{ x \in \mathbb{R}^2 \mid x_1 = 0, x_2 \in (-\tfrac{1}{2}, 0] \right\}, & g(x) &:= \tfrac{1}{2} x \end{aligned}$$

   Do the following:

   (a) Compute the reachable set from $S = \{0\} \times [0, \frac{1}{2})$

   (b) What is the ω-limit set of S?

   (c) Is the set $\{0\} \times (-\frac{1}{2}, 0]$ weakly forward invariant (from each initial condition in the set, at least one complete solution remains in the set)? Is it strong forward pre-invariant (from each initial condition in the set, all maximal solutions remain in the set)?