MAE 289A: Mathematical Analysis for Applications (F15)

Homework #3

Due on 10/15/15

1. Define the set S as follows: x is an element of S if x is an infinite sequence of the form

$$r_1, r_2, \ldots, r_n, 0, \ldots, 0, \ldots$$

with $r_i \in \mathbb{Q}$, $i = 1, \ldots, n$. In other words, from some n on, the sequence consists entirely of zeros, and the nonzero entries are rational numbers. Show that S is countable.

2. Construct a set of real numbers with exactly three limit points.

3. Let E^o denote the set of all interior points of the set E. Do the following:

   (i) Prove that E^o is always open.

   (ii) Prove that E is open if and only if $E = E^o$.

   (iii) Prove that if $G \subset E$ and G is open, then $G \subset E^o$.

   (iv) Prove that the complement of E^o is the closure of the complement of E.

   (v) Do E and $\bar{E}$ have the same interiors always?

   (vi) Do E and E^o have the same closures always?

4. A metric space is called *separable* if it contains a countable dense subset. Show that $\mathbb{R}^k$ is separable.

5. Let $X = \mathbb{R}^2$. Let $x = (x_1, x_2)$ and $y = (y_1, y_2) \in X$ be generic elements of X. Draw the ball $B((1, 1), 1)$ centered at $(1, 1)$ of radius 1 for the following metrics:

   (i) The Euclidean metric d_2 given by $d_2(x, y) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$

   (ii) The metric d_1 given by $d_1(x, y) = |x_1 - y_1| + |x_2 - y_2|$

   (iii) The metric d_∞ given by $d_\infty(x, y) = \max\{|x_1 - y_1|, |x_2 - y_2|\}$

   (iv) The metric d_4 given by $d_4(x, y) = \sqrt[4]{(x_1 - y_1)^4 + (x_2 - y_2)^4}$ (a rough sketch is fine)