MAE 289A: Mathematical Analysis for Applications (F15)

Homework #1

Due on 10/1/15

1. Given that p is T (true), q is F (false), and s is T (true), evaluate whether the following are T or F: $(\neg q \leftrightarrow p)$, $(\neg(p \to p) \vee s)$, $\neg(\neg q \vee \neg p)$, $(s \leftrightarrow \neg p)$, $\neg(\neg s \leftrightarrow (\neg s \leftrightarrow \neg q))$.

2. Evaluate whether the following are T or F:
   - $p \wedge q$ implies $p \vee q$
   - $p \vee q$ implies p
   - $(p \wedge \neg q) \to \neg p$ is equivalent to $p \to q$
   - $p \vee q$ is equivalent to $\neg p \to q$
   - $p \wedge q$ is equivalent to $\neg p \to q$

3. Check whether the following are true or false:
   (i) The statements $\neg(\forall x(p(x) \to q(x)))$ and $\forall x(q(x) \to p(x))$ are equivalent,
   (ii) The statements $(\forall x\, p(x)) \wedge (\forall y\, q(y))$ and $\forall z(p(z) \wedge q(z))$ are equivalent,
   (iii) The statements $(\forall x\, p(x)) \vee (\forall y\, q(y))$ and $\forall z(p(z) \vee q(z))$ are equivalent,
   (iv) The statements $\exists x(p(x)) \wedge \exists y(q(y))$ and $\exists z(p(z) \wedge q(z))$ are equivalent,
   (v) The statements $\neg\exists x(p(x) \vee \neg(q(x)))$ and $\exists x(\neg(p(x) \wedge q(x)))$ are equivalent.

4. Consider the following definition: If a and b are two natural numbers, we say that a divides b if there is another natural number k such that $b = ak$. Using this definition, prove directly that if a divides b and a divides c, then a divides $b + c$.

5. Prove by contradiction that if a is a rational number and b is an irrational number, then $a + b$ is an irrational number. To prove the previous statement, use the definition that a is a rational number if there are integer numbers m, n, with $n \neq 0$, such that $a = \frac{m}{n}$.

6. Typically the sums of the form $\sum_{k=1}^{n} k^m$, where m is a fixed positive integer, are proven by induction. However, to do this, you need to come up with a tentative formula in the first place. A trick to derive a formula is given by matching coefficients. We apply this for the particular case $\sum_{k=1}^{n} k^2$.
   (i) Assume that there are constants A, B, C, D such that
   $$\sum_{k=1}^{n} k^2 = An^3 + Bn^2 + Cn + D.$$
   Find what the constants should be so that when you consider $\sum_{k=1}^{n+1} k^2 = \sum_{k=1}^{n} k^2 + (n+1)^2 = A(n+1)^3 + B(n+1)^2 + C(n+1) + D$ the coefficients of powers of n are matched.
   (ii) After finding your answer to (i), prove that the result holds by induction.

7. Prove that if there are 6 people at a party, then either 3 of them are friends among themselves, or 3 of them were complete strangers before the party.

   Hint: Select one person and apply the pigeon hole argument with people who know or do not know her. Then, reason with the relationships among people in the largest group.