MAE 289A: Mathematical Analysis for Applications (F15)

Homework #4

Due on 10/22/15

1. Let $K \subset \mathbb{R}$ consist of 0 and the numbers $1/n$ with $n = 1, 2, 3, \ldots$. Prove that K is compact directly from the definition (without using the Heine-Borel theorem)

2. Show that the union of a compact set and a finite set is compact.

3. In $\mathbb{R}^3$ with the Euclidean metric consider all points on the surface $x_1^2 + x_2^2 - x_3^2 = 1$. Is this set compact?

4. Consider $\mathbb{R}^2$ with the Euclidean metric. Let S be a compact subset of $\mathbb{R}^2$. Consider the projection of S given by

   $$X(S) = \{x_1 \in \mathbb{R} \mid \exists x_2 \text{ such that } (x_1, x_2) \in S\}.$$

   Show that $X(S)$ is a compact subset of $\mathbb{R}$ (with the Euclidean metric).